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**SOCIETY
of
ACTUARIES**

Transactions



*The work of science is to substitute facts for appearances
and demonstrations for impressions.—RUSKIN*

PART I

1966

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NOTICE

The Society is not responsible for statements made or opinions expressed in the articles, criticisms, and discussions published in these *Transactions*.

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SOCIETY OF ACTUARIES

(ARTHUR A. MCKINNIE, *Executive Secretary*)

208 SOUTH LA SALLE STREET, CHICAGO, ILLINOIS 60604

OCTOBER, 1966

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TRANSACTIONS

APRIL, 1966

BOOK REVIEWS AND NOTICES*

*R. D. Eilers and R. M. Crowe, *Group Insurance Handbook*, pp. xxx, 972, Richard D. Irwin, Inc., Homewood, Illinois, 1965.

This extensive undertaking represents the collective work of over 75 authors and editors, all of whom hold important posts in the group insurance field and are well qualified to have made their respective contributions. Many are members of our Society. The book is likely to be most frequently consulted by students in various fields for educational and research purposes.

The major portion of the material is devoted to basic information, such as general underwriting principles and types of group coverages. Much of the material will have current validity for some time to come. The presentations are, in general, smoothly written, but it would be desirable, in future editions, to make a further effort to eliminate the many areas of repetition.

After a brief introduction of employee benefit plans in Part I, involving definitions, general descriptions, and the rationale of such plans, the book goes into the general nature, development, and significance of a great variety of group coverages. In Part II, the characteristics of the group technique are covered in detail, that is, the absence of individual underwriting, single master policy, experience rating, and so forth. The general principles of group underwriting are simply stated in chapter iii and should be especially useful to interested laymen. Chapters iv and v contain a concise presentation of the historical development of group life and health insurance. Chapter vi is devoted to a discussion of the legal environment of group insurance, which is especially informative with respect to federal disclosure laws but is somewhat repetitive in other respects in the light of material covered in earlier chapters, a fault that occurs elsewhere in this volume.

The next major portion of the *Handbook*, Part III, is devoted entirely to group life insurance, with chapters on group term insurance, group permanent insurance, accidental death and dismemberment insurance, dependent group life coverage, and federal employees group life insurance. A separate chapter is given to the underwriting of group life insurance and provides a very interesting presentation of this important aspect of group insurance. The last two chapters of this part provide an excellent presentation of the framework of group

* Books and other publications noted with an asterisk (*) may be borrowed from the library of the Society of Actuaries under the rules stated in the *Year Book*.

life insurance premium rates and the subject of experience rating. On the whole, the level of description in this part of the book is comparable to the Society's study notes on the subject. The Society's Education and Examination Committee should take careful note of this portion of the volume as a source of material.

Part IV presents a broad and extended treatment of the many varied group health insurance coverages. The first three chapters are devoted to the various types of health risk bearers, such as insurance companies, Blue Cross and Blue Shield, and group practice prepayment plans. The chapters covering the Blues and group practice types of "risk bearers" are well worth reading, even by those who already are acquainted with the subject.

There is a relative lack of good up-to-date information on Blue Cross and Blue Shield, especially to the extent that the practices of these organizations now parallel modern insurance company operations; the *Handbook* is particularly useful in meeting this gap. In a chapter devoted to a detailed description of current Blue Cross and Blue Shield operations, the author makes little mention of the way in which the Blue Cross reimbursement formula actually works and what items of hospital expense are usually omitted. This has been a controversial matter, with strong feelings on all sides, and is perhaps deserving of more detailed treatment than was accorded. Future editions may, however, have more detail on this point, as the existence of Medicare is likely to have a profound effect on hospital accounting and billing in general.

A chapter devoted to group practice prepayment plans gives an excellent summary of a type of risk bearer that the general public knows relatively little about. No other single source yields so much information on the principles of such plans, their historical development, and their general mode of operation. There is a rather moot point raised on page 288—that costs under these plans are lower than those under "fee-for-service health insurance programs"—but the chapter includes the pros and cons of group practice prepayment plans in a relatively frank manner.

The volume then proceeds to descriptions of the various health coverages, leading off with a lengthy presentation of group hospital expense insurance benefits. A similar treatment is given to surgical insurance, but there is an unfortunate overlap in this chapter with much of the material in the chapter on Blue Cross and Blue Shield. In a volume of this size, gathering together the efforts of so many contributors, a certain amount of repetition is perhaps inevitable, but it would seem that a description of surgical and medical service benefits which duplicates that which was given with respect to the Blues might well have been avoided. There follows an account of coverages by extended and comprehensive major medical plans. While there is little to be gained by the expert in these chapters, the educational value to students is apparent.

The next topic centers on the underwriting of group medical expense coverage, with a good summary of the problems faced in this field. Group disability income benefits are covered in three chapters which are concerned with a description, underwriting problems, premium rates, and experience rating. While

the subject matter in these three chapters is not new, it is useful to have so much information on the subject in one reference.

Part IV ends with two interesting chapters. The first is entitled "Cost Control through Cooperation with Purveyors of Medical Care," which is a current and future topic of importance, as the insurance industry is closely concerned with relationships among hospitals, doctors, and other vendors of medical care. A discussion of rehabilitation techniques is included which is, perhaps, not sufficiently appreciated by insurers, especially in connection with long-term disability insurance. The second chapter describes cost controls possible through eligibility provisions, deductibles, coinsurance, and similar items.

The next major portion, Part V, is composed of a series of chapters devoted to various routine aspects of group insurance operations, such as master contracts and certificate preparation, state laws governing such items, and group insurance marketing problems. The last of these is well written and provides good insight into the agency and group representative arrangements characteristic of many companies. The importance of the services rendered by group representatives in the dissemination of information to policyholders and in explaining the reasons for rate increases when they are necessary is well known to persons familiar with group insurance. A separate chapter sets forth the knowledge of an expert in the problems that arise in the claim administration of group health coverages of all types.

Part VI is devoted to an examination of some of the variations that have evolved over the years in the technique of providing insurance through the group approach. The various problems discussed involve insurance for trade associations, Taft-Hartley welfare funds, and the wholesale and franchise area of operations. There is an interesting chapter covering the extensions of creditor group insurance which contains very good descriptions of the experiments with this form of coverage to date. There is also a frank description of some of the important problems presently facing creditor group insurance, together with an indication of some of the ways in which certain abuses may be controlled. The two chapters devoted to medical insurance for retired individuals and descriptions of the various state 65 plans have been made mainly of historical interest by the enactment of Medicare.

The final part covers the subject of self-insurance and was included presumably for the sake of completeness. The motivations underlying self-insurance together with the methods involved in such operations are presented. The volume concludes with a very lengthy set of appendixes, including excerpts from model bills, specimen policies and certificates, and many other documents important to the group insurance business. It is useful, of course, to have all these items in one source, and their presence adds to the general utility of this volume.

As a handbook, the volume should have a wide appeal as a convenient reference. In the rapidly developing field of group insurance, however, the specialist faced with a particular problem will always want to make sure that he is aware of the most recent practice.

CHARLES A. SIEGFRIED
J. MARTIN DICKLER

*H. F. Fisher and J. Young, *Actuarial Practice of Life Assurance*, pp. viii, 450, published for the Institute of Actuaries and the Faculty of Actuaries, Cambridge at the University Press, 1965.

This is one of the series of textbooks published jointly by the Institute of Actuaries and the Faculty of Actuaries to serve as part of the official course of reading for the examinations of those bodies. It is intended to replace that by Coe and Ogborn, which was published in 1952. All four authors are Fellows of the Institute or the Faculty. The object of each text is to review the practice of life insurance and annuities in Britain from the point of view of the actuary at the dates of their publication.

When, some thirty years ago, the present reviewer was given the task of drawing up a specific course of reading for the Actuarial Society of America which, for the first time, was to be the basis of its examinations, he noted that, as in the case at present, no definite part of the examinations was intended to cover the practice of life insurance and annuities as a whole. He therefore recommended to the student that, as a preliminary to his reading for the Fellowship examinations, he read J. B. Maclean's *Life Insurance*. This is still done in the present course of reading, with special references to the chapters on dividends and governmental regulation.

This reviewer understands that it is possible that the present edition of Maclean's (the ninth) may be the last by Mr. Maclean himself, and, if so, the Society may be faced with the task in the future of undertaking a work to replace Maclean or bring it up to date in a world where change is the one factor of which we are certain. This review of Fisher and Young's text is intended to serve a double purpose: first, to comment on such of its contents as may be of particular interest to actuaries on this side of the Atlantic and, second, to give points of view regarding a later Maclean. There are a number of general works on life insurance available to the American student, and some of considerable merit in parts, but the actuarial student should be served by an actuary, as is the practice in Britain.

The present text by Fisher and Young of twenty-one chapters gives four chapters to industrial life assurance, with additional material on its history, and two chapters on group pensions and annuities. Society of Actuaries members may be surprised at this weight to industrial life insurance, for, due to greatly improved living standards in the United States and Canada, the trend is to move the business to the ordinary branch. This is not the case in Britain, where short-term endowment insurances predominate and the relatively small amounts of sums insured and home collection keep the increasing volume of business in the industrial class.

The reviewer would criticize the discussion of the origins of industrial life assurance given on pages 13 ff. The reference to "box clubs," which existed where subscriptions were placed in a box out of which benefits were paid, and the "kindly interest of the parson and squire" gives a distorted view of the origins of the thousands of friendly societies (fraternals) which existed in industrial Britain at the end of the eighteenth century, with over a thousand each in

metropolitan London and Manchester. The authors should refer to the 200-page article on the subject in Walford's *Insurance Encyclopaedia*.

In their preface the authors justify the elimination of much of the historical background given in Coe and Ogborn's text; to the reviewer this seems unfortunate. To the student, surfeited with the actuarial formulas of life contingencies, references to actual events and personages would be a relief. Life insurance and annuities have practical applications, and this text is intended to be the student's introduction to this phase. Another change noted is the omission of Coe and Ogborn's references to developments in the United States. This double curtailment makes for confusion in the references to "assessmentism." In dealing with the first part of the eighteenth century, the authors make reference to obtaining "the maximum amount of insurance cover for a given outlay" when short-term contracts were the only insurance available. There is a reference to "additional (or 'free') reserves retained from past realised profits," renewal option at an increased premium, followed by the statement that "in 1755 a common premium rate for such insurances was £5 per £100 insured for ages 20 to 50, and £6 per £100 insured for ages 50 to 60." This must be confusing to the student. In any projected Society text, it is to be hoped that the various phases of development of any subject will be referred to chronologically. There is no reference to the important part played by assessmentism in the United States, where the word originated.

Although the book is intended to be a first general text on life insurance practice for students, some of whom may never be in the service of a life insurance company, the line of approach is rather that of a management survey. This is indicated by the chapter headings: "Development and Control," "Financial Background of Life Offices," "Financial Aspects of Office Premiums." Some of these surveys are quite masterly, and some of the senior actuaries of the Society may be interested in them, although they are applied to business in a foreign country. It may even be hoped that these words may catch the eye of a Society member who may be tempted to do a paper on the differences in outlook on the practice of life insurance in Britain and North America.

But to the student who has just completed his study of life contingency formulas, one must wonder what impression the book will give. In any American text it is to be hoped that surveys will follow the details on which judgment must be based, not vice versa. Further, the absence of figures is noticeable. One might quote as an actuarial proverb, "One premium rate is worth a hundred words and one ratio, a thousand."

It is surprising that in this text the figures of ordinary life insurance in force illustrating the relative weights of participating and nonparticipating business relate to the years 1910 and 1937 only. The date of the book is 1965. An examination of recent figures would show that the trend to nonpar business has not been reversed, as the authors state.

In Section 6.7 of the text, there is a discussion of the familiar formula

$$({}_tV + P)(1 + i) = q(S - {}_{t+1}V) + {}_{t+1}V$$

and a demonstration in Table 6.4 which illustrates most clearly the fundamentals of the theory of life insurance. The reviewer, in his recently published text addressed to nonactuaries, approached the subject differently, but Table 6.4 is most interesting and valuable and should be noted by writers of actuarial textbooks.

In any actuarial text where developments associated with prominent names of the past are mentioned, the reviewer suggests that references be given. Even a student may be tempted and should be encouraged to refer to the original papers, particularly when, in Britain, they are associated with the names of C. R. V. Coutts, Sir Gerald Ryan, and Sir William Elderton, to give a few of the names mentioned, who graced every subject on which they wrote. Coe and Ogborn gave such references at the end of each chapter, with brief but valuable comments. This has been dropped in the present text, and the page or so of references at the end of the book dealing mainly with recent papers does not serve the purpose.

Coe and Ogborn's references to the ills that "flesh is heir to," forming the background of the science of selection of risks, are entirely omitted. The major contributions of United States life insurance companies to the statistics of build, blood pressure, and the various major medical impairments are entirely ignored. Further, no reference whatsoever is made to the valuable series of annual reports on mortality of insured lives published by the Society of Actuaries. It is thus not surprising that, in commenting on the practice of some British offices in rating down female lives for life insurance, the authors state, "No assured lives experience has been extracted in the case of female lives in recent years."

British offices use *liens* extensively in rating substandard risks, which is rarely done in the United States; hence their concern with the *incidence* of extra mortality which is repeatedly stressed in the text with adverse criticism of the American use of the system of extra percentage mortality ratings. It is thus illuminating to read the following: "Bearing in mind, however, the lack of precise statistics, . . . the assumption of a uniform percentage increase may often be adopted as a practical measure."

Although it is acknowledged that no statistical basis exists for the belief, there are repeated statements that the mortality of withdrawing policyholders may be lighter than those continuing. This is presumably to justify the refusal to grant generous surrender values or to guarantee them. There are references in the text to the sale of life insurance policies in the open market, presumably where the surrender value offered by the issuing company is unsatisfactory. This is anathema to any actuary in North America. Yet the concluding sentence of a whole chapter entitled "Surrender and Paid-up Policy Values" is, "The [surrender] value should compare reasonably with the premiums paid less the cost of term insurance and initial expenses." This is all we expect on this side of the Atlantic!

Society members interested in investments and aware of the success of British actuaries in that field may be examining this text. They will be sur-

prised at the emphasis given to "matching" assets with liabilities and "the general theory of immunization," that is, "that the investment of the funds as a whole may be arranged with regard to the dates of receipt of both capital and income so that the existing business is protected against changes in the general level of the rate of interest." It is acknowledged that "The concept is theoretical and there are many practical difficulties. . . ." However, conditions on this side of the Atlantic are so different that it is doubtful if the subject would be of corresponding interest to actuaries over here. No figures are given indicating the distribution of assets or the rates of interest earned, the graph of the yield of nonredeemable British government bonds (consols) being no substitute for the latter.

The two chapters on "Analysis of Surplus" and "Methods of Distribution" are of considerable interest, the former covering one of the most difficult and frustrating tasks of the actuary. The following comments on the contribution method of distributing surplus should be noted:

It is extensively followed in the U.S.A. and Canada, where, it should be remembered, premium scales do not include profit margins of the size considered appropriate in the U.K.

The claim of the method to equity rests on the way in which it keeps clearly in view the premium actually payable, the class of the policy, the age attained and duration and the changing experience of the three main sources of surplus.

There is one credit to new developments in the United States in connection with *variable annuities*, as J. B. Maclean's paper in *TFA*, Volume 23, dealt with the TIAA and CREF schemes, but it is assumed that nothing further has developed on this side since the paper was presented to the Faculty of Actuaries in 1956. Some inquiry should have been made by the authors to insure that the remarks that they made are not out of date at the time of writing, some nine years later.

A debatable point is whether a textbook presuming to cover the whole range of life insurance practice for the actuarial student who has completed his examinations on interest and life contingency formulas could not better be covered by a number of authors serving under a general editor, each a specialist in the field that he covers. Such a work was the *Insurance Guide and Handbook* (C. & E. Layton, London), of which six editions were issued, the last, in 1929, called *The Life Assurance Text-Book*; many of the outstanding actuaries of that time co-operated.

Individual textbooks covering the various parts of actuarial work do not take the place of an over-all text. Even a work limited from 400 to 500 pages would serve as a handy reference work for actuaries to refresh their memories, for our work becomes more specialized and narrow as time passes. Every ten years or so, a revised edition could be issued to bring such details as legislation and references to new developments up to date. A good index is essential for such a work; it is noted that the index in Fisher and Young is quite restricted, more so than that of Coe and Ogborn.

A final point is to stress in such a text the essential practical nature of the actuary's work and to keep before the student the admonition of S. G. Warner, one of the outstanding actuary-managers of his day who, in his presidential address to the Institute of Actuaries in 1916, stated, ". . . as the Buddhist saint tries to get touch with the infinite by fixing his gaze for a succession of years on one small object in space, so the actuary may let the stream of affairs go by him unheeded, in rapt contemplation of a decimal point."

ARTHUR PEDOE

SELECT CURRENT BIBLIOGRAPHY

In compiling this list, the Committee on Review has digested only those papers which appear to be of direct interest to members of the Society of Actuaries; in doing so, the Committee offers no opinion on the views which the various articles express. The digested articles will be listed under the following subject matter classifications: 1—"Actuarial and Other Mathematics, Statistics, Graduation"; 2—"Life Insurance and Annuities"; 3—"Health Insurance"; 4—"Social Security"; 5—"Other Topics."

The review section of the *Journal of the Institute of Actuaries* contains digests in English of articles appearing in foreign actuarial journals.

ACTUARIAL AND OTHER MATHEMATICS, STATISTICS, GRADUATION

D. Teichroew, "A History of Distribution Sampling Prior to the Era of the Computer and Its Relevance to Simulation," *Journal of the American Statistical Association*, vol. 60 (1965), pp. 27-49.

Simulation is a technique for solving problems that is frequently employed when operations research is applied to business problems. Simulation, perhaps because it seems to be a natural extension of model office computations through the introduction of random elements, has had great appeal to some actuaries.

In this paper it is shown that simulation is essentially an extension of a technique known as distribution sampling which has been used in statistics for many years. The history of distribution, or empirical, sampling and its successes and failures in contributing to the development of statistics are discussed. Recent developments in the generation of random numbers are also outlined.

The final section of the paper is concerned directly with simulation. The author's view may be summarized as follows: "Statisticians found that distribution sampling is a technique that is better than nothing but is less desirable than analytical techniques. . . . In business and other problems where simulation is being used today, it is likely that we are at the stage where only simulation can be used to attack many of the important problems. Nevertheless, in view of the history of the technique, in simulation as in distribution sampling, it is to be hoped that more emphasis will be placed on the methodological problems associated with its use, and that the output of the simulation experiments will be used to stimulate and guide theoretical analysis."

The paper contains an extensive bibliography.

R. E. Barlow and R. W. Marshall, "Tables of Bounds for Distributions with Monotone Hazard Rate," *Journal of the American Statistical Association*, vol. 60 (1965), pp. 872-90.

The statistical training of most actuaries was concentrated on solving the problems of finding point and interval estimates for the parameters of probability distributions

from specified families of distributions and on testing hypotheses concerning distributions. These statistical hypotheses were usually statements about distributions that belong to certain well-known families of distributions.

Almost from the birth of statistics as an independent discipline, there has been interest in developing statistical methods that do not require the prior specification of the family of distributions under study. Especially in recent years the field of distribution-free or nonparametric statistical methods has developed rapidly.

There also is interest in an intermediate approach in which statistical methods are developed without assuming that the distribution comes from a certain family (i.e., normal, gamma, binomial, Poisson, etc.) but in which some intuitively reasonable restriction on the class of distributions under study is imposed. For example, this approach would seek methods for solving the traditional statistical problem of estimating a distribution function and of testing hypotheses under the general assumptions that the random variables being studied are characterized by having a monotone increasing force of mortality (failure rate, hazard rate, or intensity function). Since this assumption concerning the random variable time until death has been used by actuaries at least since the time of Gompertz, it would seem that these methods might be applied to actuarial problems.

This paper is recommended as a starting point for actuaries who are interested in these developments. It contains an extensive bibliography which may guide those who wish to continue their reading. It summarizes, but it does not prove, some theorems concerning distributions with monotone forces of mortality.

The heart of the paper is a set of tables containing bounds for $1 - F(x)$ (the survival function in actuarial science), where X is a random variable with a monotone force of mortality. Some of the tables assume that the first two moments of the distribution are known, and others require only the first to be known. Along with several other possible applications, the authors suggest that the tables could be used to determine bounds for life insurance net premiums when only incomplete information concerning the distribution of time until death is known. To an actuary, the example may seem rather artificial. Seldom would an actuary have good estimates of the first two moments of time until death without also knowing a great deal more about the distribution. Yet using the tables as a check on the adequacy of the premium certainly remains a possibility.

U.S. National Center for Health Statistics, *Computer Simulation of Hospital Discharges*, pp. 35, Public Health Service Publication No. 1,000, Series 2, No. 13, Washington, February, 1966.

"A synthetic universe of 10,000 persons was established with demographic characteristics similar to those of the U.S. civilian, non-institutional population. On the basis of earlier theoretical work and empirical record-check studies, this universe was subjected to a series of stochastic operations to simulate hospital experience, and the reporting of that experience in household interviews.

"Each individual person was moved from one state to another—e.g., from not-in-a-hospital to in-a-hospital, or from in-a-hospital to discharged-alive—by a random process with probabilities which varied by such factors as age, sex, distance from death, number of days already in the hospital, and a general health index. Thus it was possible to count the simulated hospital discharges over a 12-month period, and to tabulate them in a variety of ways."

Further simulations made possible comparisons with total experience that provided a mechanism for studying a wide range of problems found in the interview data.

LIFE INSURANCE AND ANNUITIES

U.S. National Center for Health Statistics, *Changes in Mortality Trends in England and Wales, 1931-1961*, pp. 49, Public Health Publication No. 1,000, Series 3, No. 3, Washington, November, 1965.

This study is one of a series for different countries to obtain an insight into the reason why mortality rates have leveled off in recent years.¹ The report concludes that "there is no major advance in medicine or public health that is likely to alter the main trend in mortality which has failed to advance in the last few years. The present halt in the reduction is likely to be a lengthy one. . . . The best prospect to reverse this trend would be to determine why males have failed to match the improvement in female mortality and to remedy any environmental or social conditions that affect men rather than women."

U.S. National Center for Health Statistics, *Infant and Perinatal Mortality in the United States*, pp. 87, Public Health Service Publication No. 1,000, Series 3, No. 4, Washington, October, 1965.

"The outstanding characteristic of the trend in the infant mortality rate in the United States since 1950 has been the lack of large-scale reductions in the rate for any segment of the population." Between 1951 and 1962 the rate of decline in the infant mortality rate was 1 per cent per year contrasted with 4.7 per cent per year in the 1940's. The gap between rates in metropolitan and nonmetropolitan areas decreased as the rate in many large cities increased. High-risk groups, such as children weighing 2,500 grams or less at birth and nonwhite infants, showed almost no improvement during the 1950's.

The persistence of such a slow rate of decrease in infant mortality over more than a decade led to the current examination of trends and present status of infant, neonatal, and fetal mortality. Variations by sex, race, age at death, cause of death, and geographic area were considered as well as birth weight, birth defects, birth order, gestation age, gravidity, and plurality of birth. Age and pregnancy history of the mother were also studied.

Personnel, facilities, services, and financing of care are explored, and the effects of general health services, specialized health programs, and care patterns are discussed.

U.S. National Center for Health Statistics, *Infant Mortality Trends: United States and Each State, 1930-1964*, pp. 70, Public Health Service Publication No. 1,000, Series 20, No. 1, Washington, November, 1965.

Although the infant mortality rate for 1964, 24.8 per 1,000 live births, is the lowest ever recorded for the United States, it has been declining for the past fifteen years at one-quarter of the rate of the prior fifteen years. Practically every segment of the infant population of the United States, irrespective of age, color, sex, and state, has been affected by the slowdown in the rate of decline of the infant mortality rate.

The fetal mortality rate of the United States is low, the neonatal mortality rate is high, and the postneonatal mortality rate ranks close to the highest among countries with low mortality.

¹ See TSA, XVI, 116, for a corresponding report for the United States.

"This report graphs the infant, neonatal, and postneonatal mortality rates for 1930-64 of every State and the District of Columbia, as well as the infant mortality rate by color for 29 of these areas."

A. J. Steeds, "Some Considerations Affecting the Selection of Risks," *Journal of the Institute of Actuaries*, vol. 91, Part III, No. 390, pp. 231-85.

The author discusses the purposes of underwriting, methods of underwriting, and the use of statistical investigations of impaired lives as a basis for establishing ratings. Particular emphasis is given to the contrast between British and American practices in assumptions about the incidence of extra mortality and the creation of rating systems, such as numerical rating, and rating manuals. Many of the differences in methods of procedure are deemed by the author to arise from necessary differences in the environment of the business rather than from differences in theoretical approaches.

The volume of data from British sources about mortality experience associated with particular impairments is limited, whereas North American studies are much more extensive. The author indicates the need for more British studies and presents the mortality results to date of three British substandard groups, these groups being the insured lives covered in the three "pools" that have been established in Britain to facilitate the underwriting of diabetes, high blood pressure, and coronary disease, respectively.

R. E. Hayward and B. C. Lucena, "An Investigation into the Mortality of Diabetics," *Journal of the Institute of Actuaries*, vol. 91, Part III, No. 390, pp. 286-336.

This is a discussion of the problems involved in, and a presentation of the results of, a study of diabetic mortality based on hospital records. The study was carried out by the Birmingham Actuarial Society at a large clinic and covers the investigation period 1945-59. Patients' records were followed from first diagnosis to death or moving away from the clinic area, and a total of almost 33,000 life-years of exposure were recorded. The investigation is on a continuing basis.

HEALTH INSURANCE

U.S. National Center for Health Statistics, *Disability Days: United States, July, 1963-June, 1964*, pp. 53, Public Health Service Publication No. 1,000, Series 10, No. 24, Washington, November, 1965.

This report presents data on the short-term disabling effects of illness or injury. "Included are the number of days of restricted activity and bed disability, and time lost from work or school during the year, with corresponding rates of disability per person."

Rates of disability days increased with age, with the rates for females usually exceeding those for males. Residents of metropolitan areas had lower rates of restricted-activity days, bed-disability days, and time lost from work than did residents of non-metropolitan areas, though for those aged 6-16 years the rate of time lost from school was higher in the metropolitan areas. Farm residents had a higher rate of time lost from work than did nonfarm residents. Persons residing in the West Region had the highest rate of all types of disability, while residents of the Northeast Region reported the lowest rates for most types of disability. Except for time lost from school, rates of disability days due to illness or injury declined with increase in income up to \$10,000. The unemployed had higher rates of restricted activity and bed disability than did currently employed persons.

U.S. National Center for Health Statistics, *Current Estimates from the Health Interview Survey: United States, July, 1964—June, 1965*, pp. 44, Public Health Service Publication No. 1,000, Series 10, No. 25, Washington, November, 1965.

This report presents provisional statistics "on the incidence of acute illnesses and injuries and associated disability days; the percent of the civilian, noninstitutional population with one or more chronic conditions; the number of persons injured and associated disability days associated with illness; the average cost per acquisition of prescribed medicine; and a percent distribution of the cost of nonprescribed medicine by place from which it was obtained."

The following table is abstracted from the report.

DAYS OF DISABILITY PER PERSON PER YEAR, BY SEX AND AGE
UNITED STATES, JULY, 1964—JUNE, 1965

Type of Disability	Sex	All Ages	Under 17	17-24	25-44	45-64	65 & Over
Restricted-activity days.....	Both sexes	16.4	10.8	9.8	14.1	22.4	38.5
	Male	14.7	11.2	7.1	10.8	21.0	35.6
	Female	18.0	10.4	12.2	17.0	23.7	40.8
Bed-disability days.....	Both sexes	6.2	4.6	4.1	5.2	7.5	14.2
	Male	5.3	4.6	2.7	3.9	6.4	13.2
	Female	7.0	4.6	5.3	6.3	8.5	15.1
Work-loss days*.....	Both sexes	5.7		3.7	5.0	7.1	8.2
	Male	5.7		3.4	4.4	7.8	9.8
	Female	5.6		4.2	6.2	6.0	4.5

* Work loss reported for currently employed persons aged 17 years and over.

U.S. National Center for Health Statistics, *Acute Conditions Incidence and Associated Disability: United States, July, 1964—June, 1965*, pp. 61, Public Health Service Publication No. 1,000, Series 10, No. 26, Washington, December, 1965.

Incidence rates of acute illness and injury are presented by sex and age according to condition group, region, place of residence, and season. The disability days associated with each condition are subdivided into restricted-activity days, days of bed disability, and days lost from school and work. Selected incidence rates are shown in the accompanying table.

STATUS	No. ACUTE CONDITIONS PER 100 PERSONS PER YEAR	
	Males	Females
All acute conditions	203	222
Medically attended only	61	62
Medically attended and activity-restricting ..	77	86
Activity-restricting only	65	74
Infective and parasitic diseases	27	28
Respiratory conditions	108	124
Digestive system conditions	11	12
Injuries	36	24
All other	21	34
Age:		
Under 6	387	367
6-16	254	264
17-44	154	207
45 and over	131	151

U.S. National Center for Health Statistics, *Personal Health Expenses, Per Capita Annual Expenses: United States, July—December, 1962*, pp. 53, Public Health Service Publication No. 1,000, Series 10, No. 27, Washington, February, 1966.

This study includes statistics on per capita hospital, doctor, dental, medicinal, and other expenses by sex, age, education, family income, color, geographic region, place of residence, family size, activity limitation status, insurance status, and occupational status.² The accompanying table summarizes the findings.

SEX AND AGE	HEALTH EXPENSES PER PERSON PER YEAR					
	Total	Hospital	Doctor	Dental	Medicine	Special and Other
Males:						
All ages	\$111	\$25	\$37	\$17	\$22	\$10
Under 6 years	\$ 64	\$15	\$28	\$ 2	\$16	\$ 3
6-16 years	64	11	21	15	11	6
17-24 years	78	14	24	20	12	8
25-44 years	110	23	37	21	20	9
45-64 years	180	44	58	24	35	19
65 and over	203	57	62	15	48	21
Females:						
All ages	\$144	\$34	\$49	\$20	\$29	\$12
Under 6 years	\$ 56	\$11	\$25	\$ 2	\$16	\$ 2
6-16 years	65	8	19	20	12	6
17-24 years	157	45	59	24	20	9
25-44 years	175	46	64	26	29	10
45-64 years	203	48	64	26	44	21
65 and over	208	49	61	14	58	26

U.S. National Center for Health Statistics, *Dental Visits: Time Interval since Last Visit, United States, July, 1963—June, 1964*, pp. 54, Public Health Service Publication No. 1,000, Series 10, No. 29, Washington, April, 1966.³

The proportion of persons with recent dental visits varied markedly by age, race, activity status, marital status, education of head of family, and family income. "Approximately 16.6 percent of the population, the majority of whom were young persons, had never made a dental visit," an additional 14.0 per cent had not made a visit in at least five years, and 42.0 per cent made at least one dental visit during the year prior to interview.

U.S. National Center for Health Statistics, *Periodontal Disease in Adults: United States, 1960-1962*, pp. 30, Public Health Service Publication No. 1,000, Series 11, No. 12, Washington, November, 1965.

"Periodontal disease is an ailment of major proportions in the adult population of the United States. About 3 out of 4 of the entire adult population at risk—the 90 million men and women who had at least one permanent tooth—had periodontal disease, and about 1 out of 4 had destructive periodontal disease. Two out of four had gingivitis ranging from mild inflammation involving a few teeth to severe inflammation involving all their teeth."

Twenty million adults no longer had any teeth. The disease develops more rapidly in men than in women, increases with age, decreases with rise in income and years of edu-

² See also *TSA*, XVII, 590.

³ See *TSA*, XII, 210, and *TSA*, XVII, 591.

cation, and is more prevalent and severe among Negro adults than among white adults. The underlying data were collected by physical examination of a probability sample of persons 18-79 years of age.

U.S. National Center for Health Statistics, *Characteristics of Patients in Mental Hospitals: United States, April-June, 1963*, pp. 44, Public Health Service Publication No. 1,000, Series 12, No. 3, Washington, December, 1965.⁴

The median length of stay was less than one year for nongovernmental hospitals and about seven years for state and county hospitals, ranging from about three years for patients under 45 to about ten years for patients 55-64 years of age. Mental disability was more severe than physical disability, with the patients' health varying mainly with age, although some sex and color differences were present. On the average, males were younger than females and nonwhite patients younger than white, with a median age of 54 years for the group as a whole.

U.S. National Center for Health Statistics, *Utilization of Institutions for the Aged and Chronically Ill: United States, April-June, 1963*, pp. 35, Public Health Service Publication No. 1,000, Series 12, No. 4, Washington, February, 1966.⁵

Slightly under half of the residents in the institutions sampled were on public assistance, as were 54 per cent of those in proprietary institutions, 50 per cent of those in government institutions, and 32 per cent of those in nonprofit institutions. The mean lengths of stay of current residents in these institutions were 2.3 years, 3.7 years, and 4.0 years, respectively, and 3.0 years for all institutions combined. Eighty-eight per cent of the beds were occupied, about 90 per cent for institutions with nursing care or personal care with nursing and about 83 per cent for institutions with just personal care or hospital care. Proprietary and nonprofit institutions had about 90 per cent of the beds occupied, compared with 84 per cent for government institutions.

SOCIAL SECURITY

G. I. Kowalczyk, *Arkansas Titan II Missile Site Disaster: An Actuarial Analysis of Survivor Benefits Payable*, Actuarial Note No. 24, December, 1965, Social Security Administration, Washington, pp. 4.

This note presents an actuarial analysis of the benefits awarded under OASDI to the survivors of the men killed in the Arkansas Titan II missile-site fire of August, 1965. The number and types of benefits payable and the valuation of these benefits are described, in the aggregate and for a typical family.

M. A. Lannen, *Illustrative Total Monthly Family Benefits under the 1965 Amendments*, Actuarial Note No. 25, December, 1965, Social Security Administration, Washington, pp. 4.

This note presents a table showing the amounts of monthly family benefits payable under the Old-Age, Survivors, and Disability Insurance system to various family groups for each possible average monthly wage. A brief description of the methodology involved in computing the individual benefits for various types of beneficiaries is given, along with an explanation of the method used in finding the total family amount.

⁴ See *TSA*, XVII, 591 and 592.

⁵ See *TSA*, XVII, 591 and 592.

M. C. Hart and M. A. Lannen, *Additional Interest Earnings of OASDI Trust Funds Resulting from Changed Interest Rate Provided in 1960 Amendments*, Actuarial Note No. 26, January, 1966, Social Security Administration, Washington, pp. 6.

This note analyzes the interest earnings of the Old-Age and Survivors Insurance and Disability Insurance Trust Funds with regard to the effect of statutory changes in the basis of determining the interest rate for special-issues investments made by the 1960 amendments. The change increased interest earnings by a cumulative total of about \$97 million during the period from October, 1960, to June, 1965.

R. J. Myers and M. A. Lannen, *Comparison of Actual Experience under OASDI System with Short-Range Cost Estimates*, Actuarial Note No. 27, February, 1966, Social Security Administration, Washington, pp. 2.

This note compares actual experience for fiscal year 1965 under the Old-Age, Survivors, and Disability Insurance system with the short-range cost estimates given in the *Twenty-fifth Trustees Report*. The effect of actual experience on the short-range cost estimates is shown by a comparison of the fund at the end of fiscal year 1965 with the estimates made in each of the years 1962 through 1965.

F. Bayo, *Retirement Experience of Old-Age Beneficiaries*, Actuarial Note No. 28, March, 1966, Social Security Administration, Washington, pp. 3.

This note briefly analyzes the retirement experience for the years 1958-65 of workers under the social security system. The "prevalence of retirement" has been relatively stable for females during the last five years. For males, the rates have been increasing and are now almost at the same level as those of females. A continuation of the most recent experience is recommended as an assumption for future cost estimates. Data are presented by sex and single years of age. A short discussion of the proportions of workers electing an actuarially reduced benefit before age 65 is also included.

OTHER TOPICS

E. Ferguson (ed.), *Sources of Insurance Statistics*, pp. 191, Special Libraries Association, New York, 1965.

"An attempt has been made to include the recognized publications specialized to the insurance business issued by commercial publishers and by organizations within the business," including some that have ceased publication or that have merged but excluding the many handbooks of premium rates, dividend histories, and similar information. While most of the statistical data relate to private insurance companies, information is included on some voluntary organizations such as Blue Cross, fraternal associations, and Veterans Administration programs. Health insurance, life insurance, and property-liability insurance are treated separately. The information given in each reference includes the subject, source, data frequency, and period of time for which the data are usually given.

U.S. National Center for Health Statistics, *Evaluation of Psychological Measures Used in the Health Examinations Survey of Children Ages 6-11*, pp. 67, Public Health Service Publication No. 1,000, Series 2, No. 15, Washington, March, 1966.

"Because of the present state of the art of psychological measurement, studies such as those conducted by the Health Examination Survey encounter difficult problems in

attempting to estimate the prevalence of various mental health factors in the population." This report presents an evaluation of the four procedures used in the survey in terms of their validity, reliability, and applicability.

U.S. National Center for Health Statistics, *Hearing Levels of Adults by Age and Sex*, pp. 34, Public Health Service Publication No. 1,000, Series 11, No. 11, Washington, October, 1965.

This report contains findings from the National Health Examination Survey by age and sex for the right ear, left ear, and better ear for each test frequency. Estimates of hearing levels for speech are also presented. "In general the findings of the current study show substantially better hearing levels than were obtained in a number of other large-scale tests conducted under somewhat similar conditions."

U.S. National Center for Health Statistics, *Divorce Statistics Analysis: United States, 1962*, pp. 56, Public Health Service Publication No. 1,000, Series 21, No. 7, Washington, December, 1965.

"During recent years the duration of marriage at time of divorce has been comparatively stable; the median duration for the divorce-registration area was 7.3 years in 1962 and 7.1 in 1960 and 1961." The average number of children per divorce was 1.30, 1.18, and 0.85 in 1962, 1960, and 1953, respectively. The divorce rate was 2.2 per 1,000 population—highest in the West (3.6) and lowest in the Northeast (0.9).

Data are analyzed by age at time of decree, age of divorced persons at time of marriage, color or race, marriage order, residence, and birthplace. Remarriage of divorced couples, children of divorced couples, and legal variables are also discussed.

U.S. National Center for Health Statistics, *Natality Statistics Analysis: United States, 1963*, pp. 29, Public Health Service Publication No. 1,000, Series 21, No. 8, Washington, March, 1966.

"In 1963 births exceeded the figure of 4 million for the tenth consecutive year in spite of a continuing decline in the birth rate for the sixth consecutive year since the last peak was reached in 1957. The rates are still well above those observed in the period immediately prior to World War II.

"The current decline is due, in part, to the shift in the childbearing ages which occurred in the 1950's. Because the women who are now over 25 years of age had higher birth rates at younger ages, they are now having lower birth rates at the older ages. Current fertility declines at the younger childbearing ages (under 25 years) may be due to the postponement of marriage and childbearing to later ages or to a reduction in the number of children couples will have."

The sex ratio, plural births, attendant at birth, birth weight and period of gestation, season of birth, and illegitimacy are analyzed. Data are also presented by color, by state and geographic divisions, and by metropolitan areas having populations of at least one million.

Obituary

JOHN F. B. AMSDEN
RICHARD A. BOSSHART
WILLIAM CHODORCOFF
GERALD M. CROWLEY
MILTON A. ELLIS
WILLIAM H. GILBERT
DAVID HALL
WILLIAM J. HILLER
WILFRID L. HILLIARD
GRADY H. HIPPI
HAROLD A. LACHNER
ANDREW M. LENNOX
W. WARD NIPPER, JR.
RICHARD D. PERRY

1924 . . . John F. B. Amsden . . . 1966

John Francis Bekker Amsden, a Fellow of the Society of Actuaries and an Associate of the Institute of Actuaries, died at his home in the town of Mount Royal, Quebec, on March 8, 1966. Mr. Amsden was in his forty-second year.

Born in Vancouver, John Amsden received his education at Eastbourne College, England. On returning to Canada, he attended the University of Manitoba, graduating with the degree of Bachelor of Commerce, with the Gold Medal, in 1944. He served with the Canadian and British armed forces during and after World War II, terminating his service in 1948 with the rank of major.

Mr. Amsden joined the head office staff of the Sun Life Assurance Company of Canada in 1948. He was appointed a chief clerk in the actuarial department in 1950, assistant mathematician in 1951, mathematician in 1953, assistant actuary in 1955, superintendent of policy administration in 1958, and associate actuary in 1963.

He became an Associate of the Society of Actuaries in 1948, a Fellow in 1951, and an Associate of the Institute of Actuaries in 1951 also. As superintendent of policy administration he was engaged in the development and application of electronic methods to the company's ordinary insurance business, and for several years he served on the Society of Actuaries' Committee on New Recording Means and Computing Devices. When conversion to the new systems neared completion, he returned to the actuarial division of the company and had much to do with the preparations for Sun Life's entry into the individual health insurance field.

Mr. Amsden took a keen and active interest in many community endeavors, serving in many capacities with the Montreal Board of Trade and the Montreal Children's Hospital. From 1958 to 1962, he was a lecturer at Sir George Williams University in Montreal.

He is survived by his widow, the former Audrey Elizabeth Bovey; three young children, Elizabeth Karen, Barbara Jean, and Peter John; his mother, Mrs. P. C. H. B. Amsden; and a sister, Mrs. John Badham, of Ryegate, Surrey, England.

1928 . . . Richard A. Bosshart . . . 1965

Richard A. ("Dick") Bosshart, an Associate of the Society of Actuaries, died of leukemia at the age of 38 on November 23, 1965, after a relatively short period of illness.

For three years following graduation from the State University of Iowa, he was employed by insurance companies in New York City. After returning to his native Chicago, he was associated with actuarial consulting firms before joining the International Harvester Company, where he became general supervisor of retirement plans in the Employee Insurance and Pension Department. He is remembered by his company for his many contributions toward the resolution of problems in the development of its retirement programs.

To his professional associates, he was known for his valuable contributions to discussions of employee benefit plans. By applying his knowledge of the workings of a giant industrial corporation, he frequently presented a unique point of view in these discussions.

Dick Bosshart will be remembered by his friends and associates for all his fine personal qualities, but especially for his tenacity, integrity, and keen sense of humor. Despite his awareness of the discouraging diagnosis of his illness, he remained cheerful. This was not surprising to those who knew him well, since, as a child, he had recovered from

such misfortunes as the loss of a father and rheumatic fever. By conscientiously following the physical therapy prescribed for this childhood illness, he regained his physical strength and became an excellent swimmer and bowler. He was also an excellent bridge player and enjoyed light reading as well as logic puzzles and games.

He is survived by his wife, Avis, a daughter, Gail, and a son, Michael.

1910 William Chodorcoff 1966

William Chodorcoff, executive vice-president of The Prudential Insurance Company, collapsed and died April 18, 1966, in New Orleans, Louisiana, shortly after delivering a speech before delegates to Prudential's Canadian National Business Conference in the Jung Hotel there.

He was born in Winnipeg, Manitoba, on April 1, 1910, and educated at the University of Manitoba, where he received a Bachelor of Arts degree in 1930. During his last summer at the University and the summer following graduation, he worked for the Northern Insurance Company, a fire and casualty firm in Winnipeg. His interest in life insurance led him to join Prudential's home office in Newark, New Jersey, in September, 1930.

In 1941, following an eleven-year association with the company's Actuarial Department, he was promoted to assistant comptroller. In 1948 he was appointed associate comptroller. A year later, he was elected a vice-president of the company. In 1957 he was elected senior vice-president and comptroller and in 1962 was promoted to executive vice-president. Mr. Chodorcoff was head of the company's planning and control area, which comprises the Actuarial, Comptroller's and Operations Analysis, and Research Departments.

Mr. Chodorcoff was a Fellow of the Society of Actuaries and a member of the Actuaries Club of New York. He was a member of the Financial Executives Institute and the New York Insurance Accountant's Club.

For many years he was active on a number of committees of the Life Insurance Association and the American Life Convention, having served as chairman of several of these.

In 1962 he was appointed to a special fifteen-man committee of the United States Chamber of Commerce, formed at the request of President Kennedy, to study ways for improving federal budget procedures. He later served as a member of the Chamber's Government Operations and Expenditures Committee.

He was a trustee of East Orange General Hospital and the United Community Fund and Council of Essex and West Hudson. He was also a trustee of Temple B'nai Jeshurun in Newark and past president of its men's club.

He was held in highest esteem by his associates, by members of his Temple, and by the community in which he lived. In 1966 Mr. Chodoroff was a recipient of the Annual Brotherhood Award from the New Jersey Region, National Conference of Christians and Jews.

Mr. Chodoroff is survived by his wife, the former Rita Raphael of Toronto. They have two children: Michael and Karen. Their home is in Short Hills, New Jersey.

1910 Gerald M. Crowley 1965

Gerald Malcolm Crowley, an Associate of the Society, died December 1, 1965, in a New Jersey hospital shortly following a heart attack.

Mr. Crowley was born in Orangeburg, New York, on New Year's Day, 1910, and, from the age of 5, was a resident of New Jersey. He was graduated from Princeton University with a Bachelor of Arts degree in mathematics in 1930, later attending the University of London as a postgraduate student.

In 1937, after a period of employment by the College Entrance Examination Board and the editorial department of the *Encyclopaedia Britannica*, he joined the Mutual Life Insurance Company of New York, where he continued until his death except for a period of two years of military service during World War II.

In 1949 Mr. Crowley became an Associate of the Actuarial Society of America and afterward filled various positions in Mutual of New York's actuarial and group departments, the last one being that of assistant director of actuarial studies.

Aside from his business life, Mr. Crowley served as a member of the Jersey City Board of Education and as a trustee of the Jersey City Museum Association. His interest in collegiate education was shown by his frequent grants, including those to an educational trust which he commenced and to which he contributed from 1962 on.

His easy disposition, ready wit, and sound scholarship were always highly esteemed by his numerous friends and business associates.

Surviving are his mother, Mrs. Annie F. G. Crowley; a brother, Marshall Crowley; and a sister, Mrs. Kathleen C. Hannon.

1910 Milton A. Ellis 1966

Milton A. Ellis, vice-president of Metropolitan Life Insurance Company, died on March 28, 1966, after a long illness.

Mr. Ellis was born in Manila, Philippine Islands, on May 23, 1910. He received his higher education at the University of Rochester (New York), where he graduated with a Bachelor of Arts degree in mathematics in 1931.

He began his Metropolitan career as an actuarial clerk on July 22, 1931, and immediately set out upon the course of study which was to result in his qualification by examination in 1938 as a Fellow of the American Institute of Actuaries and the Actuarial Society of America. During the same year he was appointed an actuarial section head.

During World War II, Mr. Ellis served in the Navy almost three years, most of the time in the submarine service in the Pacific, where his ship was awarded the Presidential Unit Citation for four successful combat patrols. He attained the rank of lieutenant commander, which he continued to hold in the retired reserve.

Mr. Ellis returned to Metropolitan Life in 1945 and was appointed an actuarial research associate in 1946. The following year he was assigned to the company's insurance relations function and remained in that field until his death.

Mr. Ellis was appointed an administrative assistant in 1949 and was appointed an officer of the company in 1952, with the title of assistant vice-president. He was advanced to third vice-president in 1956, to second vice-president in 1960, and to vice-president in 1963.

Throughout his career Mr. Ellis was active in professional and civic affairs. He served as chairman of the legislative committees of the Bureau of Accident and Health Underwriters and its successor, the Health Insurance Association of America, and was a member of the joint legislative committee of the American Life Convention and the Life Insurance Association of America.

Among his civic activities, Mr. Ellis was president of two Parent-Teacher associations in Englewood, New Jersey, a trustee and treasurer of the First Church of Christ Scientist in Englewood, and gave over twenty years of service to Boy Scout activities at all levels.

Mr. Ellis was a greatly respected member of his profession and a warmly regarded friend to his associates within and without the insurance industry. His special delight was working and playing with children, both his own and others. He is sorely missed by his friends of all ages.

Among his hobbies, he was especially fond of those activities which got him outdoors, such as hiking, driving, and other travel.

He is survived by his widow, the former Miss Kathleen Black; two children, Fred Kimball Ellis and Mrs. Sandra Ellis Lomker; and four grandchildren.

1921 William H. Gilbert 1966

William H. Gilbert, assistant actuary of the Minnesota Mutual Life Insurance Company, died suddenly on February 10, 1966. He was forty-four years old and had been in apparent good health up to the time of his death.

Mr. Gilbert was born in Minneapolis, Minnesota, on April 9, 1921, and received his early education in that area, graduating from high school as valedictorian of his class. He was elected to Phi Beta Kappa at the University of Minnesota and received the degree of Bachelor of Arts from that institution in 1943. After serving his country in the South Pacific with the United States Navy, he returned to the University of Minnesota and received the degree of Master of Arts in 1946. He taught undergraduate mathematics while studying for his advanced degree.

He became associated with the Minnesota Mutual Life Insurance Company in 1947, which he served ably and loyally for more than eighteen years until his death. He served successively in various capacities in the company's group and data-processing departments until his election in 1959 to the position of assistant actuary in the actuarial department. He obtained his Fellowship in the Society in 1965. He was a member of the Twin Cities Actuarial Club.

The thoroughness and dispatch with which Bill Gilbert approached each task, and his warm interest in the welfare of those with whom he worked, will be sorely missed by his friends and associates, who feel a deep sense of loss by his untimely passing. His close friends particularly enjoyed his keen sense of humor. He was a very active man who enjoyed many interests, especially music, tennis, fishing, and gardening.

Mr. Gilbert is survived by his wife, Elaine, and two daughters, Helen and Jeanne, of White Bear Lake, Minnesota.

1902 David Hall 1966

David Hall, second vice-president of New England Mutual Life Insurance Company, died at his home in Wellesley Hills, Massachusetts, on April 18, 1966, at age 63.

Mr. Hall was born in Concord, New Hampshire, on August 31, 1902. After receiving an A.B. in history from Harvard University in 1922 he entered the teaching profession, first for two years at the American University of Beirut, Lebanon, then at Deerfield Academy.

He joined New England Mutual as an actuarial student in 1930 and qualified as a Fellow of the Actuarial Society in 1942. He served as secretary-treasurer of the Actuaries' Club of Boston from 1944 to 1951.

Mr. Hall was appointed assistant actuary of his company in 1939 and associate actuary in 1945. His interests, however, turned him toward administrative more than mathematical work. In 1950 he was elected an assistant secretary; operations officer was added to his title four years later. He was elected second vice-president and executive assistant in 1964.

He was extremely conscientious in church and community affairs. In the Unitarian Church of Wellesley Hills he served devotedly, most recently as chairman of its Standing Committee. He was for eighteen years a Wellesley Town Meeting member. He maintained a keen interest in music, books, chess, and bridge. He could speak French and German and was a student of the Greek language.

David Hall was a gentleman in the true meaning of that word. Unassuming and generous with a keen sense of humor expressed particularly in poetic whimsy, he was a pleasant, sympathetic companion with a sense of high ideals for himself, for his company, and for his environment. He will be remembered as a warm and likeable person.

He is survived by Kathryn, his widow; three married children, David, John, and Mrs. John P. Denio; and eight grandchildren.

1893 William J. Hiller 1966

William John Hiller, a Fellow of the Society, died March 16, 1966, at Tucson, Arizona, where he had resided because of his health since 1951.

Mr. Hiller was born in Ann Arbor, Michigan, on October 2, 1893. Following graduation from the University of Michigan, he joined Bankers Life Insurance Company of Nebraska in July, 1915, where he served as associate actuary until September, 1930. In 1917 he was commissioned a captain in the Coast Artillery and served twenty months overseas during World War I. While at Bankers Life, he spent more than ten years as a member of the faculty of the University of Nebraska, where he taught actuarial mathematics and insurance subjects.

He served Great American Life of San Antonio, Texas, as secretary-actuary from September, 1930, to January, 1940, when he joined the

Franklin Life Insurance Company of Springfield, Illinois, as vice-president in charge of underwriting, continuing in that position until a heart condition led to his early retirement in 1951.

In his earlier years, he was active at hunting, fishing, and gardening, having been president of the Vista Garden Club of San Antonio. He was active in alumni affairs and served as president of the Springfield Chapter of the Alumni Association of the University of Michigan.

Surviving are his widow, Marjorie, and son, Clark.

1896 Wilfrid L. Hilliard 1965

Wilfrid Laurier Hilliard, an Associate of the Society of Actuaries and a former associate actuary of the Dominion Life, died in Kitchener, Ontario, on October 11, 1965.

Mr. Hilliard was born at Morriston, Ontario, on February 11, 1896. After serving overseas with the Canadian Army during the first war, he returned to the University of Toronto, graduating in 1922 with a Bachelor of Arts degree.

Following graduation and a short period of service with Dominion Life, Mr. Hilliard was employed by the Commercial Life in Edmonton. In 1925, he rejoined Dominion Life, the company founded by his grandfather thirty-six years earlier. Mr. Hilliard secured his Associateship in the Actuarial Society of America in 1929. In 1940 he was appointed assistant actuary and in 1953 associate actuary. He was engaged in annual statement and reserve work for a number of years prior to his retirement in February, 1962.

Mr. Hilliard's interests were many and varied and included his church, politics, and sports activities.

He was a member of the First United Church in Waterloo, serving as an elder, a steward, and a member of the trustee board. His many years of devoted service to his church were recognized in 1963 when tribute was paid by the official board of the church to his contribution to church work.

He also had a keen interest in politics at all levels. He was a long-standing member of the North Waterloo Liberal Association, serving on the executive board for many years. In addition, he was engaged in municipal politics for a period of six years, serving Waterloo as alderman, reeve, and, during 1947 and 1948, as mayor.

Mr. Hilliard was an avid sportsman. He was an ardent curler and very prominent in lawn bowling circles. In 1950 he was a member of the provincial lawn bowling championship team.

Mr. Hilliard will long be remembered by his associates and the community for his enthusiastic, intelligent approach to his endeavors in many different spheres of activity.

He is survived by his widow, Celia, a daughter, Barbara, and his father, one brother, and four sisters.

1893 Grady H. Hipp 1965

Grady Hayne Hipp, former executive vice-president of Liberty Life Insurance Company, died at the age of 72 on June 25, 1965, in a Greenville, South Carolina, hospital following a prolonged illness. He is survived by his wife, Mrs. Thelma Bush Hipp, and a brother, C. Everett Hipp, of Toronto, Canada. Mr. and Mrs. Hipp had two daughters who died in childhood.

A graduate of Newberry College in his native South Carolina, he pursued advanced studies in economics, mathematics, and actuarial science at the University of Virginia. In 1911, Wisconsin passed the nation's first workmen's compensation act and offered a scholarship to students with related backgrounds who could staff the new department. Grady Hipp was one of these first departmental staff members and combined postgraduate work at the University of Wisconsin with his career. He became an Associate of the Actuarial Society of America in 1914 and of the Casualty Actuarial Society in 1927.

After Signal Corps service in World War I, Mr. Hipp became an actuary for the State of New York Insurance Department, and from 1930 to 1943 he was actuary for the State Insurance Fund in New York City.

At the death of one of his brothers, W. Frank Hipp, founder of the Liberty Life Insurance Company, Grady Hipp was called to its home office in Greenville to become executive vice-president, a position he held until his retirement in 1958.

The nursing division of the Greenville County Home, one of his most cherished civic interests, has been officially renamed the Grady H. Hipp Nursing Center in honor of his work in planning the home during his chairmanship of the County Home Board.

In other civic service, he was president and lieutenant governor of the Kiwanis Club, president of the Greater Greenville Chamber of Commerce, and campaign chairman and president of the United Fund. He received the honorary Doctor of Laws degree from Newberry College in 1957.

Possessed of a cheerful and friendly disposition, he will be greatly missed by his many friends and former business associates.

1909 **Harold A. Lachner** 1965

Harold A. Lachner, vice-president of the Metropolitan Life Insurance Company and a Fellow of the Society, died on December 26, 1965, after a short illness.

Mr. Lachner was born in Dubuque, Iowa, on November 13, 1909. He was educated at Yale University, where he received his Ph.B. degree, with high honors, in 1931. He also studied one year, commencing in 1928, at the University of Berlin on a fellowship from the Institute of International Education and was elected to Phi Beta Kappa in his Junior year at Yale.

He will be remembered by his associates and the members of his profession for the conscientious application of his exceptional talents to the betterment of his company, his community, and his profession. His service as treasurer of the Highlands Country Club in Garrison, New York, and as a trustee of the board of education of the Garrison Union Free School placed him among those his neighbors depended upon for wise counsel and service.

His chairmanship of the Press Committee of the Society of Actuaries, of the Actuarial and Statistical Committee of the Health Insurance Association of America, and his membership on the Joint Steering Committee on Morbidity Statistics, for both the Health Insurance Association and the Society of Actuaries, marked him as one of that small group of dependables who generally take the lead when there is work to be done.

Mr. Lachner had a varied career at Metropolitan. His efforts earned him steady advancement through clerical and supervisory positions until, in 1944, he was appointed an officer of the company with the title of assistant actuary. He was further advanced to associate actuary in April, 1952, and actuary in November, 1958. On April 1, 1963, he was advanced to second vice-president in the co-ordination division to take charge of an operation that provides company-wide management services and also to deal with matters concerning employee compensation systems. On December 1, 1965, he was again advanced, with the title of vice-president.

His assignments at Metropolitan included actuarial activities in connection with industrial, ordinary, and accident and health insurance and extended to the development of premium rates and values and to the

preparation of financial statements, analyses, and forecasts of operating results. More recently, his work involved research into systems, procedures, methods, compensation systems, and other matters that are company-wide in scope. Of his help in preparing the prize-winning text entitled *Industrial Life Insurance in the United States*, Mr. M. E. Davis, the author, wrote that his "greatest indebtedness is to Harold A. Lachner . . . who, throughout the preparation of the book, gave most generously of his time to collaboration."

Mr. Lachner earned a reputation as a top-notch bridge player among his intimates, but practically all who have had contact remember him for his sincerity, friendliness, and good humor. His friends and associates share a feeling of great loss at his passing.

He is survived by his wife, Mrs. Elizabeth Palen Lachner; a son, James Lachner; a daughter, Elizabeth Anne Lachner; his mother, Mrs. Emil S. Lachner; and a sister, Mrs. Dorothy Sheffer.

1901 . . . Andrew M. Lennox . . . 1966

Andrew Murray Lennox, an Associate of the Society of Actuaries, died suddenly at his home on January 21, 1966.

Mr. Lennox was born near Shallow Lake, Ontario, on November 7, 1901. He attended high school in Owen Sound, Ontario, then the University of Toronto, graduating in mathematics and physics from Victoria College in 1924.

After graduation, Mr. Lennox joined the actuarial department of the Imperial Life Assurance Company of Canada and became an Associate of the Society in 1926. For a number of years he served as head of the dividend section of Imperial Life Assurance Company before transferring to the group area in 1956 as group actuarial assistant and later group actuarial officer.

Mr. Lennox had an analytical mind and a quiet sense of humor which endeared him to all who worked with him. His shy, unassuming manner made many business and personal friends. For many years Mr. Lennox served on the committee for setting and marking papers for the examinations leading to the C.L.U. degree. He took a particular interest in assisting the students having difficulties with their assignments. Instead of merely indicating, briefly, the reference books involved, he often developed helpful explanations for them.

Mr. Lennox was especially devoted to his family. He leaves his widow, the former Geraldine Francis; a daughter, Mrs. Fredrick Johnston; and two grandchildren.

1931 **W. Ward Nipper, Jr.** 1966

W. Ward Nipper, Jr., assistant actuary at Connecticut General Life Insurance Company, was killed February 12, 1966, in Westfield, Massachusetts, in an automobile accident.

Born in St. Louis, Missouri, September 23, 1931, Ward knew of actuarial science from his earliest days. His father, V. W. Nipper, is senior vice-president and actuary of Mutual Trust Life Insurance Company in Chicago and has been associated with this company for many years. After attending Niles Township High School near Chicago, Ward matriculated at the University of Illinois, then transferred to Knox College, Galesburg, Illinois, where he graduated in 1954.

He then spent three years as an officer in the United States Naval Reserve, serving off the Pacific Coast on a mine sweeper and a sound laboratory ship. Released from active duty in 1958 as a lieutenant, he entered the Graduate School of the State University of Iowa, concentrating in actuarial science. He received his M.S. degree in the spring of 1960 and two months later became an Associate of the Society of Actuaries. He attained his Fellowship in 1963.

Ward began his actuarial career at Connecticut General in 1960. He worked in both the group and individual departments and was a member of the corporate actuary's department when he was appointed an assistant actuary in 1964. Here he was involved in research into the definition and analysis of profits. He had been recently appointed to the Society's Education and Examination Committee for Part 9I.

Ward Nipper possessed a restless and inquiring mind, which, combined with his analytical ability, made him a strong contributor to the actuarial work at his company.

The out-of-doors was his prime avocation; he was especially fond of skiing and sailing. He was full of vigor in both mind and body, proficient in any endeavor. His friends and associates, remembering his vitality, find it difficult to accept his passing.

Our sympathy is extended to his wife, Janet, their three children, his parents, and sister.

1939 **Richard D. Perry** 1966

Richard D. Perry, senior actuarial assistant at Connecticut General Life Insurance Company, died March 24, 1966, the result of leukemia, after a lengthy illness.

A native of New Jersey, Dick was born in Jersey City on September 13, 1939. He attended high school in Englewood, New Jersey, and upon graduation in 1957 entered LaFayette College, Easton, Pennsylvania, to study engineering. However, he soon switched to a mathematics-liberal arts curriculum and passed his first actuarial examination in 1960. While at LaFayette he was elected a member of Phi Beta Kappa and won top honors in mathematics. He graduated *cum laude* in 1961.

Dick's actuarial career commenced at Connecticut General in 1961, and within a year he left to serve six months' active duty with the Connecticut National Guard. Upon his return, he worked in both the individual and group departments and was a member of the group pension department's actuarial staff when he became a Fellow in the Society of Actuaries in 1965. Soon after he was appointed senior actuarial assistant.

Dick Perry had outstanding abilities. This was amply demonstrated by his rapid examination progress. His grasp of work assignments easily showed his penetrating analytical ability, and his potential was felt to be of the highest level.

Well liked by his friends and associates, he was a ready listener and forceful contributor. His patient and understanding nature will be difficult to emulate and hard to forget. Golf was his favorite physical activity, but he was always ready to take time for tennis, bowling, or skiing. At the bridge table he was superb, and he studied the game intently.

Our sympathy is extended to his wife, Carolyn, two daughters, Laura and Susan, and to his parents and brother.

MINUTES OF THE REGIONAL MEETING
of the
SOCIETY OF ACTUARIES

HELD AT THE FAIRMONT HOTEL
SAN FRANCISCO, CALIFORNIA
APRIL 14-15, 1966

The meeting was called to order by the President, Gilbert W. Fitzhugh, on Thursday, April 14, at 10:00 A.M. The following is a summary of the attendance at the meeting:

SUMMARY

Fellows.....	154
Associates.....	78
Guests.....	22
Total.....	254

The President introduced as distinguished guests Mr. Thomas E. Murrin, President-Elect of the American Academy of Actuaries, and Mr. Kenneth J. Hedley of Sydney, Australia, a Fellow of the Faculty of Actuaries and an Associate of the Society.

The President advised that the Society had received word of the deaths of twelve members since the last meeting: John F. B. Amsden, Milton A. Ellis, William H. Gilbert, William J. Hiller, Harold A. Lachner, W. Ward Nipper, Jr., Richard D. Perry, Fellows; Richard A. Bosshart, Gerald M. Crowley, Wilfrid L. Hilliard, Grady H. Hipp, Andrew M. Lennox, Associates. The members stood for a minute in silent respect for the memory of these men. Obituaries are printed in this volume of the *Transactions*.

The minutes of the Regional Meeting held May 27-28, 1965, as printed in the *Transactions* (XVII, 225-26, inclusive), and of the Annual Meeting held October 25-27, 1965, as printed in the *Transactions* (XVII, 603-7, inclusive), were approved.

The proposed amendments to the Constitution, which redesignated the present Article VIII as Article IX and substituted a new Article VIII entitled "Indemnification of Society Members," were approved.

The President extended a welcome to the new Fellows and Associates who were attending their first meeting of the Society after attaining their present ranks.

The meeting then proceeded to a discussion of topics of current interest. A report on United States Medicare, prepared by Mr. C. Manton Eddy, was read by the Secretary. This was followed by a report on Canadian Medicare by Mr. George N. Watson. Opening the discussion on the future of the Society, the President announced the formation of a new standing committee tentatively to be called the Committee on Future Directions, with Mr. Walter Klem as chairman. Mr. Andrew C. Webster then presented a report on the American Academy of Actuaries, and Mr. Lloyd G. Current reported on the Canadian Institute of Actuaries. This was followed by a discussion period during which many suggestions concerning the future of the Society were presented. The meeting adjourned at 12:10 P.M.

The meeting reassembled that afternoon at 2:00 P.M. in two separate simultaneous sessions. Mr. John F. Hook presided at the session on Individual Life and Health Insurance. There were informal discussion and questions from the floor on three separate major topics. The second simultaneous session was a panel discussion of Employee Benefit Plans. Mr. William Cunningham was chairman of this session, and his panel members were Messrs. Burton E. Burton, Kenneth T. Clark, Neal A. Farmer, Alan N. Ferguson, Howard H. Hennington, Henry K. Knowlton, Daniel F. McGinn, Eugene H. Neuschwander, and Walter L. Reynolds. The panel discussions were presented under three separate major topics, after each of which there were questions and discussion from the floor. These sessions adjourned at approximately 5:00 P.M.

On Friday, April 15, the meeting was called to order at 9:30 A.M. by Vice-President Edwin B. Lancaster for a panel presentation on agency matters. The moderator of the panel was Mr. Harold E. Crandall, and the panel members were Mr. Raymond A. Bierschbach and Mr. Stuart A. Robertson. Following the panel presentation, members presented discussions and a number of questions were answered by the panel.

After a brief recess, Vice-President Bruce Shepherd introduced Mr. Robert N. Powell, chairman of a panel on the subject "Implications of Title XIX of the 1965 Social Security Amendments for the Private Health Insurance Business." Panel members were Miss Josephine W. Beers and Mr. Walter L. Reynolds. After the panel presentation, there were a number of questions from the floor which were answered by the panel members.

The gavel was then turned over to President-Elect Harold R. Lawson, who closed the meeting at 12:20 P.M.

MINUTES OF THE REGIONAL MEETING
of the
SOCIETY OF ACTUARIES

HELD AT THE MAYFLOWER HOTEL
WASHINGTON, D.C.
APRIL 28-29, 1966

The meeting was called to order by the President, Gilbert W. Fitzhugh, on Thursday, April 28, at 10:00 A.M. The following is a summary of the attendance at the meeting:

SUMMARY

Fellows.....	343
Associates.....	107
Guests.....	27
Total.....	477

The President introduced as a distinguished guest Mr. G. Malcolm Murray, of Edinburgh, Scotland, a Fellow of the Faculty of Actuaries.

The President advised that the Society had received word of the deaths of fourteen members since the last Annual Meeting: John F. B. Amsden, William Chodorcoff, Milton A. Ellis, William H. Gilbert, David Hall, William J. Hiller, Harold A. Lachner, W. Ward Nipper, Jr., Richard D. Perry, Fellows; Richard A. Bosshart, Gerald M. Crowley, Wilfrid L. Hilliard, Grady H. Hipp, Andrew M. Lennox, Associates. The members stood for a minute in silent respect for the memory of these men. Obituaries are printed in this volume of the *Transactions*.

The President reported that the minutes of the Regional Meeting held May 27-28, 1965, as printed in the *Transactions* (XVII, 225-26, inclusive), and of the Annual Meeting held October 25-27, 1965, as printed in the *Transactions*, (XVII, 603-7, inclusive), were approved at the Regional Meeting in San Francisco.

The President also reported that the proposed amendments to the Constitution, which redesignated the present Article VIII as Article IX and substituted a new Article VIII entitled "Indemnification of Society Members," were approved at the San Francisco meeting.

The Secretary presented the following summary of business transacted by the Board of Governors at a meeting held on April 27, 1966:

1. Four Fellows of the Institute of Actuaries or the Faculty of Actuaries were admitted as Associates of the Society.
2. Approved as sites for future meetings:

1969 Regional Meetings:

April 21-22, 1969 Americana Hotel, New York, N.Y.
 May 1-2, 1969 The Regency Hyatt House Hotel, Atlanta, Ga.
 June 5-6, 1969 Chase-Park Plaza Hotel, St. Louis, Mo.

Annual Meetings:

Nov. 17-19, 1969 Sheraton-Boston Hotel, Boston, Mass.
 1970* Denver Hilton Hotel, Denver, Colo.
 Nov. 8-10, 1971 Royal York Hotel, Toronto, Ont.

3. Authorized the appointment of a new committee called the Committee on Future Course of the Society, with Mr. Walter Klem as chairman. The function of the committee is to look ahead to the future needs of the Society to determine what activities should be undertaken in order to promote properly and adequately the needs of the profession.
4. Heard a report from Mr. John H. Miller, chairman of the Committee To Study Pension Plan Problems. Mr. Miller reported that his committee had held two meetings and that five subcommittees had been appointed. It is hoped that a report will be ready for discussion at the fall meeting.

The meeting then proceeded to a discussion of topics of current interest. A report on United States Medicare was presented by Mr. C. Manton Eddy. This was followed by a report on Canadian Medicare by Mr. Cecil G. White. The President then discussed the formation of a new standing committee to be called the Committee on Future Course of the Society, with Mr. Walter Klem as chairman. Mr. John H. Miller presented a report on the American Academy of Actuaries, Mr. Andrew C. Webster discussed accreditation, and Mr. Richard Humphrys reported on the Canadian Institute of Actuaries.

The President extended a welcome to the new Fellows and Associates who were attending their first meeting of the Society after attaining their present ranks.

A discussion of the Future of the Society then took place, with informal discussion and questions from the floor. This was followed by a panel discussion on Long-Range Planning. Mr. Alan M. Thaler was chairman, and panel members were Messrs. Morrison H. Beach, Robert J. Lamphere, and W. James D. Lewis. Mr. Lamphere, not a member of the Society, is

* Dates to be announced at the fall meeting.

vice-president of corporate planning with the John Hancock. The meeting adjourned at 12:30 P.M.

The meeting reassembled that afternoon at 2:15 P.M. in two separate simultaneous sessions. Mr. William H. Schmidt presided at the session on Individual Life and Health Insurance. The second simultaneous session was a panel discussion of Employee Benefit Plans, with Mr. Daniel W. Pettengill as chairman. In both sessions, the panel presentations on each of three major topics were followed by informal discussion and questions from the floor. These sessions adjourned at approximately 5:00 P.M.

On Friday, April 29, the meeting was called to order at 9:30 A.M. by Vice-President Edwin B. Lancaster for a panel presentation on agency matters. The moderator of the panel was Mr. Daton Gilbert, and the panel members were Mr. Milton J. Goldberg and Mr. Ernest J. Moorhead. Following the panel presentation, members presented discussions, and questions from the floor were answered by the panel.

After a brief recess, Vice-President Bruce Shepherd introduced Mr. Richard J. Mellman, chairman of a panel on the subject "Implications of Title XIX of the 1965 Social Security Amendments for the Private Health Insurance Business." Panel members were Mr. Joseph B. Crimmins and Mr. James L. Purdy. Following the panel presentation, questions were answered by the panel members.

The gavel was then turned over to the President, who closed the session at 11:55 A.M.

A Smaller Company Forum was called to order by Chairman Dwight K. Bartlett III at 2:00 P.M. on Friday, April 29, 1966. After discussion of questions on the program, the meeting adjourned at approximately 4:00 P.M.

TRANSACTIONS

JUNE, 1966

MORTALITY EXPERIENCE ACCORDING TO
BUILD AT THE HIGHER DURATIONS

B. FRANKLIN BLAIR AND LAWRENCE W. HAINES*

INTRODUCTION

PAST studies of mortality according to build have shown rather conclusively that the extra mortality associated with overweight increases progressively during the early years after issue and that this increasing trend continues for at least 15 or 20 years in the case of policies issued at the younger adult ages. However, the published studies have not included sufficient data at the higher durations to indicate whether the relative mortality of overweights continues to increase or tends to level off or decrease. The *1955 Build and Blood Pressure Study* included no exposures after the 19th policy year and thus did not answer this question, although there was some indication of a tendency for the extra mortality of overweights to level off after approximately the first 15 policy years, even among the younger age groups. A 1948 study of "Mortality Experience According to Build on Standard Insurance in the Provident Mutual" (*TASA*, Vol. L) included experience during the first 27 years after issue. Although the exposures at the higher durations were somewhat scanty, the results suggested rather strongly that the increase in the relative mortality among overweights in general tends to level off about 20 years after issue. The main purpose of the present study is to determine, by investigating recent experience on a block of insurance exposed at the higher durations, whether the inferences derived from the earlier studies can be confirmed.

SCOPE OF THE PRESENT STUDY

The study covers original life and endowment policies issued by Provident Mutual at standard rates on white males during the years 1930-34,

* Mr. Haines, not a member of the Society, is Assistant Actuary of Provident Mutual Life Insurance Company.

exposed between the 1947 and 1964 anniversaries. All these policies were medically examined. Reinsurance ceded is included. Policies issued at ages under 15 and policies on individuals with heights below 5'0" or above 6'5" are excluded because of the scantiness of the data and the lack of reliable weight standards for those ages and heights. Except for the periods of issue and exposure, these specifications are identical with those used in the 1948 study referred to above. Issues of 1930-34 were used because this is the oldest block of issues for which our mortality records have been kept up to date. Restricting the study to a 5-year issue period increases the homogeneity of the data and reduces the chances of anomalies due to the use of different underwriting standards at different times. The period of exposure commences at the point where the 1948 study terminated.

The study is based on policies only. In the 1948 study the mortality ratios by amounts were found to be practically useless because of fluctuations caused by large death claims, in spite of an artificial limitation of individual policies to \$25,000.

The present study includes 5,408 deaths (policies), of which 5,274 are in the 16th and later policy years, compared with 5,312 deaths in the 16th and later policy years in the 1948 study.

WEIGHT STANDARD AND MORTALITY BASIS

In order to preserve consistency with the 1948 study, the "standard weights" used for classification according to degree of overweight or underweight are the same as those used in that study, namely, the average weights at age 37 at the various heights according to the Medico-Actuarial Mortality Investigation.

The expected deaths were first calculated on the basis of the 1946-49 Basic Ultimate Table. (The error introduced by applying ultimate rates to a small amount of exposure in the 14th and 15th policy years is inconsequential.) Subsequently, the expected deaths were adjusted by factors producing (for all heights and weights combined) expected deaths equal to the actual deaths in each of the three issue age groups and five duration groups studied. Thus the deviations of the mortality ratios from 100 per cent may be considered, insofar as they are statistically significant, to be related to build and to other elements of selection which have influenced the acceptance of applicants in the particular build group.

ANALYSIS BY AGE AT ISSUE AND DURATION

Table 1 presents the results of the study analyzed in broad groupings according to age at issue, policy year, and deviation from standard weight. For all ages combined, Table 1 shows that at the high durations both the favorable mortality of the underweight groups and the unfavorable

mortality of the moderately overweight group (deviations of +8 to +22 lb.) have tended to draw closer to the average rate. However, when the results in each of the three age groups are examined, it becomes apparent that these effects are confined almost exclusively to the youngest group (issue ages 15-34). In issue age groups 35-49 and 50-65, neither the underweights nor the overweights exhibit any consistent trend in their mortality ratios at the higher durations. All of this suggests that, as the individ-

TABLE 1

ANALYSIS BY AGE AT ISSUE AND DURATION

Issues of 1930-34 Exposed between 1947 and 1964 Anniversaries. Expected Deaths Based on Actual Experience in Same Period for All Builds Combined. By Policies

DEVIATION IN POUNDS FROM STANDARD WEIGHT	ACTUAL DEATHS					RATIO ACTUAL TO EXPECTED*						
	Policy Years											
	14-15	16-20	21-25	26-30	31-34	14-34	14-15	16-20	21-25	26-30	31-34	14-34
Ages at Issue 15-65												
-23 or more..	12	110	149	180	89	540	(91%)	81%	82%	86%	89%	85%
-22 to - 8..	22	257	348	390	190	1,207	(68)	83	85	86	92	85
- 7 to + 7..	51	396	461	518	208	1,634	122	98	94	101	94	98
+ 8 to +22..	40	350	397	372	148	1,307	128	121	116	110	110	115
+23 or more..	9	176	239	211	85	720		118	140	133	146	130
All weights	134	1,289	1,594	1,671	720	5,408	100%	100%	100%	100%	100%	100%
Ages at Issue 15-34												
-23 or more..	3	52	60	86	53	254		102%	78%	80%	95%	86%
-22 to - 8..	8	72	109	177	97	463		77	76	91	98	86
- 7 to + 7..	10	64	127	172	87	460	(135%)	82	107	108	108	103
+ 8 to +22..	6	70	82	94	37	289		166	131	115	94	125
+23 or more..	1	26	51	51	18	147		137	184	143	(110)	146
All weights	28	284	429	580	292	1,613	100%	100%	100%	100%	100%	100%
Ages at Issue 35-49												
-23 or more..	7	38	65	68	26	204		67%	84%	86%	72%	80%
-22 to - 8..	11	112	160	159	72	514	(68%)	79	83	77	84	80
- 7 to + 7..	28	203	227	264	88	810	124	104	91	105	86	99
+ 8 to +22..	23	177	227	212	94	733	(130)	116	117	108	119	115
+23 or more..	2	97	130	123	56	408		119	134	132	169	130
All weights	71	627	809	826	336	2,669	100%	100%	100%	100%	100%	100%
Ages at Issue 50-65												
-23 or more..	2	20	24	26	10	82		(73%)	(89%)	120%	(129%)	95%
-22 to - 8..	3	73	79	54	21	230		97	108	102	(96)	100
- 7 to + 7..	13	129	107	82	33	364	(111%)	98	87	81	88	90
+ 8 to +22..	11	103	88	66	17	285	(119)	109	102	113	(104)	108
+23 or more..	6	53	58	37	11	165		107	125	122	(127)	118
All weights	35	378	356	265	92	1,126	100%	100%	100%	100%	100%	100%

* Ratios are not shown for groups with less than 10 deaths. Ratios based on 10-24 deaths are shown in parentheses.

uals in the youngest group grow older, weight changes occur which are predominantly in the direction of more normal weights, while the weights of the two older groups are more nearly stabilized at the time of issue.

Comparison of the mortality ratios for the three age groups confirms, as found in the 1948 study, that the relative extra mortality of overweights is highest in the youngest group and lowest in the oldest group. To some extent this is probably a spurious effect resulting from use in

TABLE 2

ANALYSIS BY DURATION, SHOWING RESULTS OF 1948
STUDY AND PRESENT STUDY

1948 Study—Issues of 1920-45 Exposed between 1937 and 1947 Anniversaries
Present Study—Issues of 1930-34 Exposed between 1947 and 1964 Anniversaries
Expected Deaths Based on Actual Experience in Same Period for All Builds Combined
By Policies

DEVIATION IN POUNDS FROM STANDARD WEIGHT	RATIO OF ACTUAL TO EXPECTED DEATHS FOR POLICY YEARS:								
	1-5	6-10	11-15	16-20		21-25*		26-30	31-34
	1948 Study	1948 Study	1948 Study	1948 Study	1965 Study	1948 Study	1965 Study	1965 Study	1965 Study
-23 or more.....	104%	95%	81%	83%	81%	80%	82%	86%	89%
-22 to - 8.....	95	99	87	81	83	82	85	86	92
- 7 to + 7.....	89	92	96	94	98	102	94	101	94
+ 8 to +22.....	112	102	116	119	121	125	116	110	110
+23 or more.....	108	123	123	140	118	131	140	133	146
All weights.....	100%	100%	100%	100%	100%	100%	100%	100%	100%

* Policy years 21-27 for 1948 study.

this study of average weights at age 37 as standard weights for all ages. Using this standard, the departure from average build for their age is much greater among the young overweights than it is among the old overweights. If age-specific average weights had been used as our standard, the difference between the mortality ratios of the young overweights and the old overweights would not have been so great.

COMPARISON WITH 1948 STUDY

The 1948 study mentioned earlier covered issues of 1920-45, exposed between the 1937 and 1947 anniversaries. A comparison of the results (for all ages combined) in that study and the present one is given in Table 2. The number of deaths is not shown in this table because all the groups are of substantial size (the smallest group has 85 deaths).

Table 2 illustrates the confirmation by the present study of the tentative indications found in the 1948 study that neither the improvement of the underweight mortality nor the deterioration of the overweight mortality continues after approximately the first 20 policy years. For the policy durations covered by both studies, the results of the two studies are remarkably consistent, especially when it is considered that there was no overlapping of the exposures. Both these points are more clearly brought out when the results for the 16th and later policy years are combined into (approximately) 10-year duration groups to eliminate some of the fluctuations that occur when 5-year groups are used (see accompanying tabulation).

DEVIATION IN POUNDS FROM STANDARD WEIGHT	1948 STUDY	1965 STUDY	
	Policy Years 16-27	Policy Years 16-25	Policy Years 26-34
-23 or more.....	81%	82%	87%
-22 to - 8.....	82	84	88
- 7 to + 7.....	97	96	99
+ 8 to +22.....	121	118	110
+23 or more.....	137	129	136
All weights.....	100%	100%	100%

It should perhaps be mentioned that the proportion of policies issued on extreme overweights (+43 lb. or more) was considerably higher in the 1948 study than in the present study, mainly because of different underwriting standards in effect prior to 1930. In the 1948 study, 0.9 per cent of the expected deaths was on these extreme overweights, as compared with only 0.1 per cent in the present study. This difference, however, would not have any significant effect on the comparability of the two studies.

ANALYSIS BY AGE AT ISSUE AND HEIGHT

Table 3, which is similar to Table IV in the 1948 study, shows the material divided into three broad height groups as well as age at issue and weight groups in order to bring out any mortality differentials that may be related to height. As in the 1948 study, there is no consistent pattern of mortality related to height. For all weights combined, the only significant feature is the excess mortality on issues at ages 15-34 in the short group. For these young ages at issue, the short group had the highest mortality in each of the three weight groups in the current study and in both the underweight and average-weight groups in the 1948 study. Other

TABLE 3

ANALYSIS BY AGE AT ISSUE AND HEIGHT

Issues of 1930-34 Exposed between 1947 and 1964 Anniversaries. Expected Deaths
Based on Actual Experience in Same Period for All Builds Combined. By Policies

HEIGHT GROUP*	ACTUAL DEATHS				RATIO ACTUAL TO EXPECTED			
	Age at Issue							
	15-34	35-49	50-65	All Ages	15-34	35-49	50-65	All Ages
	All Weights							
Short.....	213	379	241	833	115%	97%	99%	102%
Medium.....	849	1,606	654	3,109	98	103	100	101
Tall.....	551	684	231	1,466	98	95	101	97
All heights....	1,613	2,669	1,126	5,408	100%	100%	100%	100%
	More than 7 Pounds under Standard Weight							
Short.....	80	93	64	237	96%	78%	111%	91%
Medium.....	362	402	179	943	84	78	96	83
Tall.....	275	223	69	567	85	86	94	87
All heights....	717	718	312	1,747	86%	80%	99%	85%
	Within 7 Pounds of Standard Weight							
Short.....	72	124	96	292	117%	92%	86%	95%
Medium.....	262	495	197	954	105	104	89	101
Tall.....	126	191	71	388	95	91	96	93
All heights....	460	810	364	1,634	103%	99%	90%	98%
	More than 7 Pounds over Standard Weight							
Short.....	61	162	81	304	154%	118%	110%	122%
Medium.....	225	709	278	1,212	119	126	112	121
Tall.....	150	270	91	511	146	107	111	117
All heights....	436	1,141	450	2,027	132%	120%	112%	120%

* Short, 5' 0" to 5' 6"; medium, 5' 7" to 5' 10"; tall, 5' 11" to 6' 5".

than this there are no clear indications of significant differences in mortality related to height.

CONCLUSIONS

The results of the current study indicate that build at issue continues to affect mortality significantly even after 30 years. Considering the results of both the current study and the previous study, the following more specific conclusions appear to be justified:

1. The favorable mortality observed among underweights in the early policy years has continued through the higher durations studied, although there appears to be a tendency for the mortality ratios to increase slightly at the high durations.

2. The high mortality ratios of overweights have increased over the first 20 years or so but at the higher durations have tended to level off or, in the case of the moderately (8-22 lb.) overweight group, to diminish.

3. Mortality among short individuals insured at ages 15-34 has been somewhat excessive, regardless of weight. For other age groups, there is no consistent evidence of mortality variations related to height.

DISCUSSION OF PRECEDING PAPER

ERNEST J. MOORHEAD:

Publication of this sequel to a 1948 paper causes one to reflect that our underwriting knowledge might expand more rapidly if other actuaries would follow the example of Messrs. Blair and Haines in making repeated, diligent studies of an individual impairment. When every company dabbles in investigations of the whole range of observable impairments, the result may often be wasteful duplication of effort.

For the underwriting of build abnormalities, these authors' extended analysis to the thirty-fourth policy year adds usefully to our knowledge. But there is yet much to be learned. Actuaries should be aware that our techniques have recently been subjected to criticism by a student of the subject from outside our industry. Canadian actuaries surely know about this, since the criticism was emblazoned in the *Financial Post* under the banner headline "Insurance Mortality Figures Dead Wrong, Says U.S. Expert."

Our critic is Carl C. Seltzer, who is a research associate in the Department of Nutrition of the Harvard School of Public Health in Boston. His ideas are to be found in an article with the short title "Ponderal Index of Mortality," published in *New England Journal of Medicine* (CCLXXIV, No. 5 [February 3, 1966]). Dr. Seltzer voices two separate and independent criticisms. First, he recommends that greater attention be paid to variations in body-build structure and body composition and to the distinction between overweight and obesity. Second, he urges that, in considering mortality purely in terms of height and weight, we use an abstraction that he has named the "ponderal index." Ponderal index is the height in inches divided by the cube root of the weight in pounds. For adults, it will normally have a value in the neighborhood of 12.5–13.0; markedly overweight individuals would have a ponderal index of 11.6 or less.

With regard to Seltzer's first point, we can readily agree that differentiation within a particular height and weight category, according to body structure, is desirable to the extent that it is feasible. But Seltzer seems to give us less credit for awareness of this than actuaries and underwriters have earned. Practice for many years has been to grant more lenient underwriting treatment if, for example, chest measurement in relation to abdomen measurement is favorable.

It remains to be seen whether Seltzer's ponderal index really fits well

the mortality curve for overweights, and, if it does, whether this is a coincidence or a basic truth. But Seltzer seems to be of the mistaken opinion that life insurance companies assume a straight-line relation of increasing mortality ratio with increasing weight at each height level. That no such linear relationship exists or is assumed for rating purposes can readily be observed by charting the figures in the 1959 *Build and Blood Pressure Study*, or by differencing the rating debits for overweight in individual companies' underwriting manuals, or most clearly by studying the excellent Tables 4-8 in Mr. Edward A. Lew's paper "Some Implications of the Build and Blood Pressure Study, 1959," published in the 1960 *Proceedings* of the Home Office Life Underwriters Association.

HARRY A. WOODMAN, JR.:

In view of the fact that so few data have been published regarding mortality data on medically substandard lives at the longer durations, we thought it would be appropriate to include some statistics on this subject as an addendum to the informative experience on build presented by Messrs. Blair and Haines.

The accompanying tabulation covers the mortality experience of New

NEW YORK LIFE SUBSTANDARD ISSUES OF 1925-34
EXPOSED BETWEEN 1946 AND 1954 ANNIVERSARIES

MEDICAL IMPAIRMENT	RATIO OF ACTUAL TO EXPECTED DEATHS* FOR POLICY YEARS:					AVER- AGE RATING
	13-15	16-19	20-24	25-29	13-29	
Syphilis.....	170.8% (5)†	143.8% (22)	115.0% (37)	101.2% (16)	120.8% (80)	145.5
Rheumatism.....	206.3 (9)	175.0 (30)	138.8 (36)	134.5 (13)	154.1 (88)	148.3
Heart murmurs.....	165.8 (33)	191.2 (171)	183.4 (301)	145.3 (98)	176.9 (603)	173.6
Rapid pulse.....	183.2 (6)	218.4 (29)	151.8 (32)	148.6 (12)	172.9 (79)	167.0
Irregular pulse.....	66.2 (3)	210.4 (35)	145.6 (44)	121.9 (16)	151.9 (98)	151.3
Elevated blood pressure	216.3 (152)	213.1 (419)	199.9 (408)	193.6 (91)	206.6 (1070)	160.0
Pulmonary tuberculosis	35.7 (2)	120.8 (35)	101.9 (57)	135.1 (31)	110.2 (125)	162.7
Asthma.....	249.5 (4)	167.3 (10)	194.9 (19)	224.6 (8)	196.3 (41)	147.3
Bronchitis.....	465.9 (10)	238.6 (15)	249.2 (24)	97.1 (3)	245.8 (52)	154.3
Gall bladder disorders.....	155.7 (6)	157.1 (26)	169.6 (41)	113.8 (7)	157.7 (80)	150.4
Albuminuria.....	280.6 (23)	247.7 (93)	223.6 (144)	203.7 (47)	230.4 (307)	158.3
Kidney disorders.....	142.9 (7)	158.8 (29)	151.9 (42)	103.0 (10)	145.4 (88)	155.9
Female disorders.....	143.7 (6)	87.5 (13)	100.5 (21)	58.0 (4)	94.0 (44)	144.7
Family history of tuberculosis.....	162.9 (22)	120.8 (68)	109.5 (100)	115.1 (42)	117.4 (232)	149.6
Nonconformity.....	273.9 (15)	252.7 (48)	216.5 (59)	170.0 (16)	225.7 (138)	159.5
Eye disorders.....	232.1 (10)	176.0 (29)	144.2 (32)	146.9 (9)	162.9 (80)	137.8
Ear disorders.....	76.9 (4)	153.2 (27)	124.9 (31)	97.4 (7)	125.8 (69)	137.2
Deformity.....	295.5 (6)	270.4 (20)	254.1 (29)	227.5 (9)	258.1 (64)	144.6
Spinal curvature.....	353.3 (3)	193.7 (7)	140.9 (10)	282.0 (8)	194.5 (28)	147.2

* Expected deaths based on Basic Table, 1935-54, policy years 16-19 (1959 *Build and Blood Pressure Study*, I, 7).

† Figures in parentheses indicate the number of deaths.

York Life medically substandard issues of 1925-34, exposed between 1946 and 1954 anniversaries. The study included single major medical impairments only and was limited to impairments symptomatic of a chronic

or degenerative condition; medical histories of acute disorders were not studied.

Because the data were not subdivided according to the size of the rating, the average rating was calculated so as to provide a basis for comparing the mortality experience. The basis selected for expected deaths was the mortality rates for durations 16-19 used in the 1935-54 Basic Table developed for the *1959 Build and Blood Pressure Study*. The period of exposure for durations 16-19 in the 1959 study was 1950-54. This duration group and exposure period are comparable to those in the study presented herein.

For many of the impairment groups studied, the mortality ratios are quite close to the average ratings. Those that produced mortality ratios significantly higher than the average ratings are elevated blood pressure, asthma, bronchitis, albuminuria, nonconformity, deformity, and spinal curvature. Those that produced mortality ratios significantly lower than the average ratings are syphilis, pulmonary tuberculosis, family history of tuberculosis, and female disorders. The results for most impairments show a decrease by duration.

These results tend to confirm previous findings in studies of mortality at the early durations.

(AUTHORS' REVIEW OF DISCUSSION)

B. FRANKLIN BLAIR AND LAWRENCE W. HAINES:

The two discussions presented have not been directed to the content of our paper but rather have presented additional information suggested by its subject. It is valuable to have this information published in the *Transactions*.

Mr. Moorhead has called attention to a criticism of the techniques used by actuaries in studying mortality by build. All actuaries would probably agree that theoretically "greater attention [should] be paid to variations in body-build structure." However, the practicalities of the situation seem to overbalance the theoretical desiderata. In the absence of data on body-build structure for insured lives, it is surely better to study the variations in mortality by height and weight alone than not to study mortality variations by build at all.

Mr. Moorhead refers to the "ponderal index" used by Seltzer. This index has been traced back as far as 1898 to an Italian anthropologist, Ridolfo Livi. He seems to have used this index in the form:

$$\text{Ponderal index} = \frac{100 \sqrt[3]{\text{Body weight}}}{\text{Stature}}.$$

This form has body weight in the numerator and thus gives larger indices as weight increases. Therefore, this form seems preferable to that used by Seltzer, where body weight is in the denominator with the anomalous result that for a given height the index decreases as weight increases.

The data presented by Mr. Woodman are helpful in extending our limited knowledge of mortality on impaired risks at the higher durations. This represents a continuation of the sharing of information on special underwriting risks, which over the years has been characteristic of New York Life.

CONCEPTS OF ADEQUACY IN PENSION PLAN FUNDING

FRANK L. GRIFFIN, JR.*

Two of the primary reasons for funding an employee retirement plan are (1) to level out what would otherwise be sharply rising future costs and (2) to promote employee security by accumulating funds to back earned pensions. The first of these makes more likely the indefinite continuation of a plan, and the second gives assurance that accrued pensions will be paid regardless of the future fortunes of the company.

In addition to long-range contribution stability, a company may seek other financial objectives in a funding program, such as the safety valve of short-range contribution flexibility. Here, again, there is a presumption of greater employee security through the improved prospect of plan permanence afforded by such flexibility.

As a general rule, therefore, there need be no basic conflict between a company and its employees in the funding of a retirement plan. Both company and employee objectives tend to enhance employee security in its broadest sense, and differences are primarily those of emphasis. Company stress is apt to be on flexible pension contributions within the framework of stable long-range costs, while employee stress (when not on increased benefits) is likely to be on the funding of accrued benefits.

The purpose of this paper is to examine certain funding guidelines which reflect both viewpoints in a logical and consistent fashion. It is concerned with principles as well as with the broad impact of actuarial assumptions and cost methods but is not concerned with specific application of the latter.

In conducting this examination, there are a number of questions for which we should seek better answers than are to be found in our formal literature. For example, what degree of funding might a company logically seek to accomplish over a period of years? How should one define the "actuarial liabilities" which should ultimately be funded, or, otherwise stated, how should one determine the amount of funds which need to be accumulated in order to provide a given measure of employee benefit

* The author wishes to acknowledge contributions made to this subject by his associate, John Hanson, F.S.A., who has developed numerous ideas and techniques in this area.

security? How are these answers affected by differing conditions relating to the employee group?

In seeking answers to these questions, it is necessary to clarify certain concepts, some of which, unfortunately, have long been couched in terms tending to confuse their real significance. For example, actuaries have become so accustomed to viewing an actuarial liability as something which of necessity ought to be amortized that they frequently overlook (1) the substantial difference in concept of actuarial liability according to its method of computation and (2) the resultant immateriality of its amortization in some circumstances. We climb the mountain because it is there—never thinking that a path around may serve just as well or even better in those cases in which the mountain is man-made and has been raised to an arbitrary height.

AN APPROPRIATE FUNDING OBJECTIVE

As has been pointed out by Preston Bassett and others (*TSA*, XVI, paper and discussion on pp. 318 ff.), a basic premise is that if a pension fund equals the value of accrued benefits the plan is fully funded. This is quite obvious in the case of a terminating plan, and is also true in the case of a continuing plan if one may assume that future contributions will exactly fund benefits accruing in the future.¹ This test, based on the one-sum cost of accrued benefits, formed the basis of "benefit security ratios" reported by the author in the *Proceedings of the Conference of Actuaries in Public Practice, 1964-65*, Volume XIV, in a forum entitled "Government Regulation of Private Pensions—Present and Future."

The most significant fact pointed up by the accrued-benefit-funding test is that the actuarial cost methods in use in a majority of plans are such as to fund benefits more rapidly than they accrue, even though no specific provision is being made for the amortization of the unfunded liability by the cost method in question. As a result, when cost methods of such a type (projected-benefit cost methods) are employed, the *unfunded costs of accrued benefits* may automatically become funded, at least in substantial part, by payment of the so-called IRS minimum contribution. Whether there is such automatic funding depends on the maturity of the group in question and whether it is stationary, increasing, or decreasing in size. However, pension plans would invariably reach an *overfunded* position if liabilities by projected-benefit cost methods were fully amortized. Thus, complete benefit security can be provided by funding less than the full amount of the actuarial liabilities by some commonly

¹ A truism in the case of most plans but later qualified.

used actuarial methods. This fact is pertinent to any consideration of the establishment of rigid rules for amortizing such liabilities.

If accrued-benefit costs were all there were to the funding of a pension plan, one might ask why all funding should not be determined in accordance with accrued-benefit cost methods. Aside from the greater contribution flexibility and greater mechanical ease of handling certain types of pension formulas by projected-benefit cost methods, the answer to this question goes back to the question of stabilizing long-range costs and, in turn, perhaps is influenced by what has long been a cardinal principle in individual life insurance funding, namely, the avoidance of rapidly rising costs under schemes such as step-rate assessment insurance. In short, the principle of substituting costs of the "level" type for costs of the "step rate" type has become so nearly second nature to most actuaries that the differences between funding problems applied to individuals and to groups are sometimes overlooked.

Be this as it may, the fact remains that costs of accrued pension benefits determined by the unit-credit method will generally increase until a constant, weighted average age is reached, and such an increasing trend is accentuated whenever there is a multiple weighting of costs with advancing age (as occurs, for example, under an "integrated" plan or a plan based on final average salary). A goal of "leveling" the aggregate costs for the group thus remains a valid objective for the employing company; on this basis alone, projected-benefit cost methods have ample justification.

As between those actuaries who feel that funding should not exceed the value of accrued benefits and those who wish to emphasize the leveling of future cost requirements, the funding concept introduced below should do little violence to either school. Essentially, this concept offers protection against the possibility that an assumption stated earlier—to the effect that future contributions to a plan will exactly fund future benefit accruals—may not be valid if the trend of future costs for the group should be markedly upward.

Assuming that the principal concern of pension funding is employee security and that the two principal guarantees of such security under a retirement plan are (1) the accumulation of funds to back accrued (or vested)² benefits and (2) the stabilization of long-range costs, the following logical long-range funding objective may be postulated. Such a long-range objective, to be reached over a reasonable period of time, would be

² Use of the phrase "accrued (or vested) benefits" is intended to imply a choice, namely, that it is not necessarily appropriate to fund *all* accrued benefits regardless of whether vested.

the larger of (a) a fund sufficient to provide in full all accrued (or vested) benefits *if the plan were to terminate* or (b) a fund sufficient (in the absence of further benefit increases) to maintain a stable contribution level *if the plan were to continue*.

The foregoing funding objective recognizes the obvious fact that a plan will either terminate or continue. A fund sufficient to satisfy either possibility would be the maximum that would ever need to be accumulated, except possibly for the convenience of a company in order to permit variation in its annual contributions.

Different actuarial assumptions may be appropriate and different cost methods are required in the proper evaluation of *a* and *b*. These questions are covered in the material which follows.

"ACTUARIAL LIABILITY" V. "COST OF ACCRUED BENEFITS"

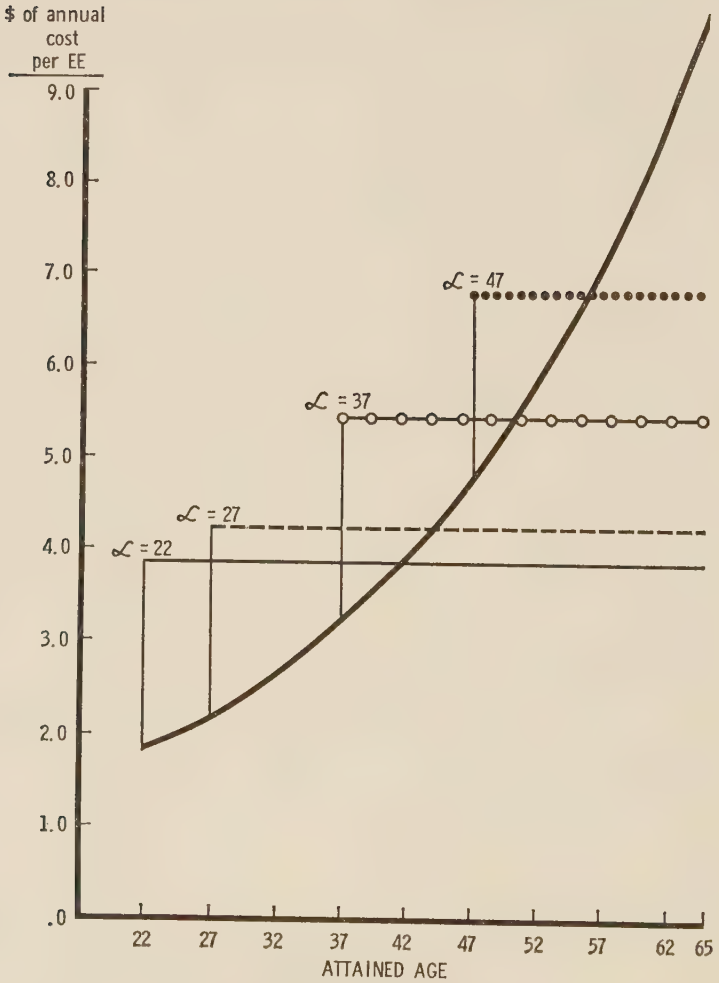
At the risk of belaboring the obvious, let us examine for a moment the difference between projected-benefit cost methods and accrued-benefit cost methods in the matter of cost and fund-accumulation structures. For illustrative purposes we will use the methods commonly known as entry-age-normal and unit-credit, respectively.

Chart I shows the curve of unit costs by attained age and superimposes the level entry-age-normal costs, at various entry ages, to provide the total projected benefit at retirement. At entry age the capitalized value of the level costs is, of course, identical with the capitalized value of the increasing unit costs payable to retirement date (here assumed to be age 65). If assumptions are the same on each basis, and exactly experienced, it is obvious that, if the "level costs" are contributed, a higher accumulation will result at any point prior to retirement date than if the increasing unit costs are contributed and that at retirement date the accumulations would be identical. Moreover, if a higher yield is experienced than that assumed, there will be a considerably greater excess of the level-cost accumulation at any point, which now will encompass all points beyond retirement as well as before retirement.

Chart II illustrates for a single entry age the comparative accumulations when assumptions are the same on each basis and exactly experienced. Since the lower curve represents the *cost of accrued benefits*, it is apparent that a liquidation or termination occurring midway in the accumulation process would result in a surplus if the level costs had been contributed throughout. The higher the yield in relation to that assumed, the greater would be this surplus. (The latter assumes that gains are either spread or used to reduce any unfunded liability rather than reflected in an immediate reduction in contributions.)

CHART I

COMPARISON OF ONE-SUM COSTS FOR A UNIT OF BENEFIT AT ATTAINED
AGE, WITH LEVEL COST FOR PROJECTED BENEFIT FROM ENTRY AGE
(Benefit at age 65 equal to \$1 a year for each year of service)

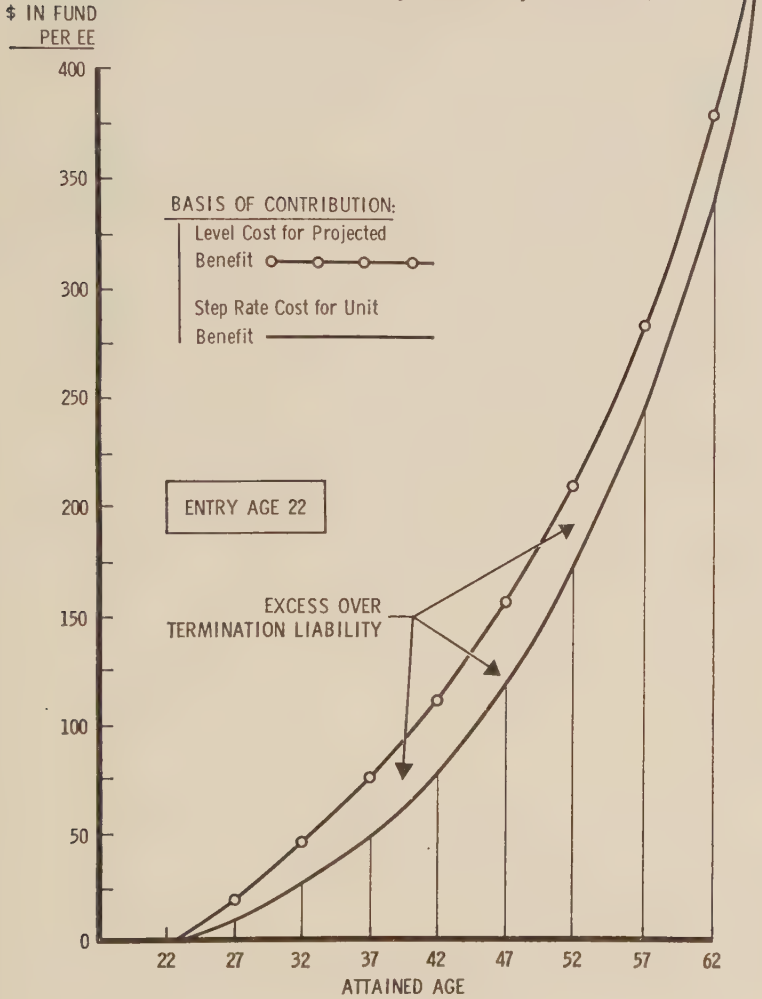


BASIS: WYCO 1965 projected mortality table with
3½% interest, no turnover, no loading

CHART II

COMPARATIVE FUND ACCUMULATIONS FROM ENTRY AGE ACCORDING TO BASIS OF CONTRIBUTIONS

(Benefit at age 65 equal to \$1 a year for each year of service)



A pension plan covering an employee group consists, of course, of a "mix" of different attained and entry ages. Therefore, the comparative cost- and fund-accumulation structures for such a group consist of composites of the relationships illustrated by Charts I and II. Since there is invariably an excess of level-cost accumulations over increasing-cost accumulations in individual cases, the same must be true for the group. Moreover, with the tendency to use conservative actuarial assumptions this excess will be greatly magnified by favorable experience.

This is exactly the pattern which has developed under many private pension plans during the past decade or two. The combined effect of using projected-benefit cost methods and of experiencing substantial investment gains, particularly under plans invested substantially in equities, is, for example, the primary reason for the very favorable "benefit security ratios" found in the Wyatt Company study, referred to above. This should not have been surprising; what is a little surprising is that actuaries should have done so little to publicize the significant distinction between actuarial liability and cost of accrued benefits, which has so profound an effect on measurements of employee benefit security.

To complete this set of illustrations, Charts III and IV show the actuarial liabilities by the entry-age-normal method and the one-sum costs of accrued benefits for four different employee distributions projected over a 30-year period. All the distributions have been equated as to initial work force, and the plan of benefits is also the same. The distributions illustrated may be characterized as follows:

Distribution I.—An initially immature work force, gradually growing in size.

Distribution II.—An initially immature work force, remaining stationary in size.

Distribution III.—A relatively mature work force, remaining stationary in size.

Distribution IV.—A relatively mature work force, shrinking in size.

The liabilities shown in Chart III for Distribution I, by the entry-age-normal method, have been indicated on two different assumptions: (1) assumptions identical to those used in computing the one-sum costs of accrued benefits—the lower of the two nearly parallel solid lines—and (2) less conservative assumptions (i.e., turnover discount introduced). It may be noted that the actuarial liabilities uniformly exceed the one-sum costs of accrued benefits. There would cease to be a difference only when there are no longer any active employees, at which time the actuarial liability would reduce to the one-sum cost of accrued benefits.

CHART III

"ACTUARIAL LIABILITIES" versus ONE SUM COST OF ACCRUED BENEFITS

Projection for Employee Distributions I and III

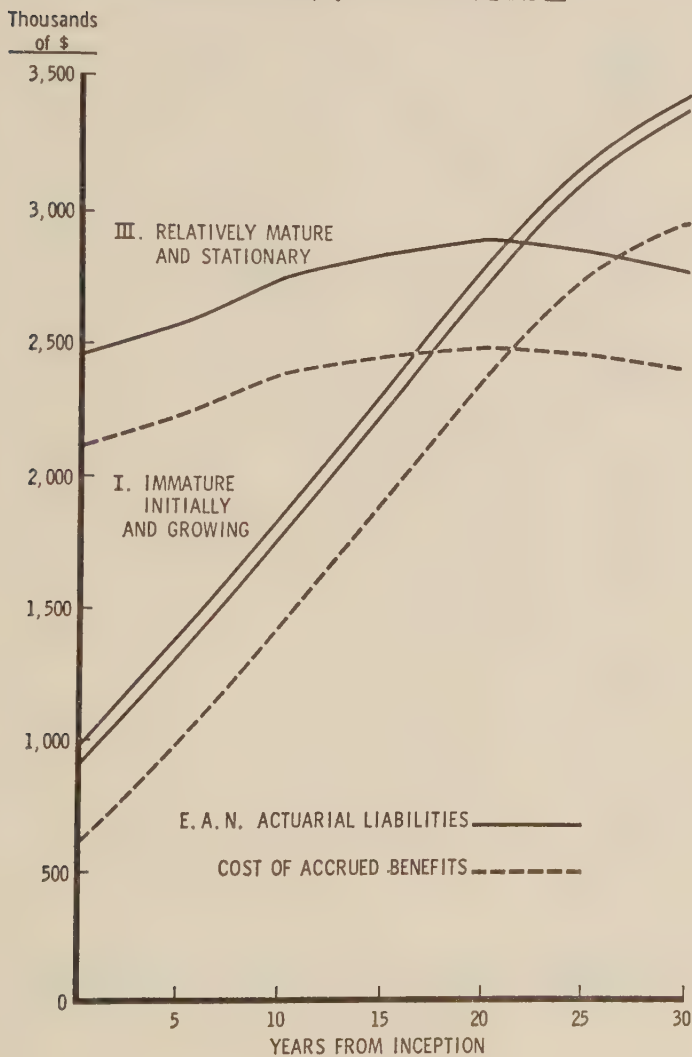
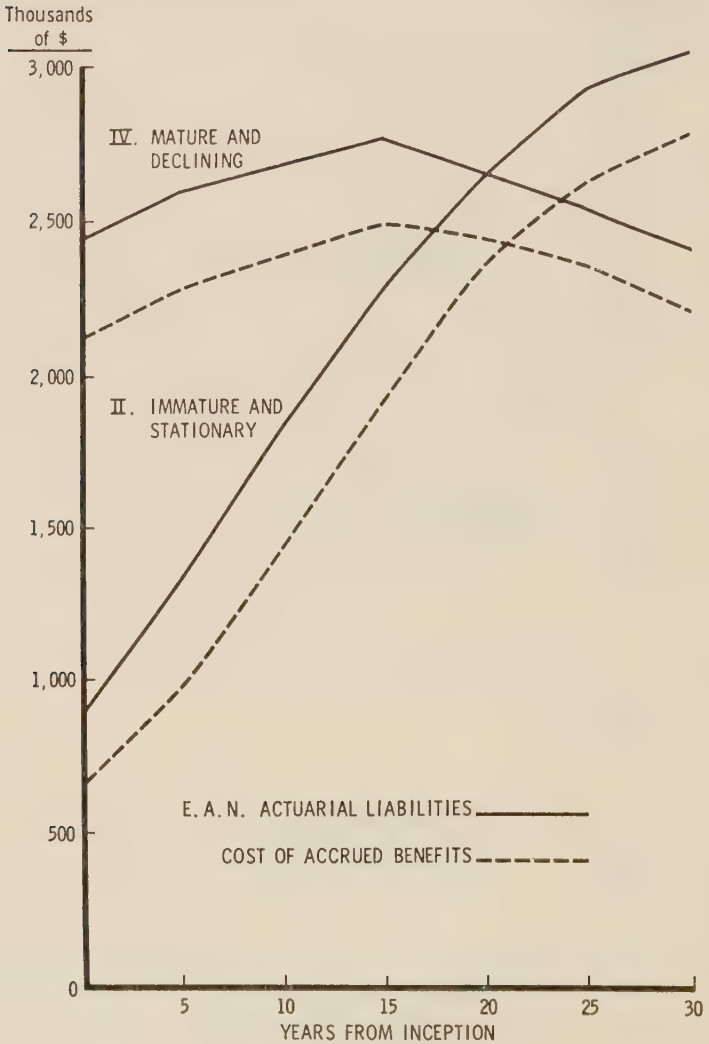


CHART IV

"ACTUARIAL LIABILITIES" versus ONE SUM COST OF ACCRUED BENEFITS

Projection for Employee Distributions II and IV



THE CONTINUING-PLAN OR "GOING CONCERN" CONCEPT

Speaking generally, a "going concern" contribution might be defined as a level contribution in perpetuity (expressed as a percentage of payroll or a per capita cost, as appropriate) whose capitalized value is equal to the value of benefit payments in perpetuity, considering both present and future generations of employees. There are, of course, difficulties with this approach. For one thing, it is impossible to predict the effect of economic and technological factors on the size of the group or its applicable payroll at some distant future point.

To minimize these difficulties, actuaries have found it convenient to adopt the concept of a mature group from which to determine the ultimate cost situation once maturity is reached. In the case of a well-established organization, the assumption of a constant work force moving toward maturity in its age distribution is probably as defensible as any other approach. However, in practice, none of the ideal conditions of a mature group (either initially or in the ultimate) will ever be found.

Despite the nonexistence of stationary and mature groups, the concept of maturity may serve a useful purpose as a limiting value in a pension projection. Since the actuary is confronted with a group of unknown future age distribution and size, practical considerations usually dictate that his valuation be limited to the group of employees existing on the date of valuation, without allowance for new entrants of the future. However, if the actuary wishes to compare the results of a valuation by any particular cost method, with a long-term projection of pension payouts or terminal funding requirements, *considering future new entrants*, the reasonableness of that method in producing long-range stabilized costs for a going concern will be tested by the comparison (see Chart V).

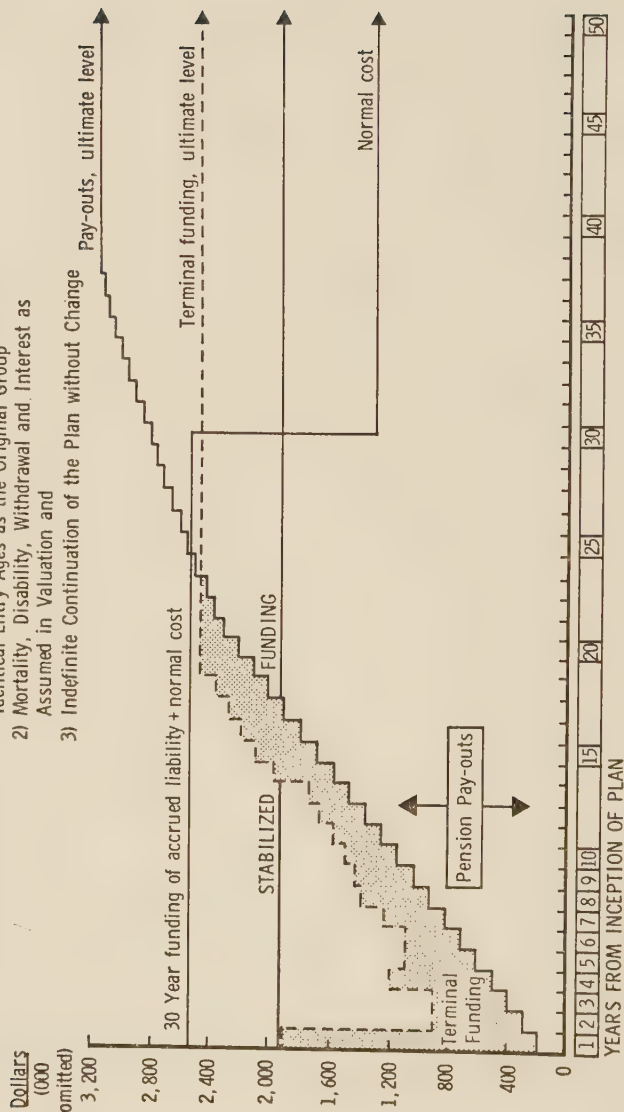
From the viewpoint of a continuing plan, the funding requirements developed by an actuarial valuation should take into account not only the past but also the future requirements on a basis which will tend to equalize long-range trends in the age distribution. The structure of projected-benefit cost methods adapts them to the requirements of a going concern by striking a balance at a given moment of time between (a) the existing funds and anticipated future income and (b) anticipated future disbursements. The entry-age-normal method does this ideally for a mature and stationary work force, but its suitability frequently extends beyond that point.

The manner in which, and the conditions under which, the entry-age-normal method will afford a reasonable representation of the long-range contribution requirements for an organization expecting to continue in business indefinitely may be illustrated by the following example. The

CHART V

PROJECTION OF PENSION CONTRIBUTIONS, PAYOUTS AND TERMINAL FUNDING REQUIREMENTS

- Assuming: 1) Constant Work Force Supported by New Hires with Identical Entry Ages as the Original Group
2) Mortality, Disability, Withdrawal and Interest as Assumed in Valuation and
3) Indefinite Continuation of the Plan without Change



figures are presented on the basis of a conventional valuation, the present values of both benefits and normal costs being with respect to present employees only, without allowance for any future new entrants.

1. Present value of future benefit payments	\$42,000,000
2. Present value of future normal costs	16,000,000
3. Balance = Gross actuarial liability	26,000,000
4. Funds accumulated	7,000,000
5. Balance = Unfunded actuarial liability	19,000,000
6. Normal cost (in addition to any payments toward unfunded actuarial liability), used in determination of item 2	1,300,000

Put in the form of a balance sheet, the asset items would be items 4, 2, and 5, usually in that order, and the balancing liabilities would be represented by item 1.

Continuing the illustration, *if future new entrants were introduced in such a manner as to maintain a constant normal cost in future years*, the balance between the asset and liability figures would not be disturbed. For example, assuming new hires sufficient to maintain a constant work force and at the same entry ages as the group being replaced each year (one of several possible assumptions), the normal cost developed by the initial valuation would be paid in perpetuity and the benefits ultimately payable to new entrants of the future would be exactly met by their normal costs. This being the case, items 1 and 2 of the above tabulation would be increased by exactly the same amount, leaving all other figures unaffected.

Thus one of the virtues of the entry-age-normal method is that, even though future new entrants are not specifically considered in a valuation, the result will be as good an approximation of the long-range level of costs for a continuing organization of constant size as it is possible to furnish. In the opinion of many actuaries, the best estimate of the long-range stabilized pension cost is the normal cost plus interest on the unfunded actuarial liability, computed by the entry-age-normal method on assumptions appropriate to a going concern.

For a group of constant size, a cost determined in this manner would support the plan indefinitely, and, even though (in the absence of substantial investment or other experience gains) it would never amortize the unfunded actuarial liability developed by this particular cost method, it might fund a substantial portion of the *unfunded cost of accrued benefits*. For groups of increasing or decreasing size, other factors enter the picture and affect the amount of contribution which is required to establish funding adequacy.

PROTECTION OF BENEFITS ON PLAN TERMINATION

As a practical matter, the objective of protecting accrued (or vested) benefits in the event of plan termination is one which, in the absence of a satisfactory system of insuring unfunded costs, must be sought through a program of amortizing initial unfunded costs over an extended period of years. Without attempting to suggest any guideposts, since companies in varying circumstances would find it necessary to tackle the problem differently, it may be noted that the President's *Cabinet Committee's Report* has recommended that companies should "amortize fully all *accrued liabilities* over a period that roughly approximates the average work life of employees but not more than 30 years" (*italics mine*). The quotation appears to use "accrued liabilities" in the sense of "actuarial liabilities" as used in this paper.

Whatever the period of years should be, it is extremely important that those becoming involved in this question be made aware that the amount to be amortized should be the *unfunded cost of accrued benefits* rather than the accrued (or actuarial) liability by a particular cost method. Moreover, these accrued benefits need not necessarily encompass all accrued benefits; while this remains a suitable question for discussion, such benefits might properly be limited to vested accrued benefits.

It is interesting to note from the Wyatt Company "benefit security ratios" developed by the author that the cost of *all* accrued benefits on date of valuation (plans ten or more years old) was only about 63 per cent of the actuarial liabilities for those plans using projected-benefit cost methods. In other words, even if this ratio were to reach 80 per cent at some time in the future, compulsory full funding of actuarial liabilities would actually mean the compulsory accumulation of funds 25 per cent in excess of the amount to insure complete security of all accrued benefits, whether vested or not.

A valuation of the cost of accrued benefits, of course, involves different factors and different treatment of benefits than a valuation designed to measure the stabilized long-range contribution on a going-concern basis. For one thing, entirely different assumptions may be appropriate. For another, a special set of rules may be required for the determination of accrued benefits in the event of plan termination, especially in the case of "final average salary" plans or plans where the pension formula does not provide a one-to-one correspondence between years of service (or earnings) and the benefit. Many actuaries are already familiar with these problems, either through their own researching of the question or through the Pension Research Council's recent memoranda relating to its proposed study of benefit-security ratios.

A CONSISTENT APPROACH

To review the funding objective outlined earlier, the maximum fund that *ever* need be accumulated to support the principal pillars of employee security would be the larger of (1) that required to provide the one-sum cost of accrued (or vested) benefits, computed on a plan "close-out" basis, or (2) that which would have resulted from the payment, year after year, of the normal cost plus interest on the initial unfunded actuarial liability by the entry-age-normal method, computed on assumptions appropriate to a going concern. In either case realistic assumptions are appropriate, and market value would be the indicated basis for comparison of assets with the theoretical fund.

(Note that this maximum required fund bears no particular relationship to the gross actuarial liability by the particular cost method and assumptions which the actuary may have selected for his valuation; in fact, it would not result specifically from the amortization of any such cost. Note also that where this maximum fund consists of [1], the goal would generally be reached only after a reasonably long period of years, during which the *unfunded costs of accrued [or vested] benefits* were to be amortized; meanwhile, the fund would consist of the larger of [2] and the partially amortized value of [1].)

As a concession to good engineering, for greater stability we might logically add a third pillar, consisting of a financial cushion (or safety factor), solely for the company's convenience in varying contributions to accord with fluctuating earnings. Additional sums thus set aside might best be considered in relation to a year's "normal" contribution, independently of the so-called adequate fund for security purposes described in the preceding paragraph.

The tests of fund adequacy set forth above appear to the author to be consistent with the realities of pension plan funding. On the one hand, a plan may continue indefinitely, and funding should be sufficient to meet requirements on a going-concern basis. On the other hand, the plan may terminate, and it would then seem desirable to protect employee security, to the extent possible, with respect to accrued benefits. Few would disagree that reasonable provision should be made, over a period of years, to establish such employee security, but in the process current stockholders should be protected against the making of redundant pension contributions which bear no close relationship to these realities.

Funding procedures commonly followed in the past, when not accompanied by an alternative valuation to measure accomplished benefit security, have in some instances led to unwitting overfunding of a plan.

This condition has frequently been accentuated by favorable investment experience, a combined circumstance which led to favorable benefit-security ratios under plans included in the Wyatt Company study. Where overfunding on a plan-termination basis has occurred, future contributions may properly be reduced to the minimum for a going concern, or lower, as long as the favorable situation continues. Further amortization of the actuarial liability would be redundant.

Referring to the two separate aspects of this test of funding adequacy, it may be noted that an organization of undisputed permanence (such as a federal or state government or possibly a large corporation in a basic industry) might properly forego measuring up to (1). However, *any* organization, even the federal government with respect to its employees, should meet the requirements of (2), if for no other reason than to avoid misleading those who foot the bill (taxpayers, stockholders, or employees) as to the true long-range costs of the benefits which have already been adopted. This perhaps classifies as "truth in labeling."

SOME ILLUSTRATIONS

There are many possible variations in the techniques which might be followed in testing the adequacy of a given level of funding. One approach which has been suggested by John Hanson (and possibly others) is to schedule contributions on a stabilized going-concern basis and to make additional contributions only if projections indicate these to be necessary in order to achieve the desired degree of funding of accrued (or vested) benefits within a selected time interval.

Let us say that the interest rate assumed in the calculation of the going-concern contribution is $3\frac{1}{2}$ per cent and that the reserve desired to cover accrued benefits (say, thirty years hence) is also computed at $3\frac{1}{2}$ per cent interest. Then, if we should expect to obtain a higher yield (including capital gains) during the next thirty years, we would test our funding on a projected basis, using the higher yield. This method is illustrated in Table 1 and in the text following.

The most favorable funding characteristics are, of course, exhibited by groups with immature age distributions and which are growing in size. For such groups, the going-concern contribution or "minimum" entry-age-normal contribution (normal cost plus interest on unfunded liability) is, under the particular circumstances illustrated, sufficient to fund all or a substantial part of the cost of accrued benefits over a reasonable period. Security in the event of plan termination is gradually and automatically being established under such circumstances.

The least favorable funding characteristics are exhibited by groups

which are initially mature (or overmature) and which are declining in size. For such groups, contributions well in excess of the going-concern contribution are required in order to fund any part of the cost of accrued benefits. Provision for security in the event of plan termination is of primary concern in these cases, and special provision must be made therefor.

For either Distribution I or II, if a yield of $4\frac{1}{2}$ per cent or better were expected during the first thirty years and excess interest were used to fund

TABLE 1

FUNDING OF ACCRUED BENEFITS ACCOMPLISHED BY PAYMENT OF GOING-
CONCERN CONTRIBUTION (NORMAL COST PLUS INTEREST)*
ACCRUED BENEFITS COMPLETELY FUNDED AT ALL AGES ABOVE AGE SHOWN

END OF YEAR	DISTRIBUTION I (IMMATURE, GROWING)		DISTRIBUTION II (IMMATURE, STATIONARY)		DISTRIBUTION III (MATURE, STATIONARY)		DISTRIBUTION IV (MATURE, DECLINING)	
	Fund Earns $4\frac{1}{2}$ Per Cent	Fund Earns 6 Per Cent	Fund Earns $4\frac{1}{2}$ Per Cent	Fund Earns 6 Per Cent	Fund Earns $4\frac{1}{2}$ Per Cent	Fund Earns 6 Per Cent	Fund Earns $4\frac{1}{2}$ Per Cent	Fund Earns 6 Per Cent
5.....	51	51	51	51	76	76	78	78
10.....	49	47	50	48	74	74	77	76
15.....	48	43	50	45	73	72	78	76
20.....	48	All	50	All	73	71	78	75
25.....	43	All	50	All	74	70	81	76
30.....	All	All	39	All	74	69		79

* Funding of accrued benefits determined on the basis of $3\frac{1}{2}$ per cent interest, no turnover, plan-close-out basis; going-concern contribution computed on the entry-age-normal method at $3\frac{1}{2}$ per cent interest with a turnover discount; fund actually earns $4\frac{1}{2}$ per cent or 6 per cent, as shown in illustration, and excess interest is used to fund rather than to reduce contributions.

benefits rather than to reduce contributions, there would be no need to contribute anything in excess of the "minimum" EAN contribution in order to accomplish a satisfactory amortization of the cost of accrued benefits, even when the latter are computed on the conservative basis of $3\frac{1}{2}$ per cent interest, no turnover, and full vesting of all benefits regardless of age or service.

For Distribution III or IV, on the other hand, no improvement whatever would be achieved in the funded status of accrued benefits by payment of the "minimum" EAN contribution, even if a $4\frac{1}{2}$ per cent yield were experienced and credited in full during the first thirty years. As a matter of fact, even with a 6 per cent yield, no improvement would be achieved under Distribution IV. For these situations, therefore, additional provision must be made for amortizing the cost of accrued benefits.

Another possible approach would be to proceed directly to a projection—say, for thirty years—in order to find the fund objective at that time if accrued benefits were to be fully funded on a plan-close-out basis. The computations needed to determine a “level” contribution might then be summarized as follows:³

Objective: Fund equal to one-sum cost of accrued benefits for all employees, 30 years hence—Distribution II

1. Projected fund required at end of 30 years, to provide all accrued benefits to employees then in service or retired (on a $3\frac{1}{2}\%$ interest, fully vested basis)				\$2,762,759
	ASSUMED YIELD DURING FIRST THIRTY YEARS			
	$3\frac{1}{2}$ Per Cent	$4\frac{1}{2}$ Per Cent	6 Per Cent	
2. Present value of item 1	\$ 984,311	\$ 737,657	\$481,024	
3. Present value of expected benefit payments first 30 years.	706,358	572,210	421,312	
4. Total present value required	\$1,690,669	\$1,309,867	\$902,336	
5. Level annual contribution to accomplish objective.	88,815	76,952	61,843	

For comparison, the following are the initial contributions determined on several alternative valuation bases, all at $3\frac{1}{2}$ per cent interest, for Distribution II:

Entry-age-normal, without turnover discount, 30-year amortization	\$99,000
Entry-age-normal, without turnover discount, “minimum” basis	83,436
Unit-credit, without turnover discount, 30-year amortization basis	78,287
Entry-age-normal, <i>with</i> turnover discount, “minimum” basis	75,374
Unit-credit, without turnover discount, “minimum” basis	67,064

The first basis develops a considerably higher contribution than is needed on a “level” basis, even if only a $3\frac{1}{2}$ per cent yield is obtained. If slightly over a $4\frac{1}{2}$ per cent yield is obtained for thirty years, any of the first four bases develops a sufficient contribution; and, if a $5\frac{1}{2}$ per cent yield is thus obtained, any of the five bases would suffice to produce the full $3\frac{1}{2}$ per cent reserve for accrued benefits at the end of thirty years.

Other techniques may, of course, be employed. These may involve a series of shorter-range goals, whereby the objective is a gradual lowering over a period of years of the age to which full funding of accrued benefits extends. A relatively simple procedure would be to perform dual purpose valuations: (1) according to the method and assumptions selected by the

³ Illustration covers a plan providing flat benefit per year of service. For a plan with benefits based on pay, it would be more appropriate to relate future contributions to future payrolls, in the usual manner.

actuary as appropriate for a continuing plan and (2) on a plan-close-out basis to determine the extent of funding of accrued benefits. Where further progress toward the latter is desired, a short-term projection (five or ten years) will indicate the degree, if any, to which the contribution on basis (1) should be increased.

In connection with periodic actuarial valuations, it may become common practice to project the funded status of a plan after a given number of years, based on different levels of contribution. Given an objective of accumulating, over a specific period of time, the full one-sum cost of accrued benefits for a particular group of employees (perhaps all employees, but more likely all employees who will then have fulfilled certain age and service requirements comparable to "vesting"), the employer can be kept advised of the contribution level required to reach that objective.

CONCLUSION

In the light of repeated benefit improvements under many pension plans in the past, which frequently have had the effect of extending the amortization of unfunded liabilities, the development of a measurement of funding adequacy may formerly have appeared somewhat academic. Yet, in a very real sense, it is not.

Aside from the possibility that benefit improvements will tend to level off in the future and thus neutralize the effect of the foregoing argument, it is important that actuaries clearly define concepts of adequacy in pension plan funding so as to avoid the possibility of confusion. Increasing pressures for regulation, which come largely from sources unfamiliar with actuarial distinctions and the requirements for benefit security, place a responsibility on actuaries to provide appropriate definitions and to direct these pressures into the right channels.

DISCUSSION OF PRECEDING PAPER

CHARLES L. TROWBRIDGE:

Mr. Griffin's "Concepts of Adequacy in Pension Plan Funding" is certainly a welcome addition to actuarial literature. My purpose in discussing his paper is twofold: (1) to demonstrate some of his conclusions (and to indicate certain reservations about others) by use of two mathematical models taken from earlier actuarial literature and (2) to offer alternates to the two specific suggestions that he makes with regard to funding patterns consistent with his stated objective.

Perhaps Mr. Griffin's greatest contribution lies in the clear statement of this objective:

Such a long-range objective, to be reached over a reasonable period of time, would be the larger of (a) a fund sufficient to provide in full all accrued (or vested) benefits *if the plan were to terminate* or (b) a fund sufficient (in the absence of further benefit increases) to maintain a stable contribution level *if the plan were to continue*.

This statement, and the background material preceding it, set forth a duality in the pension funding problem which has *not* been sufficiently emphasized. A rational and responsible approach to the pension funding problem will never be developed unless both of these two criteria—one related to employee benefit security, the other to employer cost stability—are recognized. Mr. Griffin deserves our commendation in furthering this recognition.

It is no doubt unfair to the analytical portion of Mr. Griffin's paper to reduce it to oversimplified statements. I do so (in the next paragraph) only to help point up his case and as a base from which to indicate what few reservations I have as to his conclusions. I trust that he will forgive me if I seem to make free with the *form* of his argument, hoping he will feel that I have not really distorted his meaning.

Assuming that I interpret Mr. Griffin correctly, he is telling us essentially these three things:

1. Projected-benefit methods normally produce a supplementary liability substantially in excess of the one-sum cost of accrued benefits.
2. The *complete* amortization of the projected-benefit supplemental liability has inherent in it an element of "overfunding"—and some *partial* amortization will often meet the two-pronged objective without bringing about this overfunding.

3. Nonamortization of the initial projected-benefit supplemental liability *may* result in a funding pattern essentially satisfactory by the *a* test (particularly if the initial work force is immature or if actuarial gains can be counted upon to help), and it, by its very nature, tends to satisfy the *b* test.

In the following analysis of these three statements, I will make liberal use of the mathematical models published as a part of my 1952 paper appearing in Volume IV of the *Transactions*.¹ Table II of that paper traces a mature population, stationary in size, thereby somewhat akin to Mr. Griffin's Distribution III. Table IV represents an initially immature work force stationary in size and hence similar in nature to Mr. Griffin's Distribution II. It should perhaps be pointed out that the 1952 models represent a work force subject to substantial nonvested withdrawal at the younger ages, whereas I am not sure in this regard in the Griffin illustration.

First, as to the validity of statement 1, it is entirely clear that the projected-benefit (entry-age-normal) supplemental liability is always larger than the accrued-benefit (unit-credit) supplemental liability, which is by definition the one-sum cost of accrued benefits, provided the actuarial assumptions are the same. To be absolutely precise, we must except the trivial case where the active work force has become zero. We must also except the case (which we will meet later in this discussion) where approximate or artificial entry ages have been used and the case where benefits accrue more rapidly in early years than in the later. Since none of these exceptions apply to the 1952 models, we find a pattern for them (see Chart I) similar to that shown in Mr. Griffin's Charts III and IV.

The one-sum cost tends to run at about 82 per cent of the EAN actuarial liability on the 1952 initially mature model and increases from 65 to 82 per cent by duration for the initially immature. One gets a similar impression from Mr. Griffin's charts. The fact that his Distribution III does not produce entirely horizontal lines means only that Distribution III is, as advertised, only "relatively mature." Perhaps it does not, initially, have a full complement of retired lives. The comparative steepness of the graphs for his Distribution II seems to indicate that his 1966 model is initially *more immature* than my 1952 immature model.

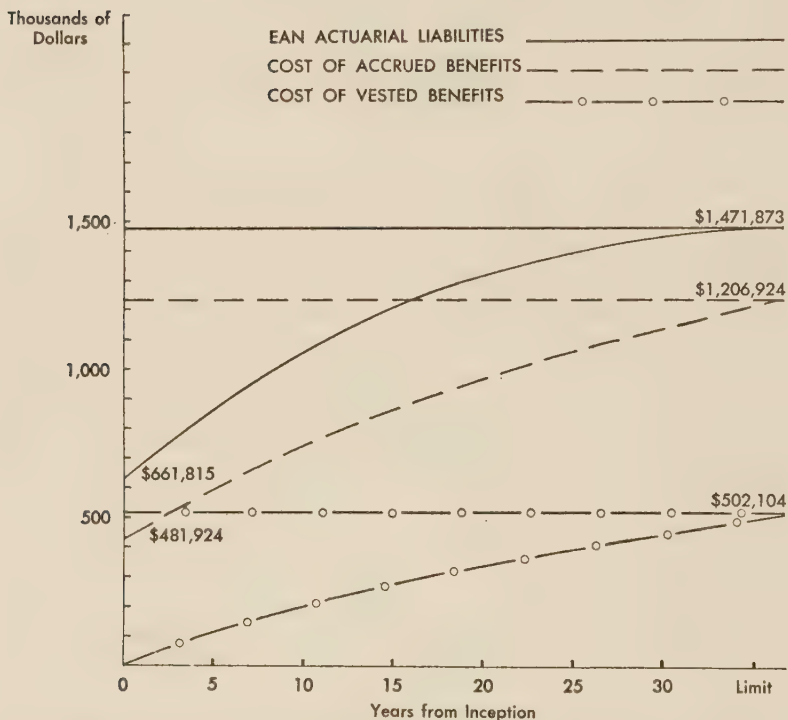
Let us now examine statement 2. If the one-sum cost of accrued benefits is only 65-85 per cent of the EAN actuarial liabilities, does it then follow that full amortization of the latter will necessarily cause overfunding as measured by the former? It would seem so—*except* for the change in actuarial assumptions that may be appropriate for the "close out" calculation. To provide for the one-sum cost of all accrued benefits

¹ "Fundamentals of Pension Funding," *TSA*, Vol. IV, 1952.

CHART I

"ACTUARIAL LIABILITIES" VERSUS ONE-SUM COST OF ACCRUED BENEFITS AND VESTED BENEFITS

MODELS FROM 1952 PAPER



Horizontal lines: Table II initially mature; *upward sloping lines:* Table IV initially immature. Both models stationary work force; both involve discount for withdrawal; no vesting until age 65.

discounted only for mortality and interest requires about \$1,415,000 for the mature model (or the ultimate phases of the immature). Thus the full amortization of the EAN actuarial liability results in overfunding of only 4 per cent *if* the criterion for employee security is the full funding of accrued benefits *with continuation of employment not a condition for receiving these benefits*. Another model with higher withdrawal rates might indicate no overfunding at all. (I can conceive of close-out situations in which future benefits might be subject to continued employment with the same or a successor employer; but the more likely situation seems to involve "vesting" at plan termination.) On the other hand, if the criterion is only the one-sum cost of *vested* benefits (Mr. Griffin deliberately gives a choice here), the overfunding would be very apparent. In these earlier models no vesting takes place until retirement at age 65 and only slightly more than \$500,000 is required.

It is my feeling that Mr. Griffin would admit this one reservation regarding statement 2. He somewhere states that "different actuarial assumptions may be appropriate . . . in the proper evaluation of *a* and *b*." I assume that this disappearance of the withdrawal discount on plan termination is at least one of the things he had in mind. It should be noted that there may be other assumption changes appropriate for plan termination that work in the opposite direction. In particular, any assumption as to future salary increase also tends to disappear.

Let us go on to statement 3. Again, the 1952 models help—this time to determine the degree of employee security arising from funding by entry-age-normal cost plus interest.

For the mature model we find that the so-called IRS minimum funding is algebraically and numerically equivalent to pay-as-you-go—hence no funding whatsoever occurs. Clearly, for a group initially mature (and this implies a mature group of pensioners as well as of workers) normal cost plus interest gives no employee security whatsoever.

On the other hand, as I think was clearly stated and illustrated in the 1952 paper, this minimum sort of funding *does* produce substantial funds in the much more meaningful immature model. Essentially, the *initial* supplementary liability is 0 per cent funded, all increases in the supplementary liability are 100 per cent funded, and the eventual result is partial funding, all as measured by the projected-benefit EAN actuarial liability. Mr. Griffin points out that under some circumstances partial funding of the projected-benefit supplementary liability may result in full funding of the accrued benefit to date.

Specifically normal cost (on EAN) plus interest eventually builds up \$810,000 in the immature model illustrated by Table IV. Retired life

benefits (the only fully vested benefits in this model) require \$502,000. The \$810,000 will therefore not only fully fund the retired lives but will also provide about 44 per cent of the accrued benefits for active lives (provided the discount for withdrawal is still appropriate) or, expressed another way, will fund accrued benefits down to about age 59. On the other hand, if our concept of plan termination causes us to lose the withdrawal discount, the 44 per cent reduces to 34 per cent. The age 59 stays unchanged, because this model has little or no discount for withdrawal above age 55.

In comparing the results obtained above with Mr. Griffin's results, we must be aware of an important difference in the handling of actuarial gains. The 1952 models were constructed in such a way that the actual experience as to mortality, interest, and withdrawal was identical to that assumed. There are, therefore, neither actuarial gains nor losses in the model; and the results obtained in no way rely upon actuarial gains to increase the degree of employee security.

Mr. Griffin's illustrations do assume actuarial gains (at least in the interest element), and the extra interest is not employed to reduce contributions. You can think of his actuarial cost method as entry age normal *with initial supplementary liability funded each year to the extent of that year's actuarial gains*. From this viewpoint, he is not really illustrating normal cost plus interest but instead is illustrating a relatively slow but accelerating amortization of the projected-benefit supplementary liability through the nonrecognition of what eventually become substantial actuarial gains. My guess is that both the $4\frac{1}{2}$ per cent and the 6 per cent illustrations on page 61 would eventually (on Distributions I and II) fund not only all accrued benefits but in time the full EAN actuarial liability. Mr. Griffin has not given us $3\frac{1}{2}$ per cent illustrations. Were he to do so, we could better separate out the effect of actuarial gains from the effect of the maturing of the groups.

For Distributions III and IV, on the other hand, poor results will be obtained no matter what rate of interest is earned. For a truly mature group (with a full complement of retired lives), we have seen that normal cost plus interest builds up no funds. Extra interest on zero principal is still zero. The reason why the ages given for Distributions III and IV in Mr. Griffin's table are not all 100 or above must be that the initial group was not really mature in the sense used in the 1952 mature model.

All in all, the above analysis fairly well substantiates statement 3. Certainly the extent to which entry-age-normal cost plus interest will (all by itself) fund accrued benefits is a function of the degree of immaturity in the initial distribution. The theoretical range of funds built up under normal cost plus interest is from absolute zero (for the initially mature

situation) to the full supplementary liability for the actuarial cost method employed (for the employee group all newly hired). In the past, some actuaries have expressed mild displeasure with my 1952 paper because I chose to classify normal-cost-plus-interest funding in Class I. Their point is well taken. I might better have indicated a *range* of class designation. Consistent with this, entry age normal, interest only could have been called Class I-IV; unit credit, interest only, Class I-III.

In addition, *if* conservative actuarial assumptions are employed and *if* the resulting actuarial gains are applied against the unfunded liabilities, then the security of the pension expectations may well approach 100 per cent in time. I must admit to a little surprise in seeing an illustration based on absorption of actuarial gains in the unfunded supplementary liability. Although Mr. Griffin's method for adjusting actuarial gains or losses (letting the adjustment for actuarial gains or losses be reflected in the *time* when supplementary liabilities are amortized) seems reasonable and practical, it is not clear that it would fit the usual interpretation of IRS regulations. Clearly, however, it is the method commonly employed with respect to "unrealized" capital gains or losses on common stocks.

I see nothing to quarrel with in his demonstration that interest-only funding is quite satisfactory from an employer-cost viewpoint. Only for the declining work force, where the fixed "interest" contribution would be an increasing charge per active employee, would its cost-stability characteristics be open to any question.

This brings me to the details of how one might set up a simple and sensible funding pattern, based on partial rather than complete amortization of the projected-benefit supplementary liability, in order to avoid Mr. Griffin's overfunding. Mr. Griffin suggests at least two ideas, both based on reaching a predetermined goal n years into the future. His two suggestions appear to be actuarially equivalent (or at least approximately so). Both appear to involve a "shifting of gears" at the end of the n th year.

Another that might appeal to some can be expressed as follows: If we can determine that entry-age-normal cost plus interest funding will eventually be about $\$D$ short of the employee-security criterion, we can *artificially raise the entry ages* of the initial employee group so that the classically calculated entry-age-normal supplementary liability is reduced by $\$D$, the present value of future normal costs increased by $\$D$. This procedure is much more legitimate than it appears at first glance (and, incidentally, shows up much of the artificiality in the entry-age supplementary liability concept). Then, if the funding proceeds on the resulting normal cost (increased) plus the resulting interest (decreased), the employee-security goal will eventually be reached, assuming (as Mr. Griffin

must in his methods, too) that a reasonably accurate projection can be made to determine $\$D$. On this score, however, I am not quite as hopeful as Mr. Griffin appears to be. Too many assumptions, particularly as to new lives, are needed.

I have previously suggested still another possibility in a 1963 paper.² Although the family of methods there suggested does not appear to involve any concept of supplementary liability, both Mr. Nesbitt and Mr. MacKinnon pointed out (in the discussion) that the actuarial cost method suggested is closely akin to a partial amortization of the entry-age supplementary liability. As a matter of interest, note, from Table II of the 1963 paper (which uses the same model as Table IV of the 1952 paper), how well a k of just over 4 per cent would approximate the present value of accrued benefits—or a k of 1.7 per cent would approximate the present value of vested benefits. The trick, of course, is to determine the k 's a priori.

I hope that Mr. Griffin's paper will stimulate thoughtful discussion. His paper is clearly aimed at certain non-actuarial circles, as well as at pension actuaries. Other actuaries can help by contributing intelligent discussion of the basic issues that he raises.

FREDERICK P. SLOAT:

Mr. Griffin merits high commendation for a very fine paper that organizes in uncomplicated and lucid style some important concepts in pension funding. It analyzes both by word and picture the distinction between various actuarial liabilities that have been indiscriminately tagged as past-service costs over the years. It is very timely in setting forth so well the influence of varying employee distributions.

In the early days of the development of pension plans in the United States, the most important funding vehicle in use was the group annuity contract using allocated funding, that is, the application of each dollar paid to the purchase of an annuity.

The accrued-benefit cost method was the natural way to utilize the group annuity approach, and it was variously referred to as the single-premium, unit-credit, or step-rate method. The latter described the effect of the method with respect to any particular employee. This step-rate characteristic created concern over the potential rising costs of a plan, unless it was for a mature group where the effect of deaths, turnover, salary changes, and retirements and of new entrants at young ages was such as to keep the aggregate costs reasonably level from year to year.

² "The Unfunded Present Value Family of Pension Funding Methods," *TSA*, Vol. XV, 1963.

After deposit administration and trust funding came into use, in order to eliminate the purchases of units of annuity and their subsequent cancellation when employees left the plan, the accrued-benefit cost method was often applied to them. Under federal income tax rulings, experience gains had to be recognized in the current or next year. This was usually not too serious with group annuity dividends because the insurance companies declared only part of any gains at one time. With nonallocated funding, however, investment and turnover gains could be quite substantial in some years, and it was very desirable to be able to spread their effect.

The popularity of the projected-benefit cost methods was prompted, in part, because, with any one of these methods, it is possible to spread experience gains and, in part, because it produces a larger amount of past-service cost. This increases the employer's funding flexibility by widening the spread between a 10 per cent payment in any year and interest only (or less, if part of the past-service cost has previously been funded). The fact that such a method eliminates the potential increasing cost resulting from advancing ages of the employees, thus tending to stabilize long-range costs, is an important reason but has not always been the most impelling reason. Further, as Mr. Griffin points out, this method is more suitable to certain types of pension formulas.

The projected-benefit cost method can be used for two very different reasons. Because of its basic nature, together with the ability to spread net gains, it is used to give a more uniform and consistent incidence of cost from year to year. But, because of its increased flexibility, it is also used to permit wider voluntary swings in amounts funded from year to year than under the accrued-benefit cost method.

Mr. Griffin has pointed to two developments over the years that have obscured the direction of pension funding. One has been the favorable investment climate, whereby most pension funds have grown much more rapidly than could have been anticipated, even without the usual actuarial conservatism for long-range pension financing. The other has been the pattern of plan improvements and amendments which has required frequent alterations of the funding status of a plan.

As he demonstrates in his paper, the entry-age-normal actuarial liabilities are always in excess of the cost of accrued benefits if computed on comparable bases and assumptions, so that, if the past service is being funded, there will be a point at which all accrued benefits will be funded. During the preparation of Ernest L. Hicks' *Research Study*, I discussed with him the impact of his conclusion that past-service cost should be taken into expense over a reasonable period. I felt that in some situa-

tions it may be undesirable or unnecessary to continue such charging of the remaining past-service cost when a total amount equal to the value of all accrued benefits has already been charged. He pointed out that, under the conclusions of his study, an employer can change the actuarial cost method used for accrual of pension costs in any year but it would require a consistency exception by the auditor. Thus, a change to the accrued-benefit cost method when the value of all accrued benefits had been charged to expense would require him to make such an exception. While this would not require the auditor to qualify his audit opinion, the inclusion of such consistency exception might appear to weaken it.

If the forthcoming opinion of the Accounting Principles Board on accounting for costs of pensions should state that the charge to expense may include either amortization of the past-service cost or interest only on unfunded past-service cost, this question might still arise. It might be desirable to charge the amortization of past-service cost only up to the point where the value of all accrued benefits is covered and to charge interest only thereafter. Further, the opinion might call for such amortization up to the point where the value of all vested benefits is covered.

Each of the usual actuarial cost methods is founded on a basic concept that makes it a rational procedure. As Charles L. Trowbridge set forth in his paper entitled "The Unfunded Present Value Family of Pension Funding Methods" (*TSA*, Vol. XV), there is a great range of possible actuarial cost methods, only a few of which can be so independently formulated. It might be very useful if a method using the long-range objective described by Mr. Griffin were deemed an actuarial cost method. It belongs in the family of projected-benefit cost methods, but that terminology presents a problem since the modification, or inside limit, is determined by the accrued-benefit cost method. It could be referred to as a "projected-benefit cost method with supplemental liability and with accrued- (vested-) benefit limit." For older terminology, we would add the expression "with accrued- (vested-) benefit limit," for example, "entry-age-normal with accrued-benefit limit." The added expression would refer to the provision that, when the amounts funded (or charged to expense) were sufficient to cover the value of all accrued- (or vested-) benefits at that time, further amounts would automatically be the normal cost and interest on the unfunded past-service cost. It should be noted that the actuarial assumptions and application of the pension formula for this limiting feature may differ from those for the main application of the actuarial cost method. This is one reason why it might seem to constitute an actuarial cost method rather than just a procedural step under one of the other methods.

PRESTON C. BASSETT:

I am pleased that Frank Griffin has found the time to set forth his thoughts on the adequacy of pension plan funding.

It is particularly important for us to recognize and to be sure that others recognize, to the extent possible, the distinction that Frank makes between actuarial or accrued liability, as normally used for valuation purposes, and the value of accrued benefits. Full funding objectives have been met when the assets valued on a realistic basis are equal to the value of the *accrued* credited benefits, also valued on a realistic basis. This determination is separate and distinct from the determination of actuarial or accrued liabilities under the usual actuarial methods and assumptions.

I was pleased to see set forth in this paper the long-range objective, to be reached over a reasonable period of time, to be

the larger of (a) a fund sufficient to provide in full all accrued (or vested) benefits *if the plan were to terminate* or (b) a fund sufficient (in the absence of further benefit increases) to maintain a stable contribution level *if the plan were to continue*.

In this quotation, it should be noted that Frank does not state that this objective should be reached by any rigorous pattern of contributions but only that it be reached over a period of time. This is important, and I wish to take a minute to emphasize this fact in discussing the adequacy of pension plan funding.

Adequacy may also include the pattern of how the company reaches the ultimate objective. The actual funding of the plan to meet the objectives must take into account how much the company may desire to contribute at any particular time. This is usually expressed by the flexibility that the company has in determining the contributions in any year or period of years. It is this flexibility that has permitted many companies to adopt and improve pension plans and fund them to the favorable level that has been reached today. A company should be informed of its cost and understand the obligations it must meet, but it should be free to meet these obligations in its own way and in its own time. Only in this way can we expect to see a continuation in the fine growth of private pension plans, both for new plans and improvements in old plans.

Flexibility in determining the current level of contributions is vitally important for most companies. We have seen it in the casualty insurance companies, where contributions were decreased in years of sizable losses such as occurred when Hazel and Betsy created such havoc. We have seen it in utility companies who have made extra contributions when the

revenues were unusually large and profits greater following cold winters or hot summers. We saw it in the case of steel companies who reduce contributions in the years in which they had prolonged industrial strikes. Flexibility in the amount of company contributions has been an important factor in the companies' financial planning.

It is fine to set forth long-range objectives and goals based upon level of funding and adequacy of the fund, but let it remain with the company to plot its own course to reach these objectives. I do not believe that it is necessary for the government, the accounting profession, actuaries, or someone else to establish rigid rules which call for set contribution levels in order to meet a desirable objective which can be reached under current regulations and practices. A company should be left free to establish its own patterns to reach these goals and even to change them as conditions change in the future. Current practices have done a fine job in the past, and I see no need for any change in this area.

There is one other comment that I would like to make in regard to adequacy of funding. I do not believe that an adequate standard of funding, as defined by Frank Griffin, can be firmly established without also considering the plan design. It seems to me that the funding objective of a plan that is negotiated with the union every few years to provide greater benefits might require a very different approach than a unilateral plan with benefits based on the average of earnings during the years just prior to retirement. In the former case, we can assume that there will be increasing liabilities in the future, whereas, in the latter case, future improvements may already have been taken care of by the formula itself. Thus, the period to reach the objective for the union-negotiated plans might be a shorter period, since we can reasonably expect additional increments to be added. This can be expressed in another fashion by asking the question "How do we set goals for adequacy of funding for an inadequate plan that must be amended in the future?" Which is more important: (a) to fund benefits adequately to protect employees in the event the plan is terminated or (b) to make the benefit adequate on a thinner funding basis so that retired employees will have better standards of living, on the assumption that the company will continue to operate in the future? It may not be the job of the actuary to answer these questions, but it is certainly the actuary's responsibility to be sure the client understands the implications of these questions.

JOHN K. DYER, JR.:

Mr. Griffin's timely paper should be of particular value to pension actuaries in inspiring them to consider more objectively and critically

the actuarial cost methods that they may adopt on behalf of their clients. At the same time, hopefully, the paper may serve some important collateral purposes—such as to demonstrate the fallacy of public criticisms of actuaries for their “uncertainty” and to give some logical direction to the thinking of those who would attempt to legislate actuarial soundness.

I agree that the author’s dual funding objective, which forms the keynote of the paper, does, in fact, represent a logical and desirable funding objective in many cases. I was a little disturbed to note that as the paper unfolded there seemed to be an increasing tendency to employ this objective as though it were the ultimate truth rather than simply one test of funding adequacy, appropriate in some but not all circumstances.

The concept of “accrued benefits,” seemingly a simple one, has many pitfalls. Only in the case of a plan which defines clearly and precisely the partial benefits that each employee may expect to receive in the event of plan termination can this concept be interpreted with certainty. In many, and perhaps most, cases the emphasis upon a continuing plan is so great that the full benefit upon retirement is the only one of which the employees are really aware. In this situation an employee’s partial benefit expectation at any point short of retirement age might be quite different from the “accrued benefit” that the actuary might determine, from a termination formula or otherwise. This raises some doubt as to whether the funding of such an “accrued benefit” can generally be considered as “full funding,” especially in the case of a plan which promises benefits based on final pay. To illustrate, there have been cases in which a final pay plan has been replaced by a career pay or profit-sharing plan and where the past-service credits representing rights accrued under the old plan have been established on a basis more favorable than would result from the application of the old plan formula to the earnings rate at the point of change.

A similar doubt arises as to the adequacy of the “stable contribution” test when applied to a career pay plan. Many actuaries feel that, for a career pay plan, a higher level of funding should be maintained than for a final pay plan—in effect, funding against the greater risk that the benefits of the career pay plan may become inadequate. Thus if “normal cost plus interest” is satisfactory as one of the tests of funding adequacy for a final pay plan, it could be maintained that for a career pay plan the corresponding test should be based upon a higher level of funding (assuming equivalent actuarial assumptions, of course).

Finally, I should like to underline the rather obvious points that “complete benefit security” is an ideal but quite unattainable objective and that “overfunding” is a technical condition determinable only by refer-

ence to some arbitrary measure of benefit security. It is from these truths that our prized funding flexibilities are derived. If we are to retain these flexibilities, it is incumbent upon us to avoid statements which, in or out of context, could be used in support of statutory limitations on funding levels.

PAUL H. JACKSON:

Mr. Griffin, in his excellent paper, takes up the fundamental question of "How much money is really necessary?" in pension funding. The paper is of particular importance in view of the current considerations being given to the accounting aspects of pension costs. The paper sets out the clear-cut logic underlying one particular funding method and demonstrates that the funds developed are adequate for benefit security, even though that method may not measure up to what accountants or others might wish to suggest as "minimum requirements."

The paper states, "As a general rule, therefore, there is no basic conflict between a company and its employees in the funding of a retirement plan." There are, of course, many situations in which the interests of the company and its employees will coincide. In a very real sense, however, there are basic differences between the interests of (a) the stockholders of a company, (b) the customers of the company, and (c) the employees. An increase in compensation costs, whether pensions or direct pay, probably results in decreased profit to current stockholders or in higher prices to current customers. Therefore, a mandatory increase in pension costs through the enforcement of unrealistic minimum funding regulations may result in a near-future depressing of the level of pension benefits or of pay rates, in a current depressing of profits, or in a current increase in price to the customer, with any such changes being counterbalanced by opposite changes in the distant future. The "general rule" stated is not, however, fundamental to either the argument or to the conclusions drawn in the paper.

Chart II of the paper illustrates the excess fund developed under a projected-benefit method with a level cost over an individual's working career as opposed to the fund that would be developed under a step-rate cost method funding only the accrued liabilities. In practice, this excess may arise from two sources: (1) a basic actuarial excess that results from the use of level costs for funding to replace the actual costs, which have an increasing pattern, and (2) an excess that may arise out of the use of actuarial assumptions for determining projected benefits that produce values larger than the termination liability. Under plans with career average formulas or flat benefits, the second item of excess does not normally arise.

On the other hand, if a final pay plan is fully funded on the unit-

credit method with a realistic salary scale and modest withdrawal rates, then a surplus may emerge at plan termination simply because the accrued benefits for funding purposes are based on a projected final average pay discounted for some withdrawal, while the benefits actually granted at plan termination would be based on current pay or a recent past-average pay. Of course, it would be possible to base close-out benefits on a projected final average by applying the salary-scale factor used for funding to current pay rates. This is rarely done directly, however, and possibly the IRS would not permit such an artificial increase in benefits. Most, if not all, of the illustrations in the paper are based on a flat benefit per year of service, so that the "unnecessary" excess funding arises only from the first source, that is, the excess funds built up by the use of a level premium. Where benefits are based on final pay, the entry-age-normal contribution with full amortization could produce a much larger degree of excess funding than the example in the paper because of this second source. And, for integrated plans with excess benefits or stepped-up benefit rates, the excess funding from this second source is greatly increased. Thus the examples used may actually understate the case.

The paper suggests that there has been a tendency in the past to use conservative actuarial assumptions. This is no doubt true over all, but, looking to the individual assumptions, the salary-scale factors have quite generally been inadequate. In the past ten years or so, actuarial losses arising from salary increases in excess of salary-scale assumptions have been less than the excess-interest gains for many mature pension funds. It remains to be seen whether this relationship will hold true in the future. If wage inflation were to occur in a period of falling stock prices, as is currently the case in some European countries, and possibly here too, the inadequacy in salary-scale factors will not conveniently be offset by realized and unrealized capital gains. Realism in salary-scale factors appears even more desirable when minimum adequate funding, as suggested in the paper, is followed.

Where an employer anticipates an interest yield of 6 per cent during the first thirty years of his pension fund, a valuation based on a $3\frac{1}{2}$ per cent interest assumption could not be considered realistic (unless there are offsetting elements in the form of nonconservative mortality, turnover, or salary-scale assumptions). Under such circumstances, even the payment of normal cost and interest only will result in deliberate plan funding based on anticipated actuarial gains. The use of realistic assumptions for valuation will eliminate such anticipated actuarial gains, and then the funding must rely on either the immaturity of the group or on specific additional payments over the going-concern level cost, or both.

When accountants propose minimum and maximum limits on the pension contributions that will be accepted for accounting purposes and when the IRS imposes further minimum and maximum limits for tax-deductibility, it becomes ever more likely that a given employer will be forced into several pension valuations, that is, a formal valuation for tax purposes, possibly another for accounting purposes, and, almost certainly, yet another conducted on realistic assumptions for management consideration. In connection with the latter type of valuation, the use of current market value for assets may be justified, but a measure of asset value that has a greater degree of stability than current market value seems desirable for the tax and accounting valuations.

The paper suggests a third reason for pension funding, namely, to provide a financial cushion that is built up as a safety factor so that a company can vary its pension contributions with fluctuating earnings. Not only do company earnings fluctuate with the economic cycle, but they are likely to depend on the relative maturity of the company and the industry. The rapid growth of companies in some industries and the concurrent decline of companies in other industries suggest that reasonable funding requirements should permit a company to pay more into its pension fund during the period when profits are at their peak in order to permit a decrease in pension costs in the company's declining years without necessitating a cutback in benefits. Unfortunately, the accountants would apparently prefer to require uniform payments from all organizations regardless of their youthfulness or senility. And, in much the same manner, there can be a temptation for actuaries to follow the funding procedure suggested in the paper for all companies. Corporations need self-discipline in their pension funding just as an individual needs self-discipline if he "buys term insurance and saves the difference." But, carrying this analogy one step further, just as an accountant should not force the individual to purchase ordinary life insurance, neither should an accountant force a corporation to build up full net level premium reserves for pensions when Mr. Griffin's "Illinois method" clearly develops adequate funds.

There are many actuaries who will personally prefer heavier funding for most of the plans that they work on, and many employers may wish to put more into their pension plans than the minimum described in Mr. Griffin's paper. If there must be regulation, however, be it direct or indirect, all concerned would seem to profit by having the widest possible range in "acceptable" pension contributions. Mr. Griffin has made an excellent case for the right of some companies to pay less than normal cost and thirty-year amortization. At the same time, it is entirely possible

that some organizations, perhaps newly established or rapidly growing, can find valid justification in adopting a pension plan on a pay-as-you-go basis, on a terminal-funding basis, or on some other funding basis that is even less strict than the basis suggested by Mr. Griffin. By denying such companies the right to embark on a pension scheme unless they pay amounts which actuaries, accountants, or others feel are "desirable," such companies may merely defer the adoption of a formal pension program. And, if IRS should ever adopt rules that deny the employer a tax deduction for his payments to a pay-as-you-go plan on an informal basis, these firms may simply shrug their shoulders when their old-timers are no longer able to work, keep them on the active payroll for a few months, and then drop them. Whether the personal heartache that results from such a procedure will be offset by the added benefits gained by regulation and soundness is questionable.

Some actuaries have apparently already concluded that a pension plan which is underfunded is worse than no pension plan at all. It is difficult indeed to argue with a logic that sparkles with such a hard gemlike flame—unless, that is, one happens to be the particular individual who is now receiving no pension at all simply because his ex-employer was unable to meet the stiff requirements for an absolutely sound plan.

Mr. Griffin's paper raises the interesting question of whether accountants or others could possibly justify forcing employers to amortize what amounts to an imaginary liability merely for the sake of comparability or consistency. Going a bit further, surely a question could be raised as to whether a formal pension fund with a positive current balance really constitutes a crime against society because the funds on hand are less than a theoretical number derived by actuarial formulas and acceptable funding principles but based in the final analysis on mere assumptions regarding the likelihood of future happenings which, for those of us who are scientifically oriented, must be considered as indeterminable.

Mr. Griffin's paper illustrates funding methods that come closer to consulting actuarial practice than anything in the extant literature. He is to be congratulated on a clear-cut exposition of the very important aspect of adequacy in pension funding.

WALTER SHUR:

Mr. Griffin has made a real contribution in identifying clearly the two basic purposes of funding a pension plan and in pointing up the difference between amortizing the unfunded cost of accrued benefits and the unfunded accrued (or actuarial) liability.

It was noted in the paper that an organization of undisputed per-

manence, such as the federal government, might properly forgo measuring up to the single-sum cost of accrued benefits but should meet the requirement of stabilizing long-range costs. In February, 1965, the President appointed a Cabinet Committee on Federal Staff Retirement Systems to review, among other things, the financial soundness of these systems. The funding recommendations of this Committee's report, as they apply to the important Civil Service Retirement System, would appear to be of interest in the light of Mr. Griffin's paper and in the light of the President's report on private pensions.

Two important factors related to the Committee's funding proposals are (1) it has been estimated that, without improved funding, the Civil Service Retirement and Disability Fund would vanish in about twenty-five years and (2) the Committee recommended a number of liberalizations³ in the program which would increase the unfunded accrued liability by about \$3 billion and the normal cost by about 1 per cent. With these changes, the unfunded accrued liability would be about \$45 billion (against a fund of about \$15 billion), and the normal cost would be about 14½ per cent.

The Committee made three basic recommendations with regard to improved funding:

1. The total of employee and government contributions should be equal to normal cost, and future benefit liberalizations that increase the normal cost should be reflected by correspondingly increased contribution rates.

2. Future increases in unfunded accrued liability resulting from benefit liberalizations, general wage increases, and so forth, should be amortized over a thirty-year period by direct government appropriations to the fund, in accordance with an arithmetically increasing amortization schedule. For example, the \$3 billion increase in unfunded accrued liability resulting from the Committee's recommendations would be financed by contributing approximately \$12 million in the first year, \$24 million in the second year, \$36 million in the third year, and, finally, \$360 million in the thirtieth year. The Committee noted that this arrangement "avoids large accumulations in the early years."

3. The Committee recognized that there were strains on the fund not covered by the first two recommendations, principally failure to pay interest on the existing unfunded accrued liability and payment directly from the fund of automatic increases in retired benefits because of increases in the consumer price index. It was recommended, therefore, that Congress make automatic payments

³ The most interesting of these proposed liberalizations is one that provides that an employee who is entitled to benefits under the Civil Service plan will receive the greater of (1) the regular benefit provided by the plan or (2) the benefit that would have been paid under social security if federal service had been considered covered employment under the social security system.

to the fund (without the need for current appropriations, that is, in the same manner that interest is paid on the public debt) whenever the fund would otherwise decrease from the prior year and whenever the fund would otherwise be less than the accumulated value of all employees' past contributions.

It is interesting that the Committee based a part of its proposal on the concept of maintaining the fund at least equal to employees' equities. As Mr. Griffin pointed out, and I would agree with him, there is no real necessity for this in a plan backed by the taxing power of the federal government. It was undoubtedly felt that, while going only part way, this would be an easier concept to sell than more difficult actuarial concepts, such as payment of interest on the unfunded accrued liability.

The real significance of the foregoing is difficult to evaluate when we recognize that the fund, which consists entirely of United States government bonds, is nothing more than formal recognition of government obligations to the retirement system, and the \$45 billion unfunded accrued liability is nothing more than failure to recognize formally an obligation that is just as real. As Mr. Griffin noted, however, appropriate funding is required, if for no other reason than to avoid misleading those who foot the bill (in this case, the taxpayer). This problem was discussed in considerable detail in my paper on the subject a few years ago (*TSA*, XVI, 265) and needs no further elaboration here.

ABRAHAM M. NIESSEN:

There is nothing more damaging to the public image of the actuarial profession than the disagreement among actuaries regarding what makes a pension plan sound and what makes it unsound. Recently, actuarial literature has begun dealing with the problem of setting down certain principles regarding the question of actuarial soundness. Mr. Griffin's paper is an important and timely contribution in this area.

The paper does much to clarify the confusion that exists in the minds of some of our professional brethren as to the intimate connection between the adequacy of a particular funding method and the type of plan to which this method applies. The author did an excellent job in stimulating clear thinking on the subject and in analyzing the true meaning of certain accrued and unfunded liability figures that are frequently considered basic to an evaluation of actuarial soundness.

For me, as an actuary serving a federally administered retirement plan, Mr. Griffin's paper has special significance in that it draws a clear distinction between plans that can be presumed to be permanent and plans whose permanence is open to question. For plans of the former type, Mr. Griffin would be satisfied with a method of financing that calls for paying entry-age-normal (EAN) costs plus interest on the unfunded

accrued liability (UAL). This is essentially the method of financing used for the railroad retirement system. We at the Board have always been saying that this method is proper for our system, but some people have at one time or another taken issue with us on this point. I am delighted to have an endorsement of our views by an authority of Mr. Griffin's stature.

Mr. Griffin's paper also lends indirect support to my view that UAL figures computed under the customary definition are not of particular relevance for the railroad retirement system. I have discussed this problem in a paper that appeared in the 1954 volume of the *Transactions* and in a special monograph published in 1963 as *RRB Actuarial Study No. 7*. In the railroad retirement case, the issue is even further confused by the fact that contributions with respect to future entrants are scheduled to be higher than EAN costs as well as by the financial co-ordination with the social security system. This, in my opinion, is a prime example of how misleading the indiscriminate use of UAL figures can be in certain circumstances.

It is my sincere hope that Mr. Griffin's excellent paper will once and for all make it unnecessary for us to defend ourselves against allegations of unsound funding based on UAL figures that we occasionally publish in the interest of full disclosure of actuarial facts.

ROBERT D. KRINSKY AND JACK M. ELKIN:

We agree in general with Mr. Griffin's suggestion of a funding objective that looks to the accumulation of assets equal to the larger of (1) the lump-sum cost of accrued (or vested) benefits computed on a plan-close-out basis or (2) the amount that would have resulted from the payment year after year of the normal cost plus interest on the unfunded actuarial liability on the entry-age-normal method.

The illustrations in Mr. Griffin's paper are based mainly on plans paying a flat benefit per year of service without limitation on the number of years of service. A substantial number of plans, however, limit the number of years that may be credited to a specific period such as 20, 25, or 30 years. In these cases, the "accrued benefit," the cost of which is to be met, can be thought of in at least two ways, namely, as (1) the flat benefit per year multiplied by the number of years of service to the maximum creditable or (2) the maximum benefit payable multiplied by the ratio of the years of service to such years increased by the years remaining to normal retirement age. The two definitions coalesce, of course, in the case of an employee whose years of service at his normal retirement age, if he continued in service until then, would not exceed the maximum creditable under the plan.

Under the first definition, the cost of accrued benefits is always lower

than the accrued liability calculated on the entry-age-normal cost method in the case of an employee whose service at normal retirement age will not exceed the maximum creditable. In other cases, the accrued-benefit cost is lower until some, usually short, time before the maximum creditable service period is completed; thereafter it is higher, until retirement, when, of course, it is equal.

Under definition 1,

$$\text{Accrued-benefit cost} = \frac{(x-w)N_r}{D_x},$$

$$\text{Accrued liability} = \frac{B_{w,r}N_r}{N_w - N_r} \cdot \frac{N_w - N_x}{D_x},$$

where

$B_{w,r}$ = benefit payable at age r on basis of service from age w ,

x = present age,

w = entry age.

The accrued-benefit cost is less than the accrued liability if

$$\frac{B_{w,r}(N_w - N_x)}{N_w - N_r} > x - w.$$

If $B_{w,r} = r - w$, the accrued liability exceeds the accrued benefit if

$$\frac{(r-w)(N_w - N_x)}{N_w - N_r} > x - w;$$

that is, if

$$\frac{N_w - N_x}{x - w} > \frac{N_w - N_r}{r - w}.$$

The left side is the average of the first $x - w$, and the right side of the first $r - w$, D values starting with D_w . Since D is a decreasing function of age and since $r > x$, the first average always exceeds the second.

For $B_{w,r}$ sufficiently (depending on the service table used) less than $r - w$, the inequality is reversed.

If this first definition of accrued benefits is used as the basis for a government-mandated vesting provision, it could, in some funds, especially those with fixed negotiated contribution rates, create serious problems by adding substantially to the cost.

From the point of view of equity, it also would seem that the second definition is preferable. Consider the case of the two workers in the same industry who begin work in neighboring cities at age 30, one under Pension Plan A, the other under Plan B. Each plan pays a normal benefit of \$100 at age 65 after twenty-five years of service. The first worker remains

covered by Plan A throughout his working lifetime; the second moves to the jurisdiction of Plan A at age 50, after twenty years under Plan B. Under the first definition of accrued benefits, the first worker would get a monthly pension of \$100, while the second, merely by the act of moving to a neighboring city, would get $\frac{2}{5}$ of \$100 plus $\frac{1}{2}$ of \$100, or \$140 per month. Under the second definition of accrued benefits, the second worker would receive $\frac{2}{3}$ of \$100 plus $\frac{1}{3}$ of \$100, or \$100 per month, the same as the first worker.

A plan providing a benefit equal to a percentage of salary per year of service produces a relationship between the cost of accrued benefits and accumulations on entry-age-normal funding analogous to that in Chart II of Mr. Griffin's paper. It should be noted, however, that, when a salary scale is used in the calculations of accruals on entry-age-normal cost funding, the accumulations are substantially larger than if no salary scale were used, and the excess of the accumulations over the cost of accrued benefits is much greater.

The remainder of our comments deal with the adequacy of pension fund contributions and the amortization of the "maximum fund that ever need be accumulated."

As Mr. Griffin suggests, it does not seem reasonable that the period of amortization should be the same for a plan covering a handful of employees of a corporation that is likely to terminate when the principals retire or die as for a plan covering the employees of a corporate giant in a basic industry. In general, the amortization period could be much longer for the latter. Similarly, for a multiemployer, industry-wide plan that is likely to exist as long as the industry exists, regardless of the fate of individual employers, a relatively long amortization schedule seems appropriate.

The fact that contributions are fixed by negotiation, however, may give rise to a special problem. A contraction of the industry, because of automation, for example, is most often at the expense of the short-service employees, with the result that the per capita unfunded accrued liability may increase. If the fixed contribution had been meeting actuarial costs on an interest-only or near interest-only schedule, it may fail to do so after the contraction. The same situation may occur, of course, in a contracting single-employer plan that is being financed by means of a fixed negotiated contribution. State and local government funds, of course, are in a class by themselves; from the standpoint of adequacy, there is seldom any need for rapid funding.

Apart from the purely actuarial considerations, we feel that mandatory funding requirements and vesting provisions as prerequisites to plan

approval may distort the correct priorities in the establishment of pension plans. Mandatory vesting may impede the establishment of new pension plans. A pension plan can be a significant item in total labor cost. To increase this cost initially—as vesting would—may increase an employer's reluctance to undertake the cost. On the other hand, if pension plans are permitted to establish themselves without the additional cost of vesting, if they are permitted to maintain their own priorities among higher initial benefits, early retirement benefits, and vesting, they may very well eventually adopt all these provisions.

That vesting is desirable is accepted by almost everyone. And most people concerned with the establishment and operation of pension plans see vesting as a present or future improvement in their plans. Greater knowledge among those covered by pension plans of exactly what rights accrue to them under the plans, and increased discussion of vesting, will accelerate the present trend toward vesting in existing pension plans. Mandatory funding will remove much of the flexibility from the development and growth of pension plans.

Most pension plan administrators, recognizing that the plans may not continue in perpetuity, that they may some day be terminated, do adopt schedules which amortize past-service liability. But actuarial gains, fluctuations in experience, changes in benefit levels, acquisition of new companies, admission of new groups of employers (in the case of multi-employer pension plans), changing cash requirements of a corporate enterprise, plus the hundreds of other changes that are likely to take place in an emerging pension program, make fixed, mandated amortization objectives difficult to achieve.

Prudent financial and actuarial policies dictate that reasonable conservatism be one of the criteria for scheduling contributions to support a program. And this often dictates amortization, since both the actuary and his client recognize that no enterprise is perpetual. However, the degree and type of amortization that is most appropriate to any business or industry can generally best be determined by the employer and the union, if any, not by a government agency.

HERBERT L. FEAY:

I have enjoyed reading Mr. Griffin's paper. One purpose of a good paper should be to disturb complacency and to cause thinking on the subject it covers. Mr. Griffin's paper has done this for me, and the extent of the discussions submitted here indicates that this is true for others.

One of the main sources of difficulty for an understanding of pension funding is the confusion regarding the purposes of the valuations made to

support employer contributions reported in federal income tax returns. The ingenuity of actuaries has produced a large number of variations in methods and standards for determining the present value of benefits and for providing for that present value by annual payments in the future. These methods and standards will give a wide distribution of annual pension plan contribution amounts for the current year and for future years. The primary concern of these calculations is support for the current deduction in the tax return and is not the determination of the proper ultimate funding costs.

Mr. Griffin is concerned about overfunding of pension plans. I doubt if there are many plans that are now overfunded, and I am of the opinion that there are now a number of plans that will be underfunded if the current levels of contributions are continued.

Mr. Griffin emphasizes the unit-credit and the entry-age-normal cost methods of calculating funding costs. The entry-age-normal cost method will usually give a greater accrued-liability total and a smaller normal-cost present-value total than the unit-credit method. The entry-age-normal method will thus usually give a greater degree of flexibility for contributions. This greater flexibility is more evident after the plan has been in effect for a few years and prior contributions have been larger than the absolute minimum. If the employer is interested in as low an initial cost as possible, the unit-credit method for normal costs plus interest only on the unfunded liability will probably give the desired results. If the employer wants to make higher contributions in the earlier years for a plan and to have a basis for greater flexibility later to meet the effects of inflation or of reduced business profits, the normal-cost method is likely to be the better funding procedure.

These same considerations can extend to the selection of the actuarial standards other than the calculation method. These other standards include the interest rate, the termination rates for active employees, retirement rates, the mortality rates for various classes of retired lives (early, disability, normal, and deferred), and the expenses of the pension plan. Conservative standards for the unit-credit method can give about the same funding costs as more realistic standards for the entry-age-normal cost method. Certainty of such an equality, of course, can only be secured by two separate valuations, using the two sets of methods and standards.

Regardless of the valuation methods and standards followed, the problems of overfunding will not come up until the assets of the fund exceed the minimum desirable funding total. At that time it should not be difficult to change valuation standards and methods so as to eliminate any

danger of future overfunding. The new standards and methods will reduce the contributions otherwise allowed as tax-return deductions from income and will probably be readily acceptable to the income tax authorities.

In this connection, I point out that the main concern of the income tax authorities is the current tax deduction for contributions and not the future guaranteed safety of the plan.

Mr. Griffin demonstrates that if a pension plan has an increasing deferred-pension liability that is supported by assets producing income at a sufficient rate in excess of valuation rate, the excess-interest earnings will fund the unfunded liability established for the plan on its effective date. Such large excess-interest gains indicate the desirability of increasing the valuation interest rate.

In no part of his discussion does Mr. Griffin mention the expenses of operation for a pension plan. These expenses are certainly part of the cost of a pension plan for which funding (payments) must be provided. Administrative expenses for active employees probably can be assumed to remain fairly constant, but expenses of paying pension benefits after retirement should be funded in advance of retirement. Certainly the present value of future expenses should be included with the single-premium value of accrued benefits in determining the minimum desirable amount of funded liability.

Mr. Griffin does not cover disability benefits or death benefits of pension plans in his determination of funding costs. Frequently these benefits are included on what is called a "one-year term basis." Actually, this is equivalent to terminal funding, since the costs for each plan year equal the reserve liabilities and the benefit payments for the claims approved in that particular year. The costs for these supplementary benefits will increase for Mr. Griffin's immature groups. These costs need to be considered in a realistic appraisal of future benefit costs for funding purposes.

The paper has no comment on the funding requirements for costs of early retirement benefits and for optional pensions. The assumption that these costs will be eliminated by "actuarial equivalent" factors is not correct for many plans.

Mr. Griffin's "another possible approach" at the end of his paper provides a suggested procedure for pension plan calculations. The actuary can make a careful and exact valuation of all benefits provided by a pension plan, using standards that he has determined to be his best estimates of future results for investment earnings, plan expenses, termination rates, disability rates, and mortality rates for active employees

and for retired employees. The calculations can have a specified target date (such as thirty years in the future) for calculations on a "close-out basis" at that future date, as suggested by Mr. Griffin. Reasonable assumptions can be made for the future size of the work force, for future new entrants, and for future compensation scales. The employer contributions for the plan costs determined on this proposed basis can then be spread over the thirty years by some method. The employer can first be given the estimated level annual contribution for the thirty-year period and then advised of how actual payments can be varied, within limits, up or down from these level amounts. Actual experience will differ from the estimates for this more exact valuation, but the differences should be much less for the exact valuation than for many valuations made for tax purposes. Such actuarial gains and losses for the exact valuation can be used to adjust the thirty-year period for the next exact valuation. These extensive valuations are probably needed only about once every five years.

After the true level annual costs have been determined to the best ability of the actuary and the employer has made a reasonable estimate of the contributions that he proposes to make for the next three or four years, valuation methods and standards can then be established for the valuations to support deductible contributions for the income tax return.

There are analogies for this proposal and the procedure used to determine gross premiums for nonparticipating life insurance. The gross premiums are calculated on realistic standards for death claims, expenses, terminations, and investment returns. Frequently a specified period, such as thirty years, is covered by the calculations and the value at the end of thirty years taken as the cash value or the cash value plus a surplus allowance. The valuations made for the state insurance departments for the insurance sold at the gross premiums so determined are made in accordance with the requirements of the insurance laws. The gross premiums have no simply defined relationship to the valuation net premiums.

Mr. Griffin states that an organization of undisputed permanence can properly fund a pension plan with contributions equal to the normal costs plus interest on the unfunded liability. He indicates that this is a minimum necessity to show "the true long-range costs of the benefits which have been adopted." I see no good reason for excluding social security from this test.

I understand Mr. Griffin's concern about ultimate overfunding of a pension plan. I doubt if there are many plans that are now overfunded, although, if contributions are continued at current levels, some of them can become overfunded. It seems to me that the greater danger currently

for many plans is underfunding, unless current annual contributions are increased in the future. Decreasing future contributions to avoid overfunding will usually be easier than increasing these contributions to prevent underfunding.

RICHARD DASKAIS:

The author has assumed that any year's pension costs (or charges against income for pensions) are equal to pension contributions for the year—that is, the employer is using cash pension accounting rather than accrual pension accounting. This is now the general, but not universal, practice in American pension plans.

It is quite possible that in the near future the principal recommendation of the Hicks report⁴ will be put into practice by accountants and that accrual pension accounting will become the rule rather than the exception. The pension contributions of an employer who accounts for pension costs on an accrual basis may be determined by considerations of employee security, opportunities for investment within and outside the pension fund, income taxes, availability of cash, and so forth. The employer need not stabilize contributions in order to stabilize pension-expense charges, since these charges are largely independent of contributions.

With accrual pension accounting, funding is divorced from accounting, and the reasons stated in the paper for a minimum *contribution* of normal cost and interest under the entry-age-normal method appear to become reasons for a *charge* of normal cost and interest.

BARNET N. BERIN:

To the outsider, one of the puzzling things about pension funding is how a particular method is chosen. The author is concerned with projected-benefit methods but evidently feels that the accrued-benefit approach may be a better measure of adequacy at a particular point in time. The questions of funding methods in general and of actuarial assumptions and their effects are not really covered. They both relate to concepts of adequacy.

In theory, at least, the choice of actuarial assumptions should be independent of the choice of funding method. Actuarial assumptions should represent appropriate estimates of rates of interest, mortality, turnover, salary scale, and disability, with rates of interest, turnover, and

⁴ *Accounting for the Cost of Pension Plans* (New York: American Institute of Certified Public Accountants, 1965).

salary increase related to the employer's probable experience. Once these are set, the funding method should be one of the six or more recognized methods which best meets the employer's objectives. All funding methods reach the same goal if we consider one employee whose retirement benefit is funded over his working lifetime. The employer's objective can lead to a funding method where the costs gradually rise (as a percentage of salary or as a dollar charge per life), where they are level, or where they gradually decrease. (The condition implicit here is that actuarial experience is exactly realized.) In the absence of an employer objective, the specifics of the case will shape the actuary's choice.

Once a set of actuarial assumptions and a funding method have been chosen, an analysis of experience gains and losses is essential. If one of the assumptions is poor—with continued actuarial losses leading to rising costs, that is, over-all losses—it should be changed. By annual examination of gain-and-loss components, the appropriateness of the assumptions can be checked over a period of time. In subsequent years, it may become important to lower costs, based upon changes in the actuarial assumptions, to meet benefit increases without changing the level of over-all costs or to enable the employer to skip contributions. Without a complete gain-and-loss analysis, such changes cannot be done in a professional manner.

The author refers to the cardinal principles of life insurance as supporting the "level premium" funding methods. This argument is weak. There are generally no surrender values comparable to life insurance policies' cash values in the funding of pension plans. The "risks" in life insurance are generally maximum at issue and decrease thereafter, while in a pension plan the "risks" gradually increase over the employee's working lifetime.

Some actuaries have felt that the accrued-benefit cost method—the unit-credit method—represents a reasonable approach to funding where a specific benefit is defined for each year of service. In practice, the accrued-benefit cost does not increase from year to year. Mr. Robert F. Link's discussion (*TSA*, IV, 665) concerning the unit-credit method and the average premium age is still relevant.

The author is concerned with the definition of accrued liabilities. The balance-sheet approach shown has the weakness that some of the asset items are at best *contingent* assets. It might be better to define the accrued liabilities as (1) — (2) and assets as (4), with the unfunded liability equal to (5). This approach can be used in all funding methods.

An interesting point is whether a stationary population will in fact ever occur. Is this concept an instructive one, or does it lead to confusion?

The accumulation of funds to back accrued benefits or to back vested benefits is a major distinction that implies very different financial objectives. Is the "vested benefits" approach feasible?

A discussion of these points would expand the material on the Part 9E syllabus and be helpful to the students.

OREST T. DACKOW:

The current dialogue now taking place in the pension area strongly suggests to me that within the next few years we will be faced with amendments in the provisions of the Internal Revenue Code dealing with pension plans. These amendments will undoubtedly reflect to some degree the recommendations of the President's Committee on Corporate Pension Funds and the ideas being put forth at the public hearings now being held by the fiscal subcommittee of the joint economic committee, headed by Representative Martha W. Griffiths.

It seems clear to me that utter chaos will result if it is decided to adopt one set of guidelines to ensure that pension plans follow some funding standards and a second, unrelated, set of standards through the Internal Revenue Code governing the employer's tax position in regard to his contributions to the plan. Those wishing to promote employee security would desire at least the funding of accrued benefits and preferably an even higher level of funding. The Treasury, on the other hand, might well feel that the maximum level of funding for which an employer should expect tax relief is the complete funding of accrued benefits. These probably diverse objectives in two sets of guidelines would make pension plan administration an even more complex matter than it currently is.

For those of you who believe that this is an idle fear, an unrealistic nightmare, I would simply remind you of the situation currently existing in Canada, where the provinces have jurisdiction over the funding standards with which pension plans must comply and the federal government regulates the employer's tax position. Under this dual set of standards, conflicts have arisen concerning the treatment of employer contributions to plans that are currently in an overfunded status.

The author concludes his paper by stating:

Increasing pressures for regulation, which come largely from sources unfamiliar with actuarial distinctions and the requirements for benefit security, place a responsibility on actuaries to provide appropriate definitions and to direct these pressures into the right channels.

I believe that this is an understatement of our responsibilities in the area.

DAVIS H. ROENISCH:

Mr. Griffin is to be commended and congratulated on his paper, particularly on making a clear distinction between the "actuarial liability" and the "cost of accrued benefits." It is easy to concur with the positions presented by his paper.

Several minor points might be noted. First, I have some comments with regard to terminology, since the terms that actuaries have used in the past to define their concepts may be the source of much of our difficulty with professionals in other fields. Thus, the term "actuarial obligation" is less likely to connote a debt which should have been paid in the past than "actuarial liability." The term "benefits earned to date" might well carry more meaning to non-actuaries than "accrued liabilities." While these changes in terminology are modest, they are advanced seriously, since the connotation of words often sets the emotional stage for discussion of funding requirements with non-actuarial groups.

The principal point of the paper is that the normal cost plus interest on the unfunded obligation calculated by the entry-age-normal method is usually adequate to fund the benefits earned to date over a reasonable period of time. Some caution is properly exercised by the author in moving from the individual case of an employee entering employment to the group which starts with employees at all ages. The strong funding position of most pension plans that have been in effect without substantial amendment over the last decade has come about primarily from favorable investment results and expanding employee groups.

Some reservation should be voiced on the author's statement (on p. 57) that the minimum contribution might fund a substantial portion of the unfunded cost of accrued benefits for a group of constant size in the absence of substantial experience gains. Unless such a group were immature, the author's Distribution III on a $4\frac{1}{2}$ per cent basis (shown on p. 61) would apply. In these conditions, very little funding of the benefits earned to date would be expected. It is true that the author qualifies his statement by saying that the contributions "might" reach this result, but, in the conditions that he cites, we believe that it would be more likely that very little funding would take place.

The value of the paper lies in its stress on the great complexity of determining the trend of funding for any given program. Not only do the benefit formula, actuarial technique, experience gains, and the trend in the size of the group play an important role in determining the speed with which the benefits earned to date are being funded, but the actuary's treatment of unrealized market appreciation and inflation must also be

taken into account. The treatment of these major imponderables has, as illustrated by the author in varying the interest rate, a very substantial effect on the appropriate contribution.

This suggests that other regulatory authorities might be best advised to satisfy themselves that the actuary has a reasonable basis for his determination of the outlays required to fund the pension plan and that this amount will adequately provide for the benefits earned to date after a reasonable length of time. Upon later review, the progress of the fund can be measured against the original assumptions, its current status compared to that which was predicted, and a conclusion reached as to the adequacy of the funding program and the prior contributions. If they content themselves with this, unsound funding procedures can be detected and highlighted without requiring the reviewer to take a course in actuarial science.

DORRANCE C. BRONSON:

"The old order changeth, yielding place to new,
.....

Lest one good custom should corrupt the world."

TENNYSON, *The Idylls of the King*

Mr. Griffin has produced, for the most part with his usual clarity, a further interesting piece for the literature on pension funding in the United States. Observers of recent pension literature will probably agree that the stalagmite of writings on retirement and allied benefits has had abnormal cubits added to its height, through actuarial papers, professorial dissertations, proposed accounting dicta, governmental reports, and so forth.

A considerable increment in these recent encrustations to pension literature (to continue the simile) has derived from the writings of certain new authors whose ignorance of private pensions is patent. "A little learning is a dangerous thing" applies quite aptly to this type of writer. The precipitates they have left (with more coming) put colors, not seen before in the United States, into our measurement and appraisal of pension literature. To some readers, these new hues appear in the bright band of the spectrum, but, for others, they are at less-brilliant locations.

The "new" is with us, willy-nilly, serious or absurd. The *serious* category is headed by politicoeconomic innovations. Evolving from the New to the Fair Deal, and across the New Frontier, we are now attempting a traverse of the Great Society's high cliffs.

Another "new" of *serious* nature (alluded to later herein) lies in ter-

minological failures. A potent example is found in C. P. Snow's much-discussed *Two Cultures*. In this work, the British author is concerned with the widening dichotomy in the ability to communicate between two great cultural branches, say, Categories A and B. Category A represents today's more-than-ever arcane disciplines of the pure and applied sciences (within which we may include "New Math" for the kiddies, on up, but from which we must bar daddy's science fiction throwaways); Category B comprehends the *serious* liberal arts and humanities, as well as the less-serious fringes thereof.

Within said "fringes" of Category B, communication is also frequently difficult, but no one seems to care or write books about it; indeed, it may be cultivated as a defense mechanism! In this area, new oddities are proliferating at an increasing rate. As illustrative of "doings" in this fringe area for those of our readers who stay out of it, I will mention only three. First, the "New Art," which, the last time I looked, was replete with (a) canvas abstractions produced by splashed-on, or dripped-down, pigments and (b) collages ingeniously composed of arranged junk (cities should encourage). A second example is from the "New Theater," within the circuit of exponential "offs," measured from Broadway; there have been (and are, I am sure) certain weird *new* productions given from time to time. Last, closer to our own profession, one learns that "New Symphonies" have been composed by E.D.P. on magnetic tape (these are, no doubt, enjoyed by E.D.P. actuaries, who can applaud with great efficiency, in bursts of FORTRAN or COBOL).

And so it is, in this relatively small, antic fringe of Culture B; anything goes, provided you are not a Harry Hypotenuse (the sum of the Squares) who, in these purlieus, is abhorred as a Vacuum abhors Nature. Any restrained levity into which I may have lapsed for this paragraph, with respect to activities imputed to fringe areas of Culture B, has only been by way of aiding the characterization of one aspect of the times, A.D. 1966.

The writings on pension subjects by qualified actuaries are all of Culture A and, I believe, aimed at offering various types of improvement in the area of private pension systems as we know them. These actuarial writings are intended for the information of employers and/or unions, as well as for the actuarial audience. In contrast to said constructive aims, there have been a few "mushroom-type" authors, out of the B-fringe, perhaps, who—through ignorance or design—sharply criticize the private system and make concomitant proposals so drastic that I do not trust my

prose to stay in decent bounds when mentioning same but turn, instead, to those twin disciplines of meter and of rhyme:

This is a sorry Scheme of Things entire,
Let's grasp and shatter it to bits—and then
Re-mould it nearer to our mind's Desire.⁵

The "shatter and re-mould" ideas, held by what, I hope, is but a small minority, are the antithesis of Mr. Griffin's views. Indeed, his paper, launched from the actuarial science post in Culture A (far from the fringes of B), is thoroughly constructive in aiming—through his rule for minimum funding—at extending the area for "shatter-proofing" private pension plans. We all, I expect, strongly support such *aims*; more doubtful, though, is that such unanimity holds over to his *ways and means*. (This lack of full agreement is a result that I am glad to find, both because it would be a dull ending and in view of the principle that beneficial "fall-out" is shed by disagreement.)

I confess to having found some difficulty in harmonizing the author's ways and means with traditional principles of pension funding. It was not clear to me at first that Mr. Griffin was writing a rulebook on *minimum* funding tests and not a tract for his idea of the *optimum* basis. The promulgation of such a tract would, indeed, have clashed in sparky opposition with subjective optimums aimed at, or attained by, various employers and their actuaries. That is, the paper does not attack these precincts as its main purpose, although it may serve to induce second looks at the current funding of a given plan and at the preconceived viewpoint of its optimum. The foregoing part of this paragraph gives my capsule summary of the paper upon restudy of it; the author will correct me if I am wrong.

In previous talks, papers, and discussions thereof, several actuaries, myself included, have pointed out the increasing difficulty of communication, both among ourselves and—even more—with those outside. The pension terminology stands little chance of being standardized, I feel, in spite of the ambitious project of the committee on pension terminology, with which project Professor McGill and others are valiantly struggling. For instance, not infrequently, an actuarial author complains about the sad state of pension terminology and then, rather promptly, either sets forth his own definitions (sometimes *ad absurdum*) or charges ahead with undefined and/or undefinable terms, each of which begs the question of "What does he mean?" or "Is this the same as he meant on page so and so?" Mr. Griffin, fortunately for the reader, has not forced his definitions

⁵ With apologies to FitzGerald's fine *Rubaiyat*, for a few words deleted with mine "dubbed in."

on us; most of his terms seem clear without definitions. He *does*, right soon and right often, use the term "overfunded," the meaning of which he should have clarified, if he could, at the start. I note that Mr. Trowbridge, in the interesting educational paper he wrote for the *Harvard Business Review*, blames the actuaries, en bloc,⁶ for the alleged existing confusion of pension funding in the business community, because of our vocabulary (the "two-culture" syndrome of C. P. Snow apparently here, too).

To turn a moment to contemporary pension funding in Great Britain, where the early pioneering papers of King, Manly, and Epps found fertile soil, I observe that a paper, fairly recent (1961), entitled "Pension Fund Valuation under Modern Conditions,"⁷ brought out what the "new," over there, consisted of under said "modern conditions," namely, a reconsideration of interest rate assumptions, certain nontraditional asset-valuation techniques, and a sharp alertness to future inflationary potential effects on actuarial methods and assumptions; in short, no basic structural changes seemed to be proposed.

The "new" actuarial funding concept⁸ enunciated in part of Mr. Griffin's paper and in certain previous writings by him and others, puts, perhaps, a stamp of "update, needs revision," on several of my various papers, books, articles, discussions, and so forth. But I doubt the demand for this and shudder to envision the job of "revision"; hence, I will await events before getting out the scissors and the paste. However, though absent the time for rewrites just now, when Mr. Griffin puts such heavy emphasis upon benefit-security ratio and its component accrued-benefit value (with a finality equivalent, it would seem, to the Laws of Moses), recollections *do* come to mind of situations where, I am sure, there were "happier endings" from the regular traditional valuation methods that had been followed (overfunding, if you will) than would have resulted under a funding history by Mr. Griffin's criterion.

My allusions above to "revisions" of past writings and to "happier endings" lead me on to recall the following random items. These references are from my own writings (which is why they came so readily to mind relative to Mr. Griffin's paper):

1. I have felt no regrets with regard to the fairly strict rule on "actuarial soundness" for which I indicated preference in a book written in 1957.⁹ I did

⁶ This, I mildly resent—one of those shotgun blasts; for example, "We are *all* to blame for the Flood, or for such and such!"

⁷ By Geoffrey Heywood and Maxwell Lander, *J.I.A.*, LXXXVII, 314.

⁸ A concept really grounded on, but with gyroscope added, the *old* group annuity single-premium technique, is it not?

⁹ *Concepts of Actuarial Soundness in Pension Plans* (Homewood, Ill.: Richard D. Irwin, Inc., 1957).

not (and do not) feel sufficiently competent or omniscient to put forward, as the sole recommendation, any explicit single definition of the term "actuarial soundness."

2. In a paper written in 1949,¹⁰ the question was raised whether "industry, in general, could *afford* full funding objectives"; Mr. Griffin's paper may imply this question, also.

3. In another paper,¹¹ with respect to the *termination of a plan*, a sizable number of benefit formulas, conditions, and termination priorities were explored. In quite a number of these, the obligation for employer funding and/or for employee equity at death or termination of service would have been quite unmet or hard to handle under Mr. Griffin's criterion, which, as he says, confines itself to normal retirement pensions. Indeed, even had he widened his focus to benefits other than normal retirement date pensions, it is hard to visualize how a measurement of all values could have been computed year after year for these variables on the plan's "benefit matrix" without unwarranted actuarial detail, effort, and fees.

4. Mr. Griffin, perforce, could not cover the pension waterfront in his paper. However, a few of the elements omitted may be listed for illustration. The paper does not explore, with respect to his funding discussion, the matter of employee contributions (e.g., how much of the accrued benefit can be imputed thereto; are there other problems?); it also omits reference to Mimeo 5717 limitations under his "closeout" value concept; and it has no allusions to any possible reshuffling of "rights" or allocations in respect to the fund or contract under special situations, such as for the euphemistic "actuarial error" or the "draw" caused by lump-sum commutation or by "freezing" certain funds through guaranteed priority (rare) of vested terminations.

5. In a paper on mergers,¹² I pointed out the probable superior position of the company which had a higher "funded ratio"¹³ than did the other company. Having later seen this very sharply brought out in practice during merger negotiations, and involving large sums of money in settling on "price," the said "superior position" impressed me greatly.

6. Last, Mr. Griffin mentions, in his final paragraph, and rather *en passant*, the possible "improvement of benefits." It appears to me, however, that, in view of the pension history over the last twenty-five years, he has understated the situation and, while he may be correct on the plateau now reached for *pension* formulas, these are neither *dollar amounts* nor the limit-of-benefit *types* possible. There surely have been great advantages and help to employers (and indirectly to employees) when benefit levels were raised, or new types added, in the existence of a valuation position which was "overfunded" under Mr. Griffin's

¹⁰ "Pensions, 1949," *TSA*, Vol. I, No. 1.

¹¹ "Pension Plans: Provisions for Termination of Plan," *TSA*, Vol. VIII.

¹² "Pension Plans under Corporate Transfers" (mimeographed paper presented to the Council on Employee Benefit Plans at New York City, October 16, 1959).

¹³ A different measure from the paper's "benefit-security ratio."

concept of the term (a term which, as mentioned earlier herein, is not clearly defined in the paper).

In the very weight, by number, of actuaries (perhaps a record) discussing this paper, the interest in and importance of Mr. Griffin's toils are evident. My own discussion is one of neither complete praise nor really critical derogation but, rather, has been set down from a quizzical, ready-to-be-converted stance. In view of the preceding sentence, it may seem paradoxical for me to say that I think Frank's paper, per se, is excellent. Sincere writings in the actuarial field are, in general, educational and thought-provoking, and, in this paper, wherein "new" funding angles provide a liberal sprinkling of *Aécent* to sharpen our taste, the author has served us a spicy dish.

* * *

To close with asterisks might indicate some terminal omission. Let me, then, expand on what I call Mr. Griffin's "parable of the mountain."

A postlecture inquiry was put to Mallory,¹⁴ the famous British climber, on *why* he wanted to climb Mt. Everest. Those of the departing audience close enough to hear the question broke out into laughter. "But, Mallory didn't laugh. He stood there . . . a little while, thinking out his answer. And then he told them, 'I want to climb it,' he said, 'because it is there.'"¹⁵ This response, extemporaneously expressed in less than ten words, has magnificent simplicity and breadth; it serves, cogently, to explain a multitude of human actions far removed from mountain heights.

But I must descend from dramatic altitudes to the pragmatic level. I inserted this vignette of Mallory's classic answer because it struck me that Mr. Griffin's paper rather invited it. Specifically, at the end of the first section, he uses a paraphrased (suitably) version of "the Mallory response, *sans pareil*," as the basis for his parable of the mountain.

The mountain (*actuarial* liability) "is there," and, traditionally, actuaries have led expeditions (amortization) to, or toward, the summit (100 per cent "funded ratio"). But in the paper, the author asserts, in

¹⁴ George Leigh Mallory attempted to climb Mount Everest three times (1921, 1922, and 1924). The last climb capped his already spectacular feats with legend. After a stop, relatively near the summit, he and a companion, Irvin, accepted the final challenge and set off for the unconquered top. Watchers saw their upward trek until the two disappeared from view into the upper mists, and clouds and ice and silence. "Disappearing from view" was "disappearing forever." Many feel that they attained the summit ere some catastrophe of the heights made fatal its strike, obliterating, for man, all traces of these brave men.

¹⁵ As described in *The White Tower*, by James R. Ullman (Philadelphia and New York: Lippincott Co., 1945).

effect, that the upward struggle is not really needed after all; there is, instead, a path (the "minimum funding" rule of the paper) *around* the mountain. Thus, many actuaries, whose eyes and minds have been busy seeking the upward trail, have overlooked the meadow route.

To this parable, there is this short addendum. Mr. Griffin, as I discussed earlier, would, as chief ranger of the region, set up no barriers or fences or any signs of "No Trespassing," against such actuaries as are upward bound. The implications of the paper, however, *do* give one (me) the feeling that the ranger has let the paint flake and grow dim on the arrows to the upward trails, while keeping those to meadow paths distinct and colorful.

And thus my discussion (rambling and digressive) comes to a close. One note alone remains, purely subjective. Both as a mountain amateur, when younger, and one with stubborn habits and conceptions (plus, I believe, a certain body of experience), I must declare that I prefer the climb, bearing in mind, before striding off, that—as it was said of old—"there are many paths (funding methods) to the top."

DOUGLAS H. MILES:

I welcome Mr. Griffin's paper to the Society of Actuaries and regret that distance will prevent my attending the discussion. I would like, however, to make certain points as comments on what is said in the paper.

In Great Britain, I have found it important when considering funding theory to consider the theory separately in relation to average salary and final salary plans. In considering final salary plans, moreover, most conventional ideas go overboard if the salary-scale increase exceeds the rate of interest used, but space prevents consideration of this here.

On the whole, it seems to me that the arguments used in Mr. Griffin's paper refer principally to average salary plans, and most of the remarks that follow also relate to these. He refers on page 55 to mature funds, and the concept of a stationary fund is useful to bring out one point. In the position of a stationary fund, the fund under unit-credit funding is smaller than the fund under prospective methods of funding (which apparently are termed "projected-benefit cost methods" in the States), the difference being (Present value of future service benefits for existing members) *less* (Present value of future contributions).

At this stage, the unit-credit cost is higher because interest on the smaller fund is less (Pensions = Contributions plus interest on fund), but at an earlier stage prospective methods have the higher cost, because a larger fund is being built up.

In Great Britain, it is the usual practice to have valuations once in

five years, although this is sometimes reduced to three. One has to set a rate of contribution, which can be maintained for at least that period and usually longer, because, once a rate of contribution has been established, it is always easier to reduce it than to increase it.

Traditionally, our methods of funding have been attained age normal cost. This has the merit that it is safe, as the rate of contribution tends to fall as new entrants at lower ages are brought in. For average salary plans, however, competition from insurance companies using single-premium costings has forced some hard thinking; it is useless to argue that the unit-credit method will eventually result in higher costs, because of the way that wages and salaries have increased since the war, with the result that virtually every fund, however old, is in fact very much in the early build-up stage. (In this respect, I would mention that we recently valued a fund that has been in force for thirty-five years and has doubled in size in the last five years.)

One method which we have used for average salary plans has been to calculate the entry-age-normal cost plus the interest on the unfunded past-service cost and express the result as a rate of contribution. As new entrants are brought in, this means that the initial past-service cost gets paid off (except in the case of a declining fund), so that on Mr. Griffin's definition the plan would be overfunded. Such a contribution, however, is usually smaller than the unit-credit cost plus the annual cost for past service paid over twenty years, so that the excess is not great.

However, there is one occasion where prospective methods of funding are needed, namely, where a fund is closed to new entrants but benefits for existing members continue to accrue. This possibility is sometimes used to justify initially a higher rate of contribution than would otherwise be called for; but such a rate of contribution need not be pressed if the employer appreciates that circumstances can arise where the rate of contribution may have to be increased.

And this brings me to the crux of my remarks, namely, that, provided the employer fully understands the position, it is possible to offer alternative immediate costs between which the employer can decide, dependent on his financial position at the time and his prospects in the future. This is not to say that one will always agree with the rate of funding adopted. Sometimes, however, the employer deliberately adopts a fairly high rate of funding, but, when it becomes possible to reduce the rate of contribution, instead of this he uses the surplus on the valuation to augment the benefits for earlier service. By this means it may be possible to have a final salary plan in practice without the guarantee that such a type of plan normally involves.

Final-salary plans present the different problem that salaries in recent years in Great Britain have been increased at rates substantially higher than have been allowed for—8–10 per cent per annum in many cases. For the most part, therefore, any margins built into the rate of contribution from whatever source have been welcome toward meeting the cost of these increases.

(AUTHOR'S REVIEW OF DISCUSSION)

FRANK L. GRIFFIN, JR.:

It is a generous custom which gives the author the last word, especially in the face of an array of experts such as have chosen to comment on this paper. To all of them my thanks for the interest, the time, and the effort which have gone into their discussions, especially those who have expanded upon my theme in bringing out additional relationships, conclusions, and exceptions.

I am pleased to have drawn a discussion from across the Atlantic and wish to thank Mr. Miles for having provided background on practices in Great Britain. There are, of course, many similarities to practices in the United States, even similarities in terminology. His reference to employers' sometimes deliberately adopting a high rate of funding and later converting excess assets into increased benefits is one which has frequently been duplicated here. Likewise, his reference " . . . it is possible to offer alternative immediate costs between which the employer can decide, dependent on his financial position at the time and his prospects in the future" strikes a familiar chord in consulting actuarial practice here.

I hope that Mr. Miles and his British colleagues understand that the guidelines in the paper are not intended to suggest an optimum upper limit on the funding which might be recommended by the actuary or which an employer might choose to do for reasons of his own. Rather, they are intended to suggest the maximum which should ever be *required* if regulation of pension funding should come to pass. These are, of course, two very different things. Contributions in excess of those designed to reach the dual objectives cited in the paper are certainly not proscribed by its language.

My colleague Lambert Trowbridge, with whom I have long been associated on projects related to the private pension movement, has come up with another of his clear expositions in testing certain of my conclusions against findings developed from "models" in his classic paper "Fundamentals of Pension Funding" (*TSA*, Vol. IV). (To those who may not be aware of his article in the *Harvard Business Review* [March–April, 1966], entitled "ABC's of Pension Funding," or who have not yet had an

opportunity to read it, I highly recommend the article as a significant contribution to better understanding of pension funding by the layman.)

I am happy to see that, for the most part, Mr. Trowbridge finds our conclusions consistent. Such differences as exist are largely matters of interpretation or emphasis. While there is no point in an extended discussion of these, it is well to stress again that changes in the actuarial assumptions which may be appropriate in establishing values upon termination of a plan—mentioned in the paper and commented upon by Mr. Trowbridge—not only include changes which may increase costs (such as the elimination of any turnover discount) but also may include changes which will decrease costs (such as a cutoff of salary-scale projection in any plan whose benefits depend on compensation and the possibility that higher interest assumptions would apply to close-out rates, as is true at the present time).

Mr. Trowbridge apparently is not kindly disposed toward projections extending over a score of years or more. While I would be the first to agree that such projections contain substantial unpredictable elements, the same is also true of the projections inherent in an actuarial valuation. I do not believe these uncertainties (on which actuaries apparently thrive!)¹⁶ constitute a serious objection to the making of projections, when it is borne in mind that these, like actuarial valuations, are adjusted to the facts as they evolve from year to year.

Mr. Trowbridge points out that certain illustrations in the paper involve the use of interest gains to accelerate the amortization of the unfunded cost of accrued benefits. He also comments that while this “seems reasonable and practical, it is not clear that it would fit the usual interpretation of IRS regulations.” This latter comment may create the impression that Treasury Department attitudes would be unfavorable to the use of gains to amortize unfunded costs rather than to reduce contributions. Bearing in mind that it is the maximum tax-deductible contribution with which the IRS is primarily concerned in the application of gains, we should find no problem so long as the contribution claimed as a deduction does not exceed the maximum. The illustrations in the paper would, I believe, satisfy IRS requirements.

Two other close associates in pension and Society affairs, Pres Bassett and Fred Sloat, have also contributed significantly to the value of this paper with their discussions. Fred Sloat, who has perhaps the closest relationship to the accounting profession and who did more than any other individual to lead the Accountants Research Staff through a maze of actuarial technicalities on its way to the publication of *ARB No. 8*

¹⁶ See *Fortune* (December, 1965, and January, 1966).

(the Hicks report), oriented his remarks to questions of accrual accounting for pension costs—in particular to the manner and extent of charges for so-called past-service costs. It is a timely contribution in view of the exposure draft on pension cost accounting to be released in August, 1966, by the Accounting Principles Board for review and comment. He has also provided us with interesting background relative to the evolution of actuarial cost methods in group annuity and trust funding.

Pres Bassett stresses the importance of funding flexibility to the sound development and financing of private pension plans, a view with which I agree entirely. I trust that there is nothing in the paper to imply that a rigorous pattern of contributions need be followed in order to reach the long-range funding objectives set forth therein, for such was certainly not intended. Mr. Bassett also logically expands the concepts of funding adequacy to encompass variations in the rate of funding according to the level of benefits in different situations. This is commented upon shortly.

Jack Dyer touches briefly on the same two points. He also expresses some reservations with regard to accrued-benefit concepts, for which he draws on an experience involving a change from a final average salary plan to a career average salary plan. The method followed in the case he illustrates apparently was selected deliberately on the generous side for industrial relations reasons and presumably because the company plan was *continuing* rather than terminating. Had it been a *terminating* plan, I feel that Jack would agree that it would have been impossible to justify promoting benefits to a compensation basis higher than employees had ever earned up to that time. Jack's example deals with a continuing plan and clearly does not relate to the accrued-benefit concept on termination of a plan, which, of course, is the sense in which the accrued-benefit test of adequacy is used in the paper.

Whatever the problems encountered in defining accrued benefits, we should avoid falling into the semantic trap of "benefit expectations," which has become a favorite expression of certain writers who have been urging compulsory vesting as well as strict regulation of funding. Arguments based on subjective "expectations" (often entirely unrelated to the rights conferred by the terms of a plan) serve to derail objective thinking on the pension question and to provide grist for the political mills. I interpret certain of Mr. Dyer's comments as making the point that those who wish to elevate expectations into legal rights may improperly cite portions of the paper in furtherance of their aims. If so, this would be most unfortunate.

I agree with Jack that care should be exercised to avoid statements which can be lifted out of context and used to support arguments by those who wish to impose statutory limitations on pension funding, whether these limitations are concerned with maximum or with minimum funding. Still, it is difficult to see just what can be done to protect one's self fully against this possibility. Remaining silent is certainly not the answer, despite the curious fact that one frequently finds the same reference cited in defense of opposing viewpoints.

Referring to certain comments by Messrs. Bassett and Dyer, as well as by Dorrance Bronson, I agree that there are circumstances where deliberate "overfunding" of a pension plan should be considered. One circumstance in which heavier funding might be desirable would be the existence of a period of unusual company prosperity, during which a cushion could be built against a possible subsequent period of poor profits or company decline. I am not quite as sympathetic to the argument that the "inadequacy" of current benefit levels (however this may be determined) dictates heavy current funding. The very smallness of the benefits may, of course, make more *practical* their rapid funding in some situations, but under a union plan a heavily funded position may itself precipitate benefit demands. It seems to me that to use the argument of benefit inadequacy as a compelling reason for a higher level of funding is just another way of saying the actuary should in fact value benefits which have not yet come into being.

The argument of benefit inadequacy implies the existence of set standards for all plans, when there are none. In any given situation an assessment of existing benefit levels will depend on many factors, some of them largely subjective. Is it the function of an actuary, or of the employer, or his unions, to decide that existing pension provisions are inadequate? Surely this involves an economic or business decision, not an actuarial one. I agree with Mr. Bassett's statement, "It may not be the job of the actuary to answer these questions, but it is certainly the actuary's responsibility to be sure that the client understands the implications of these questions."

Moreover, in deciding whether advance provision should be made for possible future escalated benefits, a number of related questions must necessarily be raised, including (a) whether the company should attempt to justify the costs of tomorrow's plan today; (b) whether investment considerations and the changing value of the dollar would make one course preferable to the other; (c) whether future changes in social security benefits may reduce the need or pressures for increasing the private plan's

benefits; and (d) whether stockholders would look favorably on any substantial overfunding which may later prove to have been done unnecessarily.

Paul Jackson corrects my oversimplification to the effect that "there need be no basic conflict between a company and its employees in the funding of a retirement plan." It is clear that there can be a conflict, although this is by no means necessarily so. I believe that it is well to recognize two factors which today bring these interests close together. The first is the fact that most funding decisions are made by management *employees* who have a personal stake in the success of the company's retirement plan. The second is that the financial welfare of the company is also vital to employee security, whether this relates to job permanence, pension funding, or pension plan permanence. The two interests are thus linked. The funding flexibility which may be vital to a company may also give the employee greater assurance of plan permanence even though the flow of contributions into the pension fund has temporarily been slowed.

I agree with Mr. Jackson's observation regarding the inadequacy of most salary-scale assumptions today (for which there are generally good reasons, it may be noted). Observations about salary-scale deficiencies are apt to be somewhat misleading within the context of this paper, however, unless one distinguishes between the effect of a salary scale on the achievement of the respective funding objectives for a terminating plan and for a continuing plan. The greatest impact of a salary-scale deficiency is on the funding for the continuing plan, and it will have considerably less effect on the adequacy of funding for accrued (or vested) benefits in the event of plan termination. Under a plan calling for the use of a salary scale in the valuation, the accrued benefits on termination, being based only on pay to that point of time, are, of course, less than a pro rata part of the projected benefits at retirement. Previous funding with a salary scale (even if deficient for a continuing plan) will generally have been done on the basis of assumed final pay higher, on the average, than actual pay achieved at termination date. Therefore, benefit security upon termination may be substantial in relation to the years of funding.

Walter Shur, whose skilful and searching appraisal of the financing of federal retirement systems appeared in *TSA*, Volume XVI, has given us some interesting comments on recent recommendations of the Cabinet Committee on Federal Staff Retirement Systems relative to improved funding of the Civil Service Retirement and Disability Fund. He mentions a connection between these recommendations and those of the President's Committee on Corporate Pension Funds (etc.). Whether there should be a "double standard" in the funding of benefits for employees

of public and private institutions is a moot question. At any rate, he and a number of other actuaries have raised their voices in support of what the paper terms "truth in labeling" of the costs of plans for public employees.

It is interesting to have Mr. Niessen's comments relating to funding in the "public" field. Even though it is an unintended result from the author's viewpoint, I am glad that he finds the paper useful in supporting the funding practice which has been followed by the Railroad Retirement Board. Apparently the funding of railroad retirement benefits is proceeding on a sounder basis than the funding of benefits for federal employees.

Messrs. Krinsky and Elkin, in their joint discussion, have added some strong arguments against legislation of mandatory vesting provisions and regulation of the funding of private plans. I wish to register my agreement with their arguments, especially those relating to benefit priorities in the establishment of plans. While these points may be self-evident to most pension consultants and actuaries, they nonetheless seem to have escaped those who are most vocal in their promotion of such legislation and regulation.

These two gentlemen also point out a basis for logical variation in the period of amortization of accrued-benefit costs, according to permanence and stability of employment in an industry or in an individual company. These observations are quite in keeping with the long-range objectives and concepts set forth in the paper.

Messrs. Krinsky and Elkin introduce alternative interpretations of accrued benefits, using as their example a plan providing a definite benefit per year of service with a maximum limit on the service credited. A decision as to the applicable alternative in a plan of this kind usually does not pose a serious problem, since most plans with early retirement provisions make clear whether or not a proration of the maximum benefit at normal retirement date is intended at an earlier date, when more than the maximum number of credited years would have been rendered at normal retirement date. The coming Pension Research Council study of benefit security sets forth in its methodology a logical method of determining accrued benefits, based in large part on the terms of the early retirement provision in the particular plan. Misunderstandings arise over this point, it is true; I have recently testified in a court case involving this question. I have found that those whose familiarity with pension plans extends primarily to plans for public employees, where retirement on full benefits is frequently permitted after a given number of years of service regardless of age, tend to relate accrued benefits under private plans to the different standards with which they are familiar.

Mr. Feay's discussion unfortunately aims beyond the boundaries and

purposes of the paper. Nonetheless, it is interesting to have his comments. The paper was not intended as a treatise on all aspects of pension funding; as was stated on the first page, "The purpose of this paper is to examine certain funding guidelines. . . . It is concerned with principles as well as with the broad impact of actuarial assumptions and cost methods but is not concerned with specific application of the latter." Thus, the illustrations were kept simple and were limited to plans providing normal retirement benefits (including, if you will, actuarially equivalent early retirement benefits and vesting in the form of deferred retirement benefits). The fact that the paper does not mention expenses, or disability benefits, or special early retirement benefits, or optional pensions, or employee contributions, or Mimeo 5717 problems, is consistent with its fundamental purpose.

On one line of reasoning, Dick Daskais' concise comment (on contributions and funding versus charges and accounting) disposes of funding objective *b* of the paper as part of the dual long-range objective—that is, provided accrual accounting for pension costs becomes standard practice (as seems likely) and provided *also* that funding thereafter is completely divorced from the costs for accounting purposes. While it is true that funding could be so divorced under accrual accounting and that there may be many situations in which it would be advantageous for companies to do so for reasons related to those enumerated by Mr. Daskais, I join many other actuaries in the belief that funding will tend to follow or parallel the accounting charges, especially if IRS limits continue in substantially their present form and certainly if compulsory funding standards should become a reality. In any such event, it will still be correct to think in terms of the dual objectives set forth in the paper.

Mr. Berin rightly stresses the importance of actuarial assumptions. This does not happen to be a primary subject of my paper except to the extent that different assumptions were pointed out to be appropriate in the case of a terminating plan and a continuing plan, respectively. That the choice of assumptions should theoretically be the same regardless of funding method may be true for a continuing plan, but it is no longer true when that plan terminates. This dual measurement is, of course, at the heart of the paper.

There are several minor points in Mr. Berin's discussion with which I do not agree, and I will but mention them. First, I do not fully agree with his oversimplification that "all funding methods reach the same goal," unless one is prepared to define the goal, in the ultimate, as no remaining funds and no persons owed benefits. Second, I believe that he has misinterpreted my reference to "cardinal principle in individual life insurance

funding." Third, I am afraid he is incorrect in generalizing that "the accrued-benefit cost does not increase from year to year." In practice, many factors can and do invalidate the latter statement, one of the most important of which is the effect of the benefit formula in changing the weighted cost age over the years.

Mr. Dackow expresses a concern which has been in many minds since the commencement of agitation for compulsory minimum funding standards under private pension plans. As he points out, we may some day be faced with conflicting governmental requirements if limitations imposed for different purposes are not geared logically to these differing purposes. Otherwise one bureau may complain of too little funding, while another seeks to restrict it. Mr. Dackow points out a comparable situation in Canada, where a dual set of standards has created conflicts.

Mr. Roenisch suggests consideration of a new term—"actuarial obligation"—as a substitute for "actuarial liability," since the former is less likely to convey the impression of a debt. Others may prefer "actuarial deficiency" or some other term, but in any event there is merit in the suggestion that "liability" be dropped. Mr. Roenisch makes a reservation with respect to the funding accomplished by IRS minimum contributions, other than for an immature group; this conclusion was also stated in connection with the illustrations near the end of the paper.

I am completely sympathetic with the views expressed in the last paragraph of Mr. Roenisch's discussion, relative to the basis on which regulatory authorities might best satisfy themselves as to the adequacy of a funding program. Actuarial certification by *accredited* actuaries, along lines similar to those which he has suggested, could provide the answer.

Almost any response would be inadequate to the nostalgic literary journey on which Dorrance Bronson has taken us in his discussion of the paper. It should be unnecessary to remind one such as he (whose own prolific works are milestones in the pension literature) that the "new" does not supplant the "old" but merely supplements it.

I am glad that Mr. Bronson has uncovered the intended emphasis on *minimum* rather than *optimum* funding guidelines and that he has also divined my purpose in presenting the matter in this way. If the presentation of the guidelines was more forthright than one would find in a "pro and con" answer to a Fellowship examination, I should think this appropriate in the circumstances.

Turning to specific points in his discussion, Mr. Bronson feels that the term "overfunded" should have been given a specific definition in the paper. I had assumed that the definition of "fully funded," given near

the beginning of the paper, took care of the matter adequately—that is, “overfunded” being anything in excess of “fully funded.”

Mr. Bronson sets forth six principal comments (including mental reservations in some cases) on the guidelines given in the paper, based on his own previous writings and conclusions. In answering each of these, I do so in the knowledge that our views are actually much closer in fact than may appear from the different emphases that we have placed on certain points.

1. I agree wholeheartedly that Mr. Bronson’s definition of “actuarial soundness” on page 14 of his well-known book *Concepts of Actuarial Soundness in Pension Plans* is a reasonable and workable one; in fact, I have recently had occasion to use it to advantage in a court case. There is no conflict between this definition and the concepts discussed in my paper.

2. Seventeen years ago, Mr. Bronson raised the question whether “industry, in general, could *afford* full funding objectives.” Mr. Bronson’s thought here is not directly related to the ideas in the present paper or even a motivation therefor. There may possibly be a parallel, but, if so, it is somewhat obscure.

3. Whether, as Mr. Bronson implies, a number of situations explored in his paper “Pension Plans: Provisions for Termination of Plan” (*TSA*, Vol. VII) would have produced unsatisfactory or awkward results if measured against the criteria of my paper would, of course, depend upon whether similar principles were followed in funding the supplemental benefits on death or termination of service. For reasons of simplicity, my paper dealt only with the funding of age-retirement benefits, but its principles could be expanded to include the funding of other types of benefit as well.

4. Mr. Bronson appreciates that the paper could not cover the “pension waterfront” in the matter of supplementary benefits and special situations. His listing of such situations will be helpful to those desiring to pursue the matter further.

5. In connection with the interesting example given with respect to mergers, it seems clear to me that a price had been paid for the “superior position” which one of the companies apparently enjoyed in the negotiations, namely, a price equal to the additional contributions paid in past years to establish the higher funded ratio, and that such position was not merely a “free” and automatic bonus of a particular cost method.

6. With regard to the advantage cited for overfunding as an aid to the financing of increased benefits at a later date, I have already made reply to Messrs. Bassett and Dyer on this point. There may well be a question whether it is an advantage, or even appropriate, to overfund one system of benefits in order to be better able to afford an expanded system later.

Finally, in Mr. Bronson’s interesting account of the Mallory incident, he has chosen to alter the meaning of the “metaphor of the mountain,” as used in the paper, turning it skilfully in defense of the ideal of complete

funding of "actuarial liabilities," no matter by what method these may be determined. I really cannot let this pass without comment. The "path around," which I suggested as an appropriate route to pension security for the funding caravan, circumscribes not the whole mountain *but only the peak*, which in this case represents the permanent excess of "actuarial liabilities" (as illustrated in the paper) over the "value of accrued benefits."

While I share with Dorrance Bronson an admiration of that indomitable spirit which accepts the challenge of a mountain "because it is there," as well as a taste for the adventure of the climb, still, in the sense in which the metaphor was introduced in the paper, I remain keenly aware that it is not we, but our clients, who must finance the expedition and take the risks.

In bringing these lengthy remarks to a close, I should like again to express my appreciation to all of the actuaries who have prepared discussions of the paper.

Obituary

M. ALBERT LINTON
EDWARD A. RIEDER
LEWIS G. SMITH

1887 M. Albert Linton 1966

The sudden death of M. Albert Linton on May 2, 1966, from a heart attack at his home in Moorestown, New Jersey, brought to a close the career of a distinguished former president of the Society whose leadership in a broad range of professional, cultural, and humanitarian activities has left lasting contributions not only to the life insurance industry but to the entire nation. Still vigorously engaged in a multitude of activities at the age of 79, Mr. Linton on the day of his death had attended a meeting of the executive committee of his company and participated actively in its deliberations.

Mr. Linton was born April 4, 1887, in Germantown, Pennsylvania. He received his education at Moorestown Friends School, Westtown School, Haverford College, the University of Michigan, and the Federal Polytechnic Institute of Zurich, Switzerland. At Haverford he earned the B.S. and M.A. degrees and was elected to Phi Beta Kappa. In 1934 he received an LL.D. degree from the University of Miami (Ohio).

His entry into the life insurance business was almost accidental. In 1909 he was working for an electrical instrument manufacturer for \$7.50 a week and was shortly to receive his Master's degree in mathematics from Haverford College. The president of the college received a telephone call from the president of the Provident Life and Trust Company (now Provident Mutual), inquiring whether he could recommend "a Quaker boy good at mathematics" who had the potential to become an actuary. Fortunately, the college president recognized the potential. "I didn't know then what the word 'actuary' meant," Mr. Linton liked to relate, "but I grabbed the chance to make \$1,000 a year."

The college president's recommendation proved more than justified. Within five years, at the age of 27, Albert Linton had attained Fellowship

not only in the Actuarial Society of America but in the British Institute of Actuaries as well. His progress in Provident Mutual was equally spectacular. In 1913 he was made an officer with the title of mathematician. He became associate actuary in 1915, vice-president in 1916, and president in 1931. From 1953 to 1958 he served as chairman of the board and remained on the board as a member of its executive committee until his death.

To the Actuarial Society of America, the American Institute of Actuaries, and the successor Society of Actuaries, Mr. Linton gave unstintingly of his time and talents. Prior to his two years as president of the Actuarial Society of America beginning in 1936, he had served eleven years as a member of the council and six years as vice-president. Following his presidency he remained on the council until the formation of the Society of Actuaries in 1949 and served as a member of the board of governors during the ensuing year. He was the author of numerous papers published in the *Transactions* and the *Record*, and he participated frequently in discussions at meetings of both the Society and the Institute. The well-known Linton A and B Termination Rates, published in 1924, are still widely used. In addition to his activities in the American societies, he served as chairman of the United States Section of the Permanent Committee for International Congresses of Actuaries and participated in several of the Congresses abroad.

The list of his offices and honors in other organizations of the life insurance industry further evidences his capacity for leadership and the esteem in which he was held. He had been chairman of the Institute of Life Insurance and the Life Insurance Sales Research Bureau and president of the Life Insurance Association of America. He played the central role in the creation and development of the Life Insurance Medical Research Fund, of which he was chairman during its first eleven years. In 1939 and again in 1948 he served on the Advisory Council on Social Security. In recognition of his outstanding services to the industry, he received the John Newton Russell Memorial Award in 1946, was named Life Insurance Man of the Year in 1950, and was elected to the Insurance Hall of Fame in 1960.

In life insurance circles Mr. Linton is probably best known as a staunch champion of cash-value life insurance. In books, pamphlets, public appearances, and radio debates, he had long been the industry's most articulate defender of permanent life insurance against the advocates of "buy term and invest the difference." His book *Life Insurance Speaks for Itself* has been an effective weapon against the detractors of life insurance.

Besides his life insurance activities, Mr. Linton was a director of several

corporations, as well as an officer or active participant in a host of cultural, civic, educational, and humanitarian organizations. An enumeration of these activities is beyond the scope of this appreciation, but two which were particularly close to his heart should not go unmentioned. These are his dedicated labors on behalf of many projects of the Society of Friends, particularly the American Friends Service Committee, of which he was a director; and his many years of distinguished service to the Academy of Natural Sciences in Philadelphia, of which he was president for more than a decade.

Amid these manifold activities he somehow found time for his many hobbies, all of which involved the outdoors. An ardent mountain-climber and skier, he scaled many of the most challenging peaks of the Alps and the Rockies. He was an internationally known amateur ornithologist; indeed, one species of Ecuadorian flycatcher has the word *lintoni* as a part of its Latin name in recognition of the man who identified it. Combining his love for birds and animals with his interest in photography, he spent much time in his later years taking colored motion pictures of wildlife in the Everglades and in Africa. He was also an enthusiastic golfer.

Mr. Linton far more than repaid his debt to his profession; indeed, he left the actuarial profession and the life insurance industry vastly in his debt. Humanity itself is the gainer from his lifetime of dedicated service.

He is survived by his wife, Mrs. Margaret Roberts Linton, a son, Morris Albert, Jr., and seven grandchildren, to whom we extend our sympathy.

1908 Edward A. Rieder 1966

Edward A. Rieder, chairman of the board of directors of the Mutual Life Assurance Company of Canada, passed away May 23, 1966, following a lengthy illness. The fortitude with which he faced the ordeal of his illness and the courageous way in which he continued to devote himself to his company's affairs as long as he was able to do so have commanded the unreserved admiration of all who knew him.

Mr. Rieder was, by nature, reserved. He did not covet honors for himself, and he shunned the spotlight of publicity. However, imbued with a tremendous sense of responsibility, he brought to the offices which he held a wealth of dignity and efficiency. His judgments and decisions were always based on high principles. His quiet friendliness, his sincerity, his keen sense of humor, and his optimistic outlook will be greatly missed.

He was born in Kitchener, Ontario, on October 6, 1908, a son of Mar-

garet M. and the late Talmon H. Rieder. He attended Kitchener-Waterloo Collegiate and graduated from the University of Toronto in 1931 with the degree of Bachelor of Commerce. He joined the actuarial department of the Mutual Life of Canada in 1931, and his entire business career of nearly thirty-five years was with this company.

He became a Fellow of the Actuarial Society of America in 1937. He was appointed assistant actuary of the Mutual Life in 1938, associate actuary in 1945, actuary in 1948, assistant general manager and actuary in 1954, and vice-president and general manager in 1958. In February, 1959, he was elected executive vice-president, and in May of that year he was elected president, which office he held until he was elected chairman of the board in 1964.

Mr. Rieder was a Fellow of the Society of Actuaries. He was also a Fellow of the Canadian Institute of Actuaries and a past president of its predecessor organization, the Canadian Association of Actuaries. He was a past president of the Canadian Life Insurance Association and served with distinction in this and other capacities for the Association.

He is survived by his mother; his wife, formerly Jean Rudell; a son, Thomas; three daughters, Sally, Mardi, and Janet; a brother and a sister—to whom our sympathy is extended.

1897 Lewis G. Smith 1966

Following a lengthy illness, Lewis Gordon Smith died in Weston, Ontario, on May 2, 1966. Born August 9, 1897, in Conewango Valley, New York, Mr. Smith came to Grimsby, Ontario, at an early age and received his education there and at the University of Toronto, graduating from the latter in Mathematics and Physics. He served in the Canadian Army in World War I.

Mr. Smith joined Confederation Life Association in 1921. He became an associate of the Actuarial Society of America in 1926 and of the American Institute of Actuaries in 1927. He worked in several of the actuarial departments and was an assistant actuary when he was forced to retire in 1953 because of ill health. He was particularly expert in reinsurance and dividends.

In spite of his long and severe illness, Mr. Smith maintained a keen interest in current affairs and an amazing cheerfulness in the face of adversity. He is survived by his widow, two sons, and seven grandchildren.

MINUTES OF THE LATER REGIONAL MEETING
of the
SOCIETY OF ACTUARIES

HELD AT THE DRAKE HOTEL
CHICAGO, ILLINOIS
JUNE 2-3, 1966

The meeting was called to order by the President, Gilbert W. Fitzhugh, on Thursday, June 2, at 10:00 A.M. The following is a summary of the attendance at the meeting:

SUMMARY

Fellows.....	173
Associates.....	260
Guests.....	32
Total.....	465

The President announced that word of the deaths of three members had been received since the last meeting: M. Albert Linton and Edward A. Rieder, Fellows; and Lewis G. Smith, Associate. The members stood in silent tribute to the memory of these deceased members. Obituaries are printed in this volume of the *Transactions*.

The President reported that the minutes of the Regional Meeting, held May 27-28, 1965, as printed in the *Transactions* (XVII, 225-26, inclusive), and of the Annual Meeting, held October 25-27, 1965, as printed in the *Transactions* (XVII, 603-7, inclusive), were approved at the Regional Meeting in San Francisco.

The President also reported that the proposed amendments to the Constitution, which redesignated the present Article VIII as Article IX and substituted a new Article VIII entitled "Indemnification of Society Members," were approved at the San Francisco meeting.

The Secretary presented a summary of the business transacted by the Board of Governors at a meeting held on April 27, 1966. This is printed in the minutes of the April 28-29, 1966, Regional Meeting in this volume of the *Transactions*.

The meeting then proceeded to the presentation and discussion of papers.

Following the presentation and discussion of papers, the President ex-

tended a welcome to 98 new Associates who were attending their first meeting of the Society after attaining their present status.

Then followed a panel discussion of agency matters. Mr. Edward G. Newcomb acted as panel chairman, with Mr. Norman F. Buck and Mr. Harold F. Philbrick as panel members. Following the panel presentations, there were informal discussions and questions from the floor.

As an introduction to a discussion of the Future of the Society, a report on the American Academy of Actuaries was presented by Mr. Edward D. Brown, Jr., and Mr. David A. Logie presented a report on the Canadian Institute of Actuaries. After discussions from members on the Future of the Society had been presented, the morning session adjourned at approximately 1:00 P.M.

The meeting reassembled at 2:30 P.M. in two separate simultaneous sessions. Mr. Maynard I. Kagen presided over the session on Individual Life and Health Insurance. The session on Employee Benefit Plans was chaired by Mr. John R. Williams, assisted by Mr. Richard B. Sieben. Following the panel presentations, there were a number of questions and comments from the floor. These sessions adjourned at approximately 4:00 P.M.

On Friday, June 3, the meeting reassembled at 9:30 A.M., with Vice-President Edwin B. Lancaster presiding. The topic of Long-Range Planning was initiated by Mr. J. Stanley Hill, who then introduced Dr. John D. Hogan, of the Northwestern Mutual Life Insurance Company, a guest, who gave a talk, illustrated by slides, on the long-range-planning activities of his company.

As an introduction to a panel discussion of Title XIX, Vice-President Bruce E. Shepherd asked the Secretary to read a report on the United States Medicare written by Mr. C. Manton Eddy. This was followed by a report from Mr. George N. Watson on recent developments in Canada in the field of health insurance. The meeting was then turned over to Mr. Richard H. Hoffman as panel chairman for a discussion of "Implications of Title XIX of the 1965 Social Security Amendments for the Private Health Insurance Business." Members of Mr. Hoffman's panel were Mr. Paul Singer, of the Continental Casualty Company, a guest; Mr. John C. Angle; and Mr. Walter Shur.

Following this panel presentation, President Fitzhugh again presided over the meeting. After a brief further discussion of the Future of the Society, the President closed the session at 12:15 P.M.

A Smaller Company Forum was called to order by Chairman Howard G. Eimers at 2:00 P.M. on Friday, June 3, 1966. After discussion of questions on the program, the meeting adjourned at approximately 4:00 P.M.

TRANSACTIONS

OCTOBER, 1966

ADDRESS OF THE PRESIDENT, GILBERT W. FITZHUGH
THE RESPONSIBILITIES AND CHARACTERISTICS
OF ACTUARIES

IT is appropriate that the President have an opportunity to address the Society at the termination of his term of office, when his feeling of indebtedness to his profession is probably at its peak. In thinking along these lines, I reviewed the recent presidential addresses of the Society and of our mother organization, the Institute of Actuaries in England. I found them very stimulating reading and commend them to you. It is said that if you steal one idea it is plagiarism, but if you steal ideas from many different people it is research. I choose to call this effort "research." Putting it another way, when I found so many good points made, I thought many of them were worth repeating. I hope that I can add a few ideas of my own to the accumulated knowledge and wisdom of my predecessors.

It is first my pleasant duty to record here my deep thanks to the Officers of the Society, the Board of Governors, and all those who have worked so hard on our committees and, in fact, to the entire membership for their wholehearted support and co-operation during my term of office. It has been a wonderful experience, and I thank all of you for making it possible.

We are all aware of the expansion of the fields requiring actuarial services and of the opportunities this affords our members. There is perhaps one characteristic that is common to all these diverse fields, and that is that practically all actuarial work involves the evaluation of long-term obligations that can reach many years into the future. Whether or not these long-term obligations will be fulfilled is determined, perhaps more than by anything else, by the ability and integrity of the actuary contributing his professional skills to the designing of the terms under which the obligations are undertaken.

In the life insurance and pension business, we are not providing a product that the user can evaluate very promptly after delivery to be sure he got what was promised. Particularly in the life insurance business

we are selling a product where the buyer knows by the very nature of the business that he may not be there when the product is delivered to see that it is "as advertised." I cannot think of any higher form of trust than to be given an important share in the responsibility for carrying out a man's hopes and dreams for himself in his retirement years or for his family when he is no longer here to provide for it himself. It seems to me that we as actuaries must never forget our share in these responsibilities in performing our day-to-day activities, regardless of our fields of endeavor.

Along with these tremendous increases in the responsibilities of actuaries, there has fortunately been a great increase in the numbers of our members. All these developments are stimulating, and we take great pride in them and the way our profession is meeting its responsibilities. This very growth, however, presents us with new problems. One of our primary objectives has always been to develop new members qualified to discharge the professional responsibilities of actuaries to society, to the Society of Actuaries, and to each other. I think that the Society of Actuaries can claim that an outstanding job has been done in this area. While the primary job is naturally one for our hard-working and effective Committees on Education and Examination, there is much more to the training and education of a truly qualified actuary than that related to preparation for and the passing of examinations. A great variety of qualities is developed through daily work on actuarial jobs and through the personal associations between younger and older members of the Society. In the past, when the Society was much smaller and the responsibilities of its members were more homogeneous, this development process was easier. A large part of it came about through a sort of osmosis. With our current size and heterogeneity of interests, it is more difficult.

For these reasons, and with apologies for any sermonizing that may appear to creep in, it occurred to me that it might be helpful to try to set down just what seem to be the characteristics of an actuary. What manner of man is he? What are the components that make a man want to enter this profession, that help to equip him to meet his professional responsibilities, and that help to make him successful in his chosen endeavor?

While actuaries come from many different backgrounds and travel many different roads to their goal, it seems to me that there are common characteristics that can be found in successful actuaries in whatever fields of work they apply themselves.

1. *Mental discipline.*—Nobody inherits a Fellowship in the Society

of Actuaries. Nobody can have one handed to him. Nobody can be forced into the study necessary to become one. The only way that it can be accomplished is for an individual to decide for himself that he wants to spend the time and effort required to go through the rigorous examination system in order to achieve the professional designation at the end of the road. The self-discipline learned in the process is one of the most useful results of Fellowship and is very helpful in any line of endeavor, even if the actuary leaves the field of actuarial work. Success in this area cannot be achieved without a genuine aptitude and a real liking for hard work. This is a characteristic that our two countries could well use in more abundance in many fields.

2. *Analytical ability.*—Related to this quality of mental discipline, and arising to some extent out of it, is the need for analytical ability. Regardless of an actuary's field, he must be able to analyze a problem and to separate the wheat from the chaff without being swayed by irrelevant matters—no matter how persuasively they are advanced.

This means that he must be able to recognize the essential ingredients and, in fact, introduce them if they have been omitted from the information supplied him.

3. *Knowledge.*—It is difficult to apply analytical ability to a specific problem without background knowledge. If it were not so difficult to assign priorities, I would put "knowing your business" up near the top of any necessary qualifications for a good actuary. No matter how smart a man is, he is not going to be very useful as an actuary if he does not have sufficient experience in his particular field to know what has gone before him and just what rules of the game apply in each instance. He must have sufficient "feel" of the problem at hand to know his way around it and to see how it fits into the broader scheme of things. In fact, he must have this knowledge and experience ingrained in his bones so that it becomes a matter of second nature to him.

To quote Mr. J. H. Gunlake from his presidential address before the British Institute in 1960, the actuary must have "the intuition to perceive the connection between apparently unrelated things."

4. *Ability to apply and express knowledge: Communications.*—It hardly needs emphasis that an actuary's usefulness is limited—no matter how good he is at coming up with the right answers—if he cannot communicate these answers to others in such a way that they can understand and accept them. Naturally, the first requirement is to get the answer, and the technician who can take a mass of information and come up with an answer is necessary and useful. How much more useful he becomes if he also possesses the ability to communicate the problem,

the solution, and the reasons therefor to others who have not had his technical training!

5. *Sound judgment*.—Each of these qualifications leads naturally to the next. Analytical ability, knowledge, ability to communicate, are all essential, but they are not of much value to an actuary unless they are coupled with sound judgment. It is possible to come up with a beautiful theoretical answer that just is not worth anything in a particular set of circumstances. One must know not only that the answer is correct but that it makes sense. I am sure that all of you know people who are very smart indeed and have all the answers, but they are seriously lacking in judgment. Sometimes they do not recognize what is important. They fight, bleed, and die for something that they are quite right about but that really does not make any difference. It is necessary to know when to fight and when not to fight, what is fundamental to the solution of the problem at hand and what can be passed over in the interests of getting to something more important. That takes practice and experience as much as anything else; it is difficult to get out of a book. In fact, we all know people who have acquired this kind of judgment even without the benefit of actuarial examinations.

6. *Creativeness*.—Here is a quality that is a little different from the preceding five very closely interrelated qualities but which is nevertheless very important. There are, of course, many areas in which actuaries can make great contributions without necessarily being creative. We must have technicians, and it in no way downgrades their importance to say that we must also have new ideas. Sometimes it is necessary to have one man or a group of men to dream up the new ideas and to have technicians to look for the holes. This often happens. On the other hand, a competent technician who is also creative, who can dream up a new kind of insurance policy, design new forms of employee benefits to meet changing needs, or develop new ways of extending coverage or of funding a pension plan in order to meet the particular objectives of an employer or union or both, and who can at the same time recognize whether it is practical as well as creative is a particularly valuable actuary.

7. *Integrity*.—Finally, and the quality that I would confidently say is the most important of all, is complete integrity. I cannot think of what use an actuary is to anyone, whether he is in the insurance field, the consulting field, the employee benefit field, or government operations, if his integrity is not above reproach. And it must not only be above reproach; it must also *appear to be* and *be known to be* above reproach. An actuary is most useful if everyone acknowledges that when he says something one can count on it. You may decide not to take

his advice for what, to you, are good and sufficient reasons, but at least you know that he is giving you a straight story as he sees it and is not cutting corners or holding anything back.

I think that it is probable that every Fellow of our Society has at one time or another been in a situation in which his personal integrity was his strongest support. This is true whatever line of actuarial work one pursues. Actuaries seem to find themselves in the middle of all problems—some people urging a decision one way and some urging a decision another way. In not a few situations he will find himself alone, with everybody pressuring him to make a decision a certain way when he knows very well he should not.

It is not only actuaries who are under pressure in this competitive world we live in. But actuaries have responsibilities of a very special nature, and, if we want to be worth our salt as professionals, we must recognize our responsibilities and stand firm when the going is tough. In the present state of the social and economic development of the United States and Canada, in fact, of the entire world, it seems to me that the actuary's responsibility extends beyond whatever particular problem he may be considering at the moment. His decisions or his advice can vitally affect the future economic and social development of the whole country. His work is closely enmeshed in the warp and woof of its economic fabric. With 130 million policyholders, together with their beneficiaries, there is hardly a family that does not have part of its economic future tied up with a life insurance company. With 35 million people having a private pension program of one form or another, we have another tremendous group of people (with considerable overlap, of course) relying on actuaries to play well their significant role in the sound design and operation of these programs for their economic security in old age. And, when we come to social security, practically everyone in the entire population, whether he knows it or not, has a stake in the quality of the actuarial advice entering into *its* design and operation.

Thus, the individual actuary must decide for himself the extent to which, in order to live up to his personal professional responsibilities, he must consider how his immediate work fits into the broader fabric. He might with truth give a technically perfect answer to a technical question, but he might, at the same time, be lending his personal prestige and dignity—and that of his profession—to an answer that might, in the long run, prove to be against the best interests of the general public, as well as those of his own employer or client. He must decide whether he is within his professional rights if he sticks to his technical

answer, or makes a technical calculation on a set of assumptions set out by someone else, without pointing out any broader implications that may be involved.

If the actuary can go one step further, he is really a paragon of virtue; that is, if he can not only resist pressures and come up with the right answers but also persuade the other fellows to accept those answers and even like them. This is a nice tightrope to walk and requires large doses of all the qualities previously outlined in order to carry it off.

To help him in some of these situations, the actuary has a set of guides. All of you are familiar with the Society's "Guides to Professional Conduct." While these can be helpful in the particular situation in which an actuary's responsibilities appear to be complicated and cloudy, they are, of course, founded on the basic rock of personal character. *Without* personal character and integrity, guides to professional conduct are meaningless. *With* basic personal integrity, they are really not necessary—but they are and can be helpful.

There is one other subject that I would like to mention today, since I believe that it fits in with this concept of an actuary's responsibilities to society, to the Society, and to each other. That is the question of whether the Society as such, its Board of Governors, or any of its committees can or should take a position on matters of public concern in areas wherein actuaries have special professional competence.

The Society's Constitution states, "No resolution expressive of opinion shall be entertained at any meeting of the Society of Actuaries." This has often been referred to as prohibiting the Society from taking a position. I am not sure whether it should or should not be expanded to imply any prohibition other than the limited prohibition it plainly states. However, rather than debate what this provision of the Constitution means, I hope that the Society will at an appropriate time have a full-dress discussion of the matter on its merits.

Mr. Walter Klem's Committee on the Future Course of the Society is giving this matter serious consideration, and you will hear further from him later in this meeting. Right now a very pertinent question is before us, arising out of the Cabinet Committee on Pension Programs appointed by the President of the United States.

The Cabinet Committee has asked Mr. John Miller, who is chairman of the Society's Special Committee on Pension Plan Problems, to designate a group of actuaries to discuss the Cabinet Committee's report. So far, Mr. Miller's group has participated in this discussion as a group of individual actuaries. However, we may soon be faced with the question of whether the Society should make any statement or appearance

before a legislative committee considering specific legislation in this area. This particular pension subject will also be open for discussion during the course of this meeting.

Without coming to any preconceived opinions, I do hope that the entire question of whether the Society, its Board of Governors, or one of its Committees should be able to take a position on a public question of particular interest to actuaries in their individual professional capacities will receive very careful, thorough, and sympathetic consideration. I recognize that the very points on which we may be asked to take a position are likely to be controversial, and it may often be quite difficult to reconcile substantial differences of opinion within the Society. However, this is undoubtedly true of other professional organizations that do take public positions on important subjects. And, if we confine ourselves to matters strictly within our professional competence, these difficulties should be minimized.

On any particular matter brought to our attention, of course, it may not be possible to obtain anything approaching a consensus. However, even if it were necessary to publicly state that there are differences of opinion within the Society and to set forth the reasons for each opinion, there might be occasions when that would be preferable to taking no position at all. Just where in this area the question of taking a position on the President's Cabinet Committee on Private Pensions would fall can better be determined after we have the benefit of the discussions later in this meeting.

Perhaps it would be appropriate to terminate this part of my remarks by quoting Andy Webster in his presidential address in 1964: "If we continue our silence about matters within our knowledge and our skill, some other, and possibly less capable, group is going to make itself heard."

From my remarks here today, you may have gathered that I think the Society of Actuaries is a great organization. In my not entirely objective viewpoint, it is made up of an outstanding group of citizens. We have inherited splendid traditions from our predecessors. It is the Society of Actuaries that gave us all the opportunity to get where we are today, and it seems to me that we owe a great debt to the Society and to the actuarial profession. Speaking for myself, if I had the choice to make over again, I can think of no better profession to have entered. I am glad and proud to be an actuary.

It is up to us to uphold these fine traditions and to do everything possible to continually improve the reputation and the character of the professional actuary and his Society. I would find it difficult to improve

on the words used by Mr. Frank M. Redington in his presidential address to the Institute of Actuaries in 1958: "The tradition which is handed to us is something alive, something to be added to, and what any one of us adds must, for better or worse, be part of himself."

As I turn over the duties of this high office to my eminent successor, Harold Lawson, and look down the future to the long line of succession beyond, and as I think about the many friends that I have made in the Society of Actuaries, I know that the future of the Society is in good hands. I commend it to all of you.

EXPANSION OF PROBABILITY DENSITY FUNCTIONS AS A SUM OF GAMMA DENSITIES WITH APPLICATIONS IN RISK THEORY

NEWTON L. BOWERS, JR.

INTRODUCTION

THE basic idea in this paper follows from the observation that, in calculations involving the distribution of total claims in a risk-theory setting, several authors have found it desirable to approximate a risk-theoretic distribution by use of the incomplete gamma function. In a recent paper, D. K. Bartlett [1] evaluated an excess loss ratio premium by use of a gamma distribution whose first and second moments agreed with those of the risk-theoretic loss distribution. Professor D. A. Jones, in his discussion of the Bartlett paper, pointed out that a translated gamma distribution allowed one to make the approximate distribution agree with the risk-theoretic distribution as far as the first three moments are concerned. The method presented in this paper allows one to make as many moments of the approximate distribution as desired agree with those of the risk-theoretic loss distribution.

In Section I of this paper, the formulas for an expansion of a probability density function as a sum of gamma densities will be developed. The formulas presented will allow one to make the first five moments agree, but the method is easily extended to include more moments. In Section II, the result of Section I is adapted so as to be of use in a risk-theoretic setting. Formulas are developed for the probability distribution function and for the stop-loss or excess loss ratio premium. In Section III, two sample calculations are made: the first will be an example given by Cramér for which exact values are known, and the second will be the Bartlett example. Finally, in Section IV, a peculiarity in the result for the Bartlett example will lead to a criteria for a "best" stop-loss level. Net premiums for two such coverages will then be presented. An appendix to the paper shows another characterization of the type of approximation developed in Section I, and several remarks are made regarding the accuracy of the approximation.

I. METHOD OF EXPANSION

The idea for this expansion is based on the Gram-Charlier series which is described in Mood [6, p. 118]. In the Gram-Charlier series, a density function is expanded in terms of the normal density and its derivatives.

The polynomials which multiply $[1/\sqrt{(2\pi)}] \exp \{-x^2/2\}$ are the Hermite polynomials described in some detail in Jackson [4, p. 176]. They are known to be orthogonal on the entire real axis with respect to the weight function $\exp \{-x^2/2\}$.

Consider the Laguerre polynomials given by

$$L_n^{(a)}(x) = (-1)^n x^{1-a} e^x \frac{d^n}{dx^n} (x^{n+a-1} e^{-x}).$$

The form of the Laguerre polynomials involves a slight change in notation from that usually used and is tied in with the form of the gamma density used in this paper:

$$f(x) = \frac{x^{a-1} e^{-x}}{\Gamma(a)}.$$

One advantage of this form of the gamma density as opposed to

$$g(x) = \frac{x^a e^{-x}}{a!}$$

is that the mean and variance of a random variable with the density $f(x)$ are both a . This change seemed to cut down the length of the formulas in the paper slightly. The first six of these polynomials as defined above are shown below:

$$L_0^{(a)}(x) = 1,$$

$$L_1^{(a)}(x) = x - a,$$

$$L_2^{(a)}(x) = x^2 - 2(a+1)x + (a+1)a,$$

$$L_3^{(a)}(x) = x^3 - 3(a+2)x^2 + 3(a+2)(a+1)x - (a+2)(a+1)a,$$

$$L_4^{(a)}(x) = x^4 - 4(a+3)x^3 + 6(a+3)(a+2)x^2 - 4(a+3)(a+2) \times (a+1)x + (a+3)(a+2)(a+1)a, \quad (1)$$

$$L_5^{(a)}(x) = x^5 - 5(a+4)x^4 + 10(a+4)(a+3)x^3 - 10(a+4)(a+3) \times (a+2)x^2 + 5(a+4)(a+3)(a+2) \times (a+1)x - (a+4)(a+3)(a+2) \times (a+1)a.$$

The pattern of these polynomials is clear at this point. Further, these polynomials are known [4, p. 184] to be orthogonal on the positive real axis with respect to the weight function

$$\frac{x^{a-1} e^{-x}}{\Gamma(a)}.$$

This means that

$$\frac{1}{\Gamma(a)} \int_0^\infty z^{a-1} e^{-z} L_n^{(a)}(z) L_m^{(a)}(z) dz = 0, \quad \text{if } m \neq n.$$

Furthermore,

$$\frac{1}{\Gamma(a)} \int_0^\infty z^{a-1} e^{-z} [L_n^{(a)}(z)]^2 dz = \frac{n! \Gamma(a+n)}{\Gamma(a)}.$$

Therefore, if we assume that a given density function $f(x)$ may be written as

$$f(x) = \frac{x^{a-1} e^{-x}}{\Gamma(a)} [A_0 L_0^{(a)}(x) + A_1 L_1^{(a)}(x) + A_2 L_2^{(a)}(x) + \dots],$$

we may use the above orthogonality conditions to determine the A_n . For

$$\begin{aligned} \int_0^\infty f(z) L_n^{(a)}(z) dz &= \int_0^\infty \frac{z^{a-1} e^{-z}}{\Gamma(a)} [A_0 L_0^{(a)}(z) + A_1 L_1^{(a)}(z) \\ &\quad + \dots] L_n^{(a)}(z) dz \\ &= A_n \frac{n! \Gamma(a+n)}{\Gamma(a)}, \end{aligned}$$

as all other terms equal zero. Therefore,

$$A_n = \frac{\Gamma(a)}{n! \Gamma(a+n)} \int_0^\infty f(z) L_n^{(a)}(z) dz.$$

Now let us assume that we have a nonnegative valued random variable Y with a sufficient number of moments. Further, let us define a second random variable X by $X = \beta Y$, where β is chosen so that the mean of X equals the variance of X . We then pick a equal to the common value of the mean and variance of X and evaluate the various constants A_n , where we assume for the purposes of developing these formulas that X has a density function $f(x)$.

$$A_0 = \int_0^\infty f(z) L_0^{(a)}(z) dz = \int_0^\infty f(z) dz = 1,$$

$$A_1 = \frac{1}{a} \int_0^\infty f(z) L_1^{(a)}(z) dz = \frac{1}{a} \int_0^\infty f(z)(z-a) dz = 0,$$

since a is the mean of X .

$$\begin{aligned} A_2 &= \frac{\Gamma(a)}{2! \Gamma(a+2)} \int_0^\infty f(z) L_2^{(a)}(z) dz, \\ &= \frac{\Gamma(a)}{2! \Gamma(a+2)} \int_0^\infty f(z) [z^2 - 2(a+1)z + (a+1)a] dz = 0, \end{aligned}$$

from the fact that α is the mean and variance of X . Similarly,

$$A_3 = \frac{\Gamma(\alpha)}{3!\Gamma(\alpha+3)}(\mu_3 - 2\alpha),$$

$$A_4 = \frac{\Gamma(\alpha)}{4!\Gamma(\alpha+4)}(\mu_4 - 12\mu_3 - 3\alpha^2 + 18\alpha),$$

and

$$A_5 = \frac{\Gamma(\alpha)}{5!\Gamma(\alpha+5)}[\mu_5 - 20\mu_4 - (10\alpha - 120)\mu_3 + 60\alpha^2 - 144\alpha],$$

where μ_n is the n th moment about the mean of the random variable X . Therefore, the approximate density, a partial sum of the series

$$\frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}[1 + A_3L_3^{(\alpha)}(x) + A_4L_4^{(\alpha)}(x) + A_5L_5^{(\alpha)}(x) + \dots], \quad (2)$$

may be used to evaluate details of the distribution of X and hence corresponding details for Y .

The interesting point of the above formulas is that the A_n may be calculated from one's knowledge of the moments of X even though the exact form of $f(x)$ is unknown. This situation is particularly applicable to calculations on the distribution of the amount of claims paid by an insurance company. The true distribution of claims paid is unknown. However, there are several models of the insurance company, two of which are the collective-risk model and the individual-risk model, and it is possible to evaluate moments of the distribution of claims paid under these two models. Obtaining numerically exact results even of the models themselves is extremely difficult [2, p. 184] except in a few special cases. The method proposed here is a method of approximating the distribution of the claims in the model and thus attempts to do what the Esscher approximation does. The exact distribution of the claims in a model is what is called a risk-theoretic loss distribution in other parts of this paper.

II. FORMULAS FOR THE DISTRIBUTION FUNCTION AND THE STOP-LOSS PREMIUM

We now apply the expansion (2) developed in Section I to obtain formulas for the distribution function and for stop-loss premiums. For simplicity, we shall adopt the following notation:

$$\begin{aligned} A &= \frac{\mu_3 - 2\alpha}{3!}, \\ B &= \frac{\mu_4 - 12\mu_3 - 3\alpha^2 + 18\alpha}{4!}, \\ C &= \frac{\mu_5 - 20\mu_4 - (10\alpha - 120)\mu_3 + 60\alpha^2 - 144\alpha}{5!}. \end{aligned} \quad (3)$$

Then, if we take four terms in the partial sum, the approximation (2) may be written as

$$\frac{x^{a-1}e^{-x}}{\Gamma(a)} \left[1 + A \frac{\Gamma(a)}{\Gamma(a+3)} L_3^{(a)}(x) + B \frac{\Gamma(a)}{\Gamma(a+4)} L_4^{(a)}(x) + C \frac{\Gamma(a)}{\Gamma(a+5)} L_5^{(a)}(x) \right]. \quad (4)$$

If we now substitute the actual polynomials (1) for $L_n^{(a)}(x)$, we obtain

$$\begin{aligned} & \frac{x^{a-1}e^{-x}}{\Gamma(a)} (1 - A + B - C) + \frac{x^a e^{-x}}{\Gamma(a+1)} (3A - 4B + 5C) \\ & + \frac{x^{a+1}e^{-x}}{\Gamma(a+2)} (-3A + 6B - 10C) + \frac{x^{a+2}e^{-x}}{\Gamma(a+3)} (A - 4B + 10C) \quad (5) \\ & + \frac{x^{a+3}e^{-x}}{\Gamma(a+4)} (B - 5C) + \frac{x^{a+4}e^{-x}}{\Gamma(a+5)} (C), \end{aligned}$$

which is just the sum of six gamma densities. In general, if it is desired to have the approximate distribution agree with the risk-theoretic distribution for n moments ($n > 2$), the sum of $n + 1$ gamma densities is required. We may thus approximate the distribution function

$$F(x) = \int_0^x f(z) dz$$

by

$$\begin{aligned} F(x) & \doteq \Gamma(x, a)(1 - A + B - C) + \Gamma(x, a+1)(3A - 4B + 5C) \\ & + \Gamma(x, a+2)(-3A + 6B - 10C) \\ & + \Gamma(x, a+3)(A - 4B + 10C) \\ & + \Gamma(x, a+4)(B - 5C) + \Gamma(x, a+5)(C), \end{aligned} \quad (6)$$

where $\Gamma(x, a)$ is the incomplete gamma distribution function given by

$$\Gamma(x, a) = \frac{1}{\Gamma(a)} \int_0^x z^{a-1} e^{-z} dz.$$

Values of this function have been tabulated by Pearson [8] and Salvosa [9].

Integration by parts shows that

$$\Gamma(x, a+1) = \Gamma(x, a) - \frac{x^a e^{-x}}{\Gamma(a+1)}.$$

By use of this, we arrive, after a long calculation, at an alternate expression for $F(x)$. It is

$$\begin{aligned}
F(x) \doteq & \Gamma(x, a) - A \left[\frac{x^a e^{-x}}{\Gamma(a+1)} - \frac{2x^{a+1} e^{-x}}{\Gamma(a+2)} + \frac{x^{a+2} e^{-x}}{\Gamma(a+3)} \right] \\
& + B \left[\frac{x^a e^{-x}}{\Gamma(a+1)} - \frac{3x^{a+1} e^{-x}}{\Gamma(a+2)} + \frac{3x^{a+2} e^{-x}}{\Gamma(a+3)} - \frac{x^{a+3} e^{-x}}{\Gamma(a+4)} \right] \\
& - C \left[\frac{x^a e^{-x}}{\Gamma(a+1)} - \frac{4x^{a+1} e^{-x}}{\Gamma(a+2)} + \frac{6x^{a+2} e^{-x}}{\Gamma(a+3)} - \frac{4x^{a+3} e^{-x}}{\Gamma(a+4)} + \frac{x^{a+4} e^{-x}}{\Gamma(a+5)} \right].
\end{aligned} \tag{7}$$

In this form the close relationship between the proposed approximation method and that implied by the Bartlett method is revealed. Essentially Bartlett worked with the first term on the right-hand side of formula (7).

The problem actually discussed in Bartlett's paper is the evaluation of the stop-loss or excess loss ratio premium, given by

$$\Pi(x) = \int_x^\infty (z-x) f(z) dz.$$

Starting again from the approximation (4) for the density and using the integration-by-parts argument above, we obtain after a long calculation that

$$\begin{aligned}
\Pi(x) \doteq & a[1 - \Gamma(x, a+1)] - x[1 - \Gamma(x, a)] \\
& - A \left[\frac{x^{a+1} e^{-x}}{\Gamma(a+2)} - \frac{x^{a+2} e^{-x}}{\Gamma(a+3)} \right] \\
& + B \left[\frac{x^{a+1} e^{-x}}{\Gamma(a+2)} - \frac{2x^{a+2} e^{-x}}{\Gamma(a+3)} + \frac{x^{a+3} e^{-x}}{\Gamma(a+4)} \right] \\
& - C \left[\frac{x^{a+1} e^{-x}}{\Gamma(a+2)} - \frac{3x^{a+2} e^{-x}}{\Gamma(a+3)} + \frac{3x^{a+3} e^{-x}}{\Gamma(a+4)} - \frac{x^{a+4} e^{-x}}{\Gamma(a+5)} \right].
\end{aligned} \tag{8}$$

The first two terms, $a[1 - \Gamma(x, a+1)] - x[1 - \Gamma(x, a)]$, are those used by Bartlett. Another convenient form for these two terms is

$$\frac{ax^a e^{-x}}{\Gamma(a+1)} - (x-a)[1 - \Gamma(x, a)]. \tag{9}$$

III. TWO EXAMPLES

In the following two examples, we shall use the above approximation method to evaluate details of certain risk-theoretic distributions. In both cases the model is from collective-risk theory, and we shall assume that the number of claims is given by a Poisson distribution. In other words, given that the expected number of claims equals t , the probability that n claims occur equals $e^{-t} t^n / n!$. The assumptions on a population which lead to the Poisson distribution for the number of claims are given by Kahn [5, p. 414]. Further, let $\hat{\mu}_n$ be the n th moment about the origin

of the distribution of the claim amount, given that a claim has occurred. Then, the moments about the mean, $\hat{\mu}_1 t$, of the distribution of total claims, given that the expected number of claims is t , are as follows:

$$\begin{aligned}\mu_2 &= \hat{\mu}_2 t, \\ \mu_3 &= \hat{\mu}_3 t, \\ \mu_4 &= \hat{\mu}_4 t + 3\hat{\mu}_2^2 t^2, \\ \mu_5 &= \hat{\mu}_5 t + 10\hat{\mu}_2 \hat{\mu}_3 t^2.\end{aligned}\tag{10}$$

A derivation of the first four moments is given by Bartlett [1, p. 451], and the fifth moment may be derived in the same manner.

TABLE 1
VALUES OF $10^6 F(y)$

y	z	Exact Value	2d Moment Gamma	3d Moment Gamma	4th Moment Gamma	5th Moment Gamma	Edgeworth Expansion	Esscher Approximation
0	0	0	0	0	0	0	-64	0
4	2	342	110	144	183	216	264	341
8	4	6,039	5,110	5,683	6,086	6,255	6,165	6,033
12	6	25,385	25,589	26,048	25,861	25,518	25,375	25,370
16	8	53,540	54,687	54,067	53,362	53,166	53,526	53,526
20	10	77,387	77,990	77,146	77,032	77,324	77,373	77,376
24	12	91,172	91,054	90,763	91,120	91,338	91,241	91,168
28	14	97,150	96,839	96,974	97,263	97,234	97,134	97,150
32	16	99,218	99,000	99,231	99,306	99,204	99,160	99,218
36	18	99,814	99,711	99,878	99,840	99,782	99,991	99,814
40	20	99,961	99,922	100,009	99,955	99,944	100,010	99,960

A. An Example from Cramér [3]

Let the distribution function for the amount of the claim, given that a claim has occurred, be $1 - e^{-y}$. It is easy to verify that $\hat{\mu}_n = n!$. Further assume that the expected number of claims, t , is 16. Then the moments of the distribution of total claim amount, Y , may be calculated by use of formula (10). They are found to be

$$\mu = 16, \mu_2 = 32, \mu_3 = 96, \mu_4 = 3,456, \mu_5 = 32,640.$$

The scaling necessary to make the mean equal to variance is one-half. After this scaling the moments of X are

$$\mu = 8, \mu_2 = 8, \mu_3 = 12, \mu_4 = 216, \mu_5 = 1,020.$$

Applying formula (3), we see the constants to be used in the approximation are $\alpha = 8$, $A = -\frac{2}{3}$, $B = 1$, $C = -1.1$.

Table 1 shows values of $F(y)$ for various values of y . The "exact value"

figures are adapted from some in Cramér [3, p. 42], as are those for the two approximation methods. One is a four-term Edgeworth expansion based on the normal distribution and its derivatives, and the other is an Esscher approximation using as the approximating distribution another four-term Edgeworth expansion. Both of these methods are described in some detail by Cramér [3, pp. 31–40]. The remaining figures give the results of the approximation method that we are considering correct to 2, 3, 4, and 5 moments.

If we compare the results, we see the gamma approximations are not particularly good near the mean, 8, of the scaled variable X . In fact, the correction term corresponding to the fifth-moment correction, the C term, is still quite large near the mean. However, the gamma approximations appear superior to the Edgeworth expansion near the tails of the distribution. The Esscher approximation is much more accurate but also much more difficult to apply.

B. *An Example from Bartlett*

This example from Bartlett [1] is for a health insurance coverage where the claim amount is proportional to the number of days of disability. From a table in the Bartlett paper, one can evaluate the moments of the distribution of the number of days of disability.

$$\begin{aligned}\hat{\mu}_1 &= 31.35211, & \hat{\mu}_2 &= 1,861.705, & \hat{\mu}_3 &= 139,531.08, \\ \hat{\mu}_4 &= 11,453,147.7, & \hat{\mu}_5 &= 979,479,188.\end{aligned}$$

The distribution of the number of claims is assumed to be Poisson, with the expected number of claims equal to 14.63. Application of the formulas (10), therefore, gives us the following moments of the total claim amount Y measured in days.

$$\begin{aligned}\mu &= 458.68, & \mu_2 &= 27,237, & \mu_3 &= 2,041,300, \\ \mu_4 &= 2,393,100,000, & \mu_5 &= 570,320,000,000.\end{aligned}$$

The scaling necessary to make the mean equal the variance is $X = Y/59.381$. After making this adjustment, the moments of X are

$$\begin{aligned}\mu &= 7.7244, & \mu_2 &= 7.7244, & \mu_3 &= 9.7491, \\ \mu_4 &= 192.4737, & \mu_5 &= 772.4700,\end{aligned}$$

and the constants of the approximation are

$$a = 7.7244, \quad A = -0.950, \quad B = 1.480, \quad C = -1.604.$$

To calculate the stop-loss premium associated with 120 per cent of the pure-risk premium, we evaluate formula (8) with $a = 7.7244$ and $x =$

$1.2a = 9.2693$. The accompanying tabulation gives the stop-loss premium correct to various numbers of moments.

Number of Moments	Stop-Loss Premium $\Pi(x)$	Stop-Loss Premium Scaled to Y
2.....	0.55429	32.91
3.....	.56163	33.35
4.....	.54111	32.13
5.....	0.53632	31.85

The figure 32.91 compares with the 32.24 figure which Bartlett obtained by linearly interpolating between figures corresponding to $a = 5$ and $a = 10$ to get the figure for $a = 7.7244$. The figures presented here are based on the tables by Salvosa [9], where in effect the values of a are closer together.

The most interesting feature of this example is the fact that improving the approximation by making calculations correct to the third moment increases the stop-loss premium. In this particular example, the third moment of the gamma distribution fitted by use of the first two moments of the risk-theoretic distribution is $2a = 15.4488$, and the third-moment correction is made to reduce this third moment to 9.7491. In other words, the adjustment for the third moment is made to reduce the skewness of the approximation, and making this change still increases the stop-loss premium. An answer to this apparent paradox is made in Section IV.

IV. SOME "BEST" STOP-LOSS REINSURANCE AGREEMENTS

It was noted in Section III that, in one case at least, an adjustment to reduce skewness increases a stop-loss premium. To see that this is not a peculiarity of the approximation method used, assume that the density function for claims is the gamma density $[1/\Gamma(a)]x^{a-1}e^{-x}$, which has mean and variance equal to a . The stop-loss premium for claims in excess of the mean is given by formula (9) to be $a^{a+1}e^{-a}/\Gamma(a+1)$. For the normal density with mean a and variance a , the stop-loss premium for claims in excess of the mean is $\sqrt{(a/2\pi)}$. But it is known [7, p. 528] that $\Gamma(a+1) = a! > (a/e)^a\sqrt{(2\pi a)}$ for $a = 1, 2, 3, \dots$. Therefore, the stop-loss premium is greater under the normal density (unskewed) than it is under the gamma density, which is skewed positively.

If we examine the third-moment correction in the formula (8) for the stop-loss premium, we notice that this term may be written as

$$-A \frac{x^{a+1}e^{-x}}{\Gamma(a+2)} \left(1 - \frac{x}{a+2}\right).$$

Thus for $x < a + 2$, the stop-loss premium is increased when $-A = (2a - \mu_3)/3!$ is positive, that is, when the third moment of X is less than $2a$. However, if the stop-loss level is chosen to be $x = a + 2$, then the third-moment correction is equal to zero. In that case, the fourth-moment correction in the formula (8) for the stop-loss premium would be

$$+B \frac{x^{a+1} e^{-x}}{\Gamma(a+2)} \left[1 - \frac{2x}{a+2} + \frac{x^2}{(a+2)(a+3)} \right],$$

which, when $x = a + 2$ is substituted, becomes

$$-B \frac{(a+2)^{a+2} e^{-(a+2)}}{\Gamma(a+4)}.$$

In both cases examined in Section III, this quantity was negative. Similarly, the fifth-moment-correction term at $x = a + 2$ in formula (8) may be reduced to

$$+4C \frac{(a+2)^{a+2} e^{-(a+2)}}{\Gamma(a+5)},$$

which was also negative in both examples. This suggests then that we should use the stop-loss level of $x = a + 2$ and base the premium on only the first two moments. There would be no third-moment correction, and our two examples suggest the fourth- and fifth-moment corrections are usually negative.

Now let us assume that we are studying some group or small insurance company and have set up an appropriate collective-risk or individual-risk model for it. Let us assume that μ is the expected amount of claims of this group under our model and that σ is the standard deviation of the amount of claims. Then σ^2 is the variance, and the required scaling is found by setting $\beta\mu = \beta^2\sigma^2$, so that $\beta = \mu/\sigma^2$. Therefore, $a = \beta\mu = \mu^2/\sigma^2$, so that the stop-loss level of $a + 2 = \mu^2/\sigma^2 + 2$ for the scaled variable. Thus, the stop-loss level for the original unscaled variable is $\mu + 2\sigma^2/\mu$. The stop-loss premium for the scaled variable is by formula (8) equal to

$$\Pi(a+2) = a[1 - \Gamma(a+2, a+1)] - (a+2)[1 - \Gamma(a+2, a)],$$

which is a function of a alone. But a is equal to μ^2/σ^2 , so $\Pi(a+2)/a$ may be looked on as a function of σ/μ . Since $100 \Pi(a+2)/a$ is the stop-loss premium as percentage of the expected claims of the scaled variable and is equal to the same ratio for the original unscaled variable, we may tabulate the stop-loss premium for this coverage in the quite simple manner shown in Table 2.

In what sense is this a "best" stop-loss reinsurance agreement? Pri-

marily because only two moments, the mean and variance of expected total claims, must be calculated for any group or reinsured company to calculate the stop-loss premium. Wide variation in the skewness of the distribution of total claims makes no difference in the stop-loss premium calculated by an approximation correct through three moments. Whether such a three-moment approximation is generally conservative is problematical—it was conservative in the two examples in Section III.

This stop-loss reinsurance scheme was designed from considerations arising from the expansion described in Section I. If a Gram-Charlier expansion is made and the development of Sections I and II followed through, it may be shown that the stop-loss premium has a zero third-moment correction term only if the stop-loss level is the expected claims.

TABLE 2
PREMIUM FOR STOP-LOSS COVERAGE
EXCESS CLAIMS OVER $\mu + 2\sigma^2/\mu$
PREMIUM AS PERCENTAGE OF μ

σ/μ	Premium as Percentage of μ	σ/μ	Premium as Percentage of μ
0.00.....	0.000%	0.35.....	5.746%
.05.....	1.758	0.40.....	5.832
.10.....	3.092	0.45.....	5.852
.15.....	4.076	0.50.....	5.825
.20.....	4.777	0.55.....	5.768
.25.....	5.257	0.7071.....	5.494
0.30.....	5.565	1.00.....	4.979

For this case, the stop-loss premium was found to be $\sigma/\sqrt{(2\pi)}$, as indicated at the beginning of this section.

The author would like to take this opportunity to thank Professors C. J. Nesbitt and D. A. Jones, who have read various drafts of this paper and have made many helpful suggestions.

APPENDIX

It is known [4, p. 27] that, in an expansion of a suitable function in a Fourier series, the resulting partial sum at any stage is a trigonometric polynomial which minimizes a certain integral. A similar statement can be made about the expansion developed in Section I.

Assume α is given. We then form the integral

$$G = \Gamma(\alpha) \int_0^\infty \frac{1}{z^{\alpha-1} e^{-z}} \left[f(z) - \frac{z^{\alpha-1} e^{-z}}{\Gamma(\alpha)} \sum_{k=0}^n B_k z^k \right]^2 dz.$$

G measures the error between $f(x)$ and an approximating density, which is a gamma density times a polynomial. G is formed by squaring this error, multiplying by a weight function $\Gamma(\alpha)/x^{\alpha-1}e^{-x}$ for each x , and then integrating. For $\alpha > 1$, this has the effect of strongly weighting the errors at both tails of the distribution. For $\alpha \leq 1$, the weight function has small effect near $x = 0$ but strongly weights error for large values of x . An attempt is then made to select the B_k so that G is minimized. It proves convenient in this calculation to change the polynomial

$$\sum_{k=0}^n B_k x^k$$

into the form

$$\sum_{k=0}^n A_k L_k^{(\alpha)}(x),$$

which is again a polynomial of degree at most equal to n . To effect the minimization, we take the partial derivatives $\partial G/\partial A_k$ and set them equal to zero. But $\partial G/\partial A_k = 0$ implies

$$-2 \int_0^\infty f(z) L_k^{(\alpha)}(z) dz + 2 A_k \int_0^\infty \frac{z^{\alpha-1} e^{-z}}{\Gamma(\alpha)} [L_k^{(\alpha)}(z)]^2 dz = 0,$$

where again use is made of the orthogonality of the Laguerre polynomials relative to the weight function $x^{\alpha-1}e^{-x}/\Gamma(\alpha)$. This shows that A_k should be chosen so that

$$A_k = \frac{\int_0^\infty f(z) L_k^{(\alpha)}(z) dz}{\int_0^\infty \frac{z^{\alpha-1} e^{-z}}{\Gamma(\alpha)} [L_k^{(\alpha)}(z)]^2 dz} = \frac{\Gamma(\alpha)}{k! \Gamma(\alpha + k)} \int_0^\infty f(z) L_k^{(\alpha)}(z) dz.$$

In the sense of the error integral G , adding an additional term improves the approximation as A_{n+1} correctly chosen gives a lower value to the integral than the choice of $A_{n+1} = 0$. Unfortunately, no estimates are known to the author as to bounds on the error in the approximation using this criteria or some other. However, the form of the weight function, low near the mean, suggests that the approximation is less accurate in that range, and the example by Cramér studied in Section III, A , bears this out.

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DISCUSSION OF PRECEDING PAPER

PAUL MARKHAM KAHN:

Mr. Bowers' paper represents a very fine contribution to the analysis of claims distributions and the approximations to them using standard functions. Recently this subject has received considerable attention, a notable example of which is the Bohman-Esscher report based on the work of a Swedish committee of actuaries.

One stimulus to the use of the gamma distribution in these investigations is its relative simplicity compared with the Esscher approximation, which, though complicated, gives good results, as shown in Table 1 of Mr. Bowers' paper. The purpose of this discussion is to call attention to a very recent paper by two Finnish actuaries ("Approximations of the Generalised Poisson Function," by Lauri Kauppi and Pertti Ojantakanen), presented to the 1966 ASTIN Colloquium. Kauppi and Ojantakanen give a simple formula for the amount of claims corresponding to a given probability level, and this formula produces results that agree very well with the Esscher approximation.

One note of caution, however, should be made concerning any of these approximations. It seems unreasonable to look for accuracy beyond a few significant figures, especially when calculating levels of security.

WILLIAM D. BERG:

I wish to congratulate Mr. Bowers on his well-written paper. The student of mathematical statistics will find it a pleasure to read, especially as an introduction to the mathematical aspects of collective risk theory. His presentation of the sum of the gamma functions is elegant and illuminating.

Because of the central importance of the expression (9), the insight gained from what I shall call a quadrature interpretation is useful. By writing $1/\Gamma(a)$ for $a/\Gamma(a+1)$, the first term, which is the first approximation to the stop-loss net premium, is more readily seen as the area of the rectangle whose length is x and whose height is the ordinate of the density function at x (x is the scaled-down claim amount). The next approximation reduces this by an amount represented by the area of the rectangle whose length is the excess of x over its expected value a and whose height is the fraction represented by the area of the tail of the p.d.f. beyond the value x . The quadrature interpretation helps one to visualize the effect of a variation in x on the stop-loss net premium.

I trust that the author will catch the typographical error in the galley proof that uses lower case y 's for capital Y 's in the heading of Table 1. The same slip with x is not material.

B. GEORGE ISEN:

Actually the process of fitting a smooth curve to a sample does not add anything to our information about $f(x)$ that is not contained in the sample. The fitted curve may, in fact, give one an entirely misleading impression of the real density function.¹

It has been interesting and enlightening to follow the recent incursions made into the hazy never-never land of stop-loss reinsurance. This problem area, and the methods which have been proposed to establish a "science-proves" solution, are related to many other important problems to the actuary, including among them the establishment of contingency reserves, the chance of ruin, the establishment of retention limits for life insurance policies, and certain problems related to group insurance policies and their experience rating.

Mr. Herbert Feay² has certainly made the members of the Society indebted to him "for his courage, talent, and industry"³ in this field. After Mr. Feay thoroughly classified various approaches to reinsurance, its purposes, and some general considerations, he proceeded into the misty bog in the section entitled "Determination of Premiums." This section of the paper evoked many other suggested pathways through the murk and elicited much erudite, though critical, discussion (which, if not critical, is probably not constructive). I hesitate to add my discussion of Mr. Bowers' ably written and informative paper to the long list of writings stimulated by Mr. Feay's paper, including those most important papers by Dr. Paul Markham Kahn⁴ and Mr. Dwight K. Bartlett.⁵

However, the line of papers seems to be bearing down on one basic problem: What is the true underlying nature of the risk, how may it best be expressed mathematically, and how may this "mathematic" be applied in day-to-day work? (This is, in reality, only one basic problem, analyzed into its fundamental parts, much as the atom, which is the basis of all matter, may be analyzed into its fundamental parts, etc.) Mr. Feay

¹ Mood, *Introduction to the Theory of Statistics* (New York: McGraw-Hill Book Co., Inc., 1950), p. 120.

² Herbert L. Feay, "Introduction to Nonproportional Reinsurance," *TSA*, XII, 22.

³ Irving Rosenthal, in his discussion of Feay's paper, *TSA*, XII, 54.

⁴ "An Introduction to Collective Risk Theory and Its Application to Stop-Loss Reinsurance," *TSA*, XIV, 400.

⁵ "Excess Ratio Distribution in Risk Theory," *TSA*, XVII, 435.

seemed interested in establishing the basic, simple mathematical framework of his subject⁶ and then, justifying the use of the normal distribution, proceeded to calculate some illustrative premiums. As I intimated above, the discussions then went on to challenge the use of the normal distribution as the basic underlying function describing the true density of the claim distribution. It is not necessary to mention each discussant separately, but it may be of help to partially and hastily (and probably badly) outline the alternative roads to Oz (all varying in their shades of yellow, but all leading, brick by brick, through that country, strange, uncharted, and inhabited only by tin woodmen and straw scarecrows).

The methods which have been suggested include the direct application of collective risk theory (with the use of the Esscher approximation), Monte Carlo techniques, the substitution of the Poisson formula for the normal distribution, and such, thereby stamping Mr. Feay as misguided in the shade of the bricks on his road.

To save the day, Dr. Kahn⁷ gave us a summary of the weapons developed and used by our European brethren (seriously, his and Mr. Feay's papers led this poor actuary to the sources so valuably accumulated in the rich bibliographies of their papers), and again Mr. Feay's normal distribution came under attack as being abnormal. Dr. Kahn's pathway is tortuous and narrow, and only the most adroit at compound-Poisson processes may follow in safety.

Mr. Bartlett⁸ ventured into the mire, indicating the importance of finding a path and an easy one—like a gamma density function⁹ of the type found in chapter vi of Mood.¹⁰ Fitting the first two moments of the actual distribution of claim amounts to those moments of the density function would of necessity determine the value of the parameters of the function and excess ratio probabilities, and, hence, stop-loss premiums could be determined. Mr. Bartlett has not let us travel easily, but, as with the previous scholarly papers of Feay and Kahn, his introduces new concepts which can be evaluated with only the closest attention. The road seems to be narrowing. But, again, out of the discussions can be heard the pleas for

$${}^6 R^H_L = \int_{t=L}^{t=H} (t-L) f(t) dt \times \frac{R}{M}.$$

⁷ *Op. cit.*

⁸ *Op. cit.*

$${}^9 f(x) = \begin{cases} \frac{1}{a! \beta^{a+1}} x^a e^{-x/\beta} & x > 0 \\ 0 & x < 0 \end{cases}.$$

¹⁰ *Op. cit.*, p. 112.

the use of the logarithmico-normal distribution, the Pareto distribution, the T-gamma (with three parameters, and hence the use of three moments) for use as the "secondary" distribution, and weakly, but determinedly, the path opened by a spin of the roulette wheel is again offered for consideration (Monte Carlo and his random numbers). It seems to have been settled that the Poisson distribution leads to the right distribution of the number of claims (except for Mr. Jackson's minor dissent in his discussion of Mr. Bartlett's paper).¹¹

Mr. Bowers now invites us to follow along yet another path. He describes very accurately and lucidly the land to which he leads us. He takes advantage of Mr. Bartlett's lead of the use of the gamma function for the density function in question but tends to increase our confidence by invoking the use of more than two or three moments, this being possible through the use of an expansion similar to the Gram-Charlier series, except that the Laguerre polynomials are used as weight functions (this making the solutions neat by their being orthogonal polynomials) in the place of the Hermite polynomials used in the Gram-Charlier series. Mr. Bowers must be congratulated for his thorough analysis and his offer of yet another path to the Emerald City.

In Table 1 of Mr. Bowers' paper a comparison is made of various approximations to an exact value resulting from an explicit function. The Esscher approximation seems to yield the closest values to the exact values, while fifth-moment gamma is certainly not as reliable an estimate to the values in this table. However, it is possible that the fifth-moment gamma function may be justified on other grounds, though it seems that the mathematical work involved in applying the fifth-moment gamma function is approaching that needed to complete the approximation by means of Esscher functions.

As I stand at the edge of this land fraught with dangers, preparing to reach the goal—that city of which only the Wizard of Oz himself knows the location—I see before me the beckoning of many paths: all are yellow and all are brick, but I am at a loss, for my instruction to follow the yellow brick road has guided me to indecision.

It seems that the literature is clear as to what price must be paid when a road is to be selected.¹² In life insurance mathematics, the secret pathway was sought by men of the stature of Gompertz and Makeham. Today, no actuary pretends that these mathematical functions for the force of mortality are anything but techniques of graduation. In the graduation of our most recent mortality table for use in valuation, these were abandoned

¹¹ Paul H. Jackson, *TSA*, XII, 459.

¹² Mood, *op. cit.*, p. 120.

for, as we all know, the use of Jenkins' fifth-difference modified osculatory interpolation formula for most of the range of ages in the table. At the upper ages, where the data are scarce, artificial methods were used to close off the table.

Mr. Paul Jackson, in his discussion of Kahn's paper,¹³ illustrates the possibility of a growing and confusing number of roads to the goal. But he also brings forward a point which should be enlarged upon. The fitting of these many frequency functions depends on the data involved. Of necessity the data are most plentiful at or near the mean of these distributions of actual claim amounts, but there is a paucity of data at the extreme or tail, and here precisely is that magic city that we are trying to reach—the underlying, basic, inherent density function at the tail of the distribution. The problem of evaluating the “true” tail by comparison of a density to the actual data has limited reliability.

The Monte Carlo technique has been advanced, but this technique still relies on a basic underlying probability being associated with various claim amounts, and the analysis of hypothetical samples exposed for a large number of hypothetical time periods, with the occurrence of large amounts being estimated therefrom and probably being small in number at any rate (that is, duplicating the paucity of data in the tail). Because of my limited knowledge of this area, it seems to me that to apply the Monte Carlo technique to, say, a group of lives, requires that ages, probabilities conforming to those ages, and amounts conforming to those lives must be selected and then the results of the simulation analyzed. Here the road is not only a different shade of yellow but has many side lanes in sight even before we enter upon it.

The problem, which seems to be losing its disguise, is not what the underlying function is but rather what is a good graduation formula for the data observed, so that the stop-loss premiums (or other related functions) calculated therefrom may be adequate, equitable, consistent, and competitive. The more parameters added to a graduation formula, the closer we expect the curve to fit to the data. Hence, adding moments and using the “method of moments” (if you will) may give a better fit to the observed data, but does it predict the nature of the tail any better when the data there are so sparse? (Of course, if the basic function does not describe the data closely, then added parameters may cause greater deviation from the observed, this being true where parameters are contained in the terms of a series which itself diverges or converges very slowly.)

Before turning to a conclusion, I would like to review the assumption underlying the Poisson distribution as being representative of the prob-

¹³ *TSA*, XIV, pp. 443-44.

ability of the number of claims incurred. Using standard actuarial notation, the probability that (x) dies (or becomes hospitalized or disabled) before age $x + h$ is ${}_h q_x$. Also, let ${}_h p_x = 1 - {}_h q_x$. Then the probability that (x) survives to $x + kh$ and dies before age $x + (k + 1)h$ is ${}_k h p_x \cdot {}_h q_{x+kh}$. If it is assumed that $0 \leq h \leq 1$ and k is an integer such that $0 \leq kh < 1$, then ${}_k h p_x \cdot {}_h q_{x+kh}$ represents the probability that (x) dies before $x + 1$, and exactly between $x + kh$ and $x + (k + 1)h$. Usually an assumption of uniform distribution of deaths is made. This results in the following:

$$\begin{aligned} {}_k h p_x \cdot {}_h q_{x+kh} &= \frac{l_{x+kh}}{l_x} \cdot \frac{l_{x+kh} - l_{x+(k+1)h}}{l_{x+kh}} \\ &= \frac{k h l_{x+1} + (1 - k h) l_x - (1 - k h - h) l_x - (k + 1) h l_{x+1}}{l_x} \\ &= \frac{h (l_x - l_{x+1})}{l_x} \\ &= h \cdot q_x. \end{aligned}$$

Hence, under this assumption the probability that (x) dies in any small part, h , of the year of age is the same as for death in any other part of duration h in that same year. However, the probability of death in a small part of the year, h , if (x) has lived part of the year, increases throughout the year, and this without regard to the size of h . That is:

$${}_h q_{x+kh} < {}_h q_{x+(k+1)h},$$

if

$$h \cdot q_x < {}_k h q_{x+(k+1)h},$$

$$h \cdot q_x < \frac{h \cdot q_x}{p_{x+kh}},$$

or

$$p_{x+kh} < 1,$$

which it is in ordinary meaning of probability.

Hence, the assumption that the probability of a claim in a short period of time is independent of what happened prior to that time (at least respecting life insurance) is not true, even using the assumption of uniform distribution of claims. (It is obvious where the UDC is not used, because μ_x , the force of mortality, is not considered constant with respect to time in any case respecting life contingencies of mortality or morbidity.) However, where the probability of occurrence in that period is very small and the number exposed to the hazard very large, the Poisson is an excellent approximation to the true probability related to the number of claims. A small number exposed, as in small groups, challenges the accuracy of using

the Poisson distribution. I regret that I have not carried forward these thoughts to the point of calculations and comparisons due to lack of time. Mr. Jackson had touched on this problem from another point of view¹⁴ but had not referred to the fact that claims have to be not only independent of each other but also of the time of their occurrence, in order for the Poisson distribution to be exactly representative.

I have digressed sufficiently from making a decision regarding the path to take. In attempting to choose wisely, I have re-examined many basic texts in statistics. Wolfenden's text¹⁵ directed to the problems of actuaries in curve fitting, graduation, and statistics, indicates on page 64 that variations in the Gram-Charlier series are available for use with data that are very skewed. The normal functions may be replaced by the Poisson exponential as the generating function giving rise to the Poisson-Charlier series. Also, the Gram-Charlier series may be used with the logarithmico-normal as the generating function.¹⁶

In conclusion then, Dorothy was much more successful in reaching the Emerald City than I, for she had only one yellow road to choose. I am still looking for the road with the right shade of yellow. Perhaps we can look forward to added papers on this subject, where these many functions and Monte Carlo techniques will be applied to actual data derived from the sources under study, such as hospitalization claims under group insurance contracts or results of actual reinsurance treaties. Perhaps each problem requires a unique road of its own.

(AUTHOR'S REVIEW OF DISCUSSION)

NEWTON L. BOWERS, JR.:

I wish to thank Dr. Kahn, Dr. Berg, and Mr. Isen for their discussions of my paper. Their discussions bring up some of the problems of risk theory and a new solution to one of these problems.

Dr. Kahn in his discussion draws attention to a new paper by Kauppi and Ojantakanen, presented at the 1966 ASTIN Colloquium. The paper describes a method of using the standard normal distribution to approximate closely the well-known Esscher approximation. The technique involves a simple change of variable which takes account of the third moment. I highly recommend this paper to the interested reader.

Dr. Berg brings up a point about a typographical error which I would call a difference in notational preference. I use capital letters, Y , to indi-

¹⁴ *Ibid.*, p. 442.

¹⁵ Hugh H. Wolfenden, *The Fundamental Principles of Mathematical Statistics* (The Actuarial Society of America, 1942).

¹⁶ *Ibid.*, p. 312.

cate the random variable and small letters, y , to indicate numerical values that the random variable assumes. While this notation is not universal, it has been used in several recently published textbooks.

Dr. Berg also shows a quadrature interpretation of the stop-loss net premium where the distribution of total claim amount is assumed to be a gamma distribution. A general interpretation of this type is possible. Looking at the integral for the stop-loss net premium, we see

$$\begin{aligned}\Pi(x) &= \int_x^{\infty} (y-x)f(y)dy = \int_x^{\infty} [(y-\mu)-(x-\mu)]f(y)dy \\ &= \int_x^{\infty} (y-\mu)f(y)dy - (x-\mu)[1-F(x)].\end{aligned}$$

If the gamma density $f(y) = 1/\Gamma(\alpha)e^{-y}y^{\alpha-1}$ is substituted, the integral term may be evaluated as $x f(x)$, as Dr. Berg has pointed out. If $f(y)$ is instead the standard normal density, the integral term may be shown to equal simply $f(x)$.

Mr. Isen's comments raise some interesting questions, and I will answer them by giving my view of risk theory and by restating what I consider to be the place of my paper.

As I see it, there are two problems in risk theory. The first problem is to construct the probability model. There are two main classes of models. The first type, the collective model, divides the problem of calculating the distribution of total claim amounts into two stages. First a distribution of the number of claims is calculated. Certain assumptions, outlined in Dr. Kahn's paper, lead one to a Poisson distribution for the number of claims. Mr. Isen discusses these assumptions but seems to object to the stationarity assumption for the collective model on the basis of an analysis pertaining to an individual life. In any case, I agree with him that the Poisson assumptions seem inappropriate for small groups. Other actuaries have noted that in certain branches of the insurance industry the negative binomial distribution seems to fit the number of claims much more closely. The negative binomial distribution can, in fact, be obtained on theoretical grounds under the assumption that the Poisson parameter itself is a random variable with a gamma distribution.

The second part of the construction of the collective model is to obtain a probability distribution on the amount of a single claim. This is a problem which seems to me to be quite difficult and one which must be faced in casualty, health, and disability coverages. In these fields of insurance a statistical analysis of the distribution of individual claim sizes would be necessary. This analysis is then used to construct the so-called secondary distribution, the form of which is often assumed to be log-normal or

Pareto or gamma. It is possible in some of the so-called distribution-free methods to assume nothing about the distribution of individual claims but simply to use the data on claims to evaluate moments. If the data are sparse, the sample may give misleading results as to the distribution of large claim amounts.

The second model in general use is the individual model. The total claim amount is viewed as the sum of the claims on the various individual lives in the portfolio. The distribution of total claim amounts is the distribution of a sum of a fixed number of independent random variables. For life insurance where the amount of claim is, in general, fixed given the time of claim, this individual approach appears to be the preferred one.

Once the model is constructed, the second problem comes into play. How does one use the model to evaluate probabilities and expectations? It is virtually impossible to evaluate these probabilities directly because of difficulties in evaluating convolutions of functions.

There are several solutions to this second problem. One of the most popular methods at present is to use Monte Carlo techniques of simulation. If a large number of simulations are used, details of the distribution can be evaluated. Modern computers make such a scheme quite feasible.

Another method is to use the probability model to evaluate moments of the distribution of total claim amounts. A standard curve is then fitted to these moments and used to evaluate probabilities. Since the normal distribution and the gamma distribution are two parameter families of distributions, the values of these parameters may be chosen so that the mean and variance of the approximating probability curve are equal to those of the underlying model. If the third moment is known, the Pearson Type III curve may be used. Bohman and Esscher have shown that this last distribution gives much better results than the normal distribution for details of the tails of the distribution. The Gram-Charlier series and the Edgeworth series both allow one to use even more additional information in the form of knowledge of additional moments to improve the approximating density. The series suggested in my paper is like the Gram-Charlier series in that it makes use of additional moments. However, the underlying distribution used in generating the series is the gamma distribution rather than the normal distribution, and the results of the Bohman-Esscher paper suggested to me that this gamma series might give better results.

Another solution to the problem of evaluating various probability statements is by the use of the Esscher approximation. This involves evaluating moments of a special distribution function containing a multiplicative factor e^{hx} . It is a little unhandy to deal with, but it does give good

results. Furthermore, the paper mentioned by Dr. Kahn shows a very good and very easy approximation to the Esscher method, which does not involve the evaluation of moments of the special distribution function.

Again I would like to thank those who took the time to prepare discussions of my paper and to allow me to indicate where I feel it fits into risk theory.

PROJECTION OF OPERATIONS

MELVIN L. GOLD

FOR the life insurance executive, an indispensable tool in the decision-making process is the "projection." The proliferation of companies and increased competition for the consumers' dollars have made the need for knowledge and better planning most compelling.

By a "projection" we mean the forecasting of a company's future financial experience. The mature life insurance company is, in some ways, ideally suited to make such a prognostication: mortality is fairly predictable, the investment rate cannot really fluctuate too widely since most investments are long term and for a fixed income, expenses ordinarily do not change rapidly, and, of course, there is generally a large inventory of existing business whose nature has been rather carefully charted. Yet projections, when they are made, are all too often a haphazard thing. With the continuing pressures on profit margins, it has become a vital necessity for companies to plan what they want to happen and to know how they will make it happen rather than to rely on illusory premium redundancies to mitigate errors in judgment.

A projection is called for when life insurance management wonders about:

- a) The necessity for additional corporate financing.
- b) How much the company can afford to invest in new business.
- c) Expanding its operation into new benefit or geographical areas.
- d) Any basic change in field compensation, such as the converting from a general agency to a branch-office operation or vice versa.
- e) The effect of a change in premium rates and/or cash value or dividend structure on profits and taxes.
- f) Decentralizing a company's operation.
- g) The effect of a merger with another life insurance company.
- h) The setting-up of a subsidiary life insurance company.
- i) The utilizing of net operating loss carry-forward deductions.

A projection may also be prepared for:

- a) An investment banking firm as a prerequisite to the raising of new or additional capital.
- b) An investor thinking of buying a substantial interest in a life insurance company.
- c) An insurance department as a prerequisite to licensing or the raising of additional capital.

Very little has appeared in the actuarial literature about life insurance company projections. With this thought in mind, it was felt that the setting-forth of the mechanics of such projections would be useful. This paper will discuss projections for both new and existing companies.

NEW-COMPANY PROJECTION

The organizers of a new life insurance company should be required to prepare a detailed plan of its proposed mode of operation, together with an actuarial projection. While a long-range projection (say, for 10 years) is a mercurial operation at best, it does force some realistic thinking on the entrepreneurs, who, by their very nature, are an optimistic lot. This will help to determine the capital structure of the new corporation so as to insure that there is sufficient surplus to finance the early operation and to preclude the possibility that the company's loss carry-forwards will expire. In addition, the projection will illustrate the results of actions considered by management. It can help management achieve its goal of investing surplus rather than dissipating it.

A new-company projection essentially involves the setting up of a model office and the determination of the concomitant parameters. The various parameters entering into the model office of a new company include, among other things, the following:

- Capital and surplus
- Production
- Distribution of business (new and in force)
- Investment return
- Gross premium rates
- Premium taxes
- Commissions and other field expenses
- General administrative expenses
- Mortality rate
- Morbidity rate
- Persistency rate
- Cash-value basis
- Reserve basis
- Policyholder dividends
- Premium-mode distribution
- Reinsurance cost
- Federal income tax

Varying each of these elements would produce an almost impossible array of possibilities. As a practical matter, many of these parameters are usually set before the projection begins and the net effect of changes determined empirically. Such preset parameters might include commission rates, interest rates, premium taxes, expenses, capital and surplus,

and so forth. The effect of variations in gross premiums, persistency, slope of production, and distribution of business is more difficult to predict. Therefore, separate model offices will frequently be required. Often, "high," "medium," and "low" projections are prepared in order to dramatically illustrate the effect of relatively minor changes in the parameters.

Capital and Surplus

Interest on the capital and surplus account should be kept apart from the insurance operation. These earnings arise because of the initial (and subsequent) investment made by the stockholders and are not part of the earnings arising from the insurance operation. Thus new companies have started with \$100,000, and others have commenced operations with \$10,000,000. It is manifestly unfair to lump them together and examine only the change in surplus without some indication of the surplus level itself. Accordingly, the investment return should be separated into interest on the capital funds and interest on the insurance operation (reserves). Capital gains and losses should also be kept separate from insurance earnings. The surplus (positive or negative) arising from the insurance operation would then be added to the capital funds to produce the annual statement surplus.

Another approach for an investor might be to credit the initial capital funds with interest based on the risk assumed. Thus, risk capital might be considered to earn 10-15 per cent. The stockholders' investment accumulated at an assumed "risk" interest rate can then be compared with the actual capital funds plus the value of the in-force business and the value of the "going concern."

Production

The level and slope of estimated production should be carefully thought out since they have an important bearing on the evaluation of a company's projected operation. Production levels will depend in part on the type of field organization, the amount of money invested therein, the portfolio of policies and rates, and so forth. In recognition that promoters, by their very nature, consciously or unconsciously, tend to overestimate future production and underestimate expenses, some insurance departments request new companies to prepare projections based on a high, medium, and low level of production in order to emphasize the various levels of investment capital which might be required to put the company into the black. Of course, management should be interested in seeing expected results under various levels of production regardless of such outside request.

Composition of New Business

Variations in the composition of new business can be made by plan, issue age, policy size, premium mode, underwriting, percentage substandard, and so forth. As a practical matter, generally only variations in the first two items are employed, combined with the appropriate average policy size for each plan-age combination. A large array of plan-age combinations does not necessarily make for more sensitive results. Different proportions of permanent and term insurance, high early cash value and minimum cash-value plans, par and nonpar insurance, high and low issue ages, will produce significantly different earnings patterns. On the other hand, one plan can sometimes conveniently represent 20-pay life, life at 65, 20-year endowment, and endowment at 65. The validity of this assumption depends on variations in profit levels from plan to plan. The data in the *Life Insurance Fact Book* may be used as a first approximation of the composition of new business. Generally, however, the *raison d'être* underlying the new company will better indicate what pattern of new business might be expected.

Investment Income

In spite of the continuing investment advantage of new money over old money the net investment rate of most new companies is generally below the industry average. The reasons are many: (1) proportionately high liquidity needs, particularly where a negative cash flow exists; (2) a higher proportion of government securities, particularly in order to meet state deposit requirements; (3) unavailability or impracticability of securing the more lucrative investments, that is, direct placements, mortgages; (4) extreme conservatism; and (5) lack of investment astuteness.

The effect of a $\frac{1}{4}$ of 1 per cent variation in investment earnings can be easily calculated since the final projection will set forth the company's projected capital funds and reserves. The interest rate is generally applied to the mean funds available for investment.

Gross Premiums

Variations in nonpar gross premium levels can often be employed with dramatic effect for management. A mere 5-10 per cent increase in whole life gross premium may double profits, and, conversely, a corresponding reduction may almost eliminate them. For simplicity, premiums are often assumed to be collected on an annual basis. The overstatement of premium income is offset to some extent by the overstatement of commissions, premium taxes, and reserves.

Premium Taxes

A flat percentage, say, $2\frac{1}{4}$ per cent, is generally valid for a "broad based" company. A new company, however, generally does most of its business in its home state. Its premium taxes may therefore differ significantly.

Commissions

It is essential to separate commissions and other sales expenses by first-year and renewal, agent, general agent, branch office, expense reimbursement, and so forth. This is particularly true of a company operating in New York, where a significant proportion of the sales compensation in the form of expense reimbursement or development allowances goes into Exhibit 5 of the Annual Statement. The effect of different methods of sales compensation can then be more readily determined. In addition, it facilitates the determination of whether a company licensed in New York will comply with the requirements of Section 213 of the New York Insurance Law. A prospective investor would also want to know whether a company intends to make a heavy investment in agency-financing, whatever the descriptive tag.

Expenses

This is a most difficult figure to estimate. The general expenses of new companies vary widely. Much, of course, depends upon the projected mode of operation. Companies in their first year of operation have had expenses varying from \$100,000 to \$1,000,000. Arthur Pedoe's article (*TSA*, Vol. XIII) and the discussions thereof are an important source of expense rates in the ultimate situation. In the early years, before expenses have reached the going-concern level, discussions with the promoters and the chief executive officers will give some indication of personnel requirements, salary levels, amount of space to be rented, attitudes toward furnishings, and so forth. In any case, a budget should be prepared to show how much will be spent on executive and clerical salaries, furniture, rent, printing, legal, accounting and actuarial fees, advertising, and the rest.

There may be some surprises in store. Since the unexpected expense is to be expected, the projection should so provide. This is particularly true in this day of SEC prospectuses and audits. The projection can also compare anticipated expenses with the expense loading implicit in the gross premium structure. While general expenses will undoubtedly exceed the "amount released" for this purpose, it is worthwhile to constantly ham-

mer home the theme that "excess expenses" come from surplus. Ultimately, the expense level will depend on the quality of management and on the scope and wisdom of their ambitions.

Mortality Rate

Since a new company will take many years to develop a credible mortality experience, industry tables such as the 1955-60 Select Basic Table are frequently used as the underlying mortality assumption in projections. Where appropriate, an adjustment should be made to reflect the higher mortality under nonmedical and "guaranteed issue" business.

Lapse Rate

Lapses are of particular concern for the newer companies although the "minimum deposit" sale and the growth of new companies have made it a problem for almost everyone. The new-company president is generally bombarded by all sorts of agents with all sorts of deals. These "musical chair" agents know how desperate the company is for business. The common requirement is financing, and the common result is a high lapse rate. Since a new company's lapse rate is so crucial a factor and yet so hard to forecast, the actuary is often well advised to illustrate graphically the effect on profits of various persistency levels.

Mr. Buck, in Volume XII of the *Transactions*, presented a lucid study of the characteristics of the policyholder, policy, and agent and their correlation with the first-year lapse rate. Moorhead's excellent treatise on persistency also appears in the same volume.

Reserves

The reserves employed in a projection should provide, where appropriate, for the immediate payment of claims, for nondeduction of deferred fractional premiums, and for the return of the "unearned" portion of the premium in the year of death. In order to minimize early losses, preliminary term reserves are almost always employed, although not necessarily the Commissioners Reserve Valuation Method.

Reinsurance

Reinsurance is an important cost factor since generally a relatively high proportion of a new company's business is reinsured. This is particularly true where companies attract successful producers and the concomitant large policies. The key questions here are: (1) What is the projected retention of the company? (2) What proportion of the business will exceed the company's retention? (3) Reinsurance method? Retention limits should be reviewed periodically and increased as soon as possible.

Federal Income Tax

Since a new company will generally operate in the red for a good many years, prudent management means tax-planning to insure that the loss carry-over benefits can be utilized. This generally means, among other things, the use of minimum reserves and the postponement of depreciation allowances where possible.

Mechanics

In asset share calculations, premiums are generally assumed to be collected at the beginning of the year and terminal reserves set up at year end.

In a projection, premiums can be assumed to be collected at the beginning of the year, at midyear, or at some other point of time, and the appropriate reserves set up. Assuming that premiums are received at the beginning of the year is a relatively simple and straightforward approach but rarely corresponds with reality. The premiums collected by a company generally build up over the year and, therefore, more nearly correspond with a midyear approach. If a new company goes into operation at the beginning of a calendar year, the projection year will correspond with the calendar year, that is, the annual statement. The actuary should always be prepared to explain, however, why the change in surplus as revealed by the annual statement differs from the projection.

A simplified projection of a hypothetical new company has been set forth in the Appendix for illustrative purposes.

EXISTING COMPANY

Projecting the operations of an existing company can be an extremely difficult and frustrating task. The complexity depends on the type of operation that must be projected. Are we talking about:

1. A one-year-old company that may go in almost any direction?
2. A medium-sized company that has been growing rather smoothly and sells only individual life policies?
3. One of the giants of the industry, which operates in all states and sells a broad spectrum of coverages through a variety of agency arrangements?

For an existing company, one of a number of approaches can be used: (1) model office, (2) analysis of operation, or (3) employment of unit profit and cost factors, or an "over-all" method.

Model Office Method

Under this approach, separate model offices are created for the in-force business and for future production. The experiences of both segments are then projected and combined.

A most important first step is the determination of the plan, age and duration elements making up the model office. To keep the model office within manageable proportions, only those plans that are truly indicative of the in force should be used. The validity of the model office of existing business is measured by whether it reproduces the financial experience of the company. This is a necessary but not sufficient condition.

A projection of earnings on an existing closed block of business is essentially a transformation (producing earnings year by year instead of discounting) of a gross premium valuation, and such a valuation must be based on assumptions which reflect past experience. All the accouterments of a new-company projection are present here (mortality rate, interest rate, lapse rate, commission rate, expenses, etc.), except that we are concerned with an additional dimension—duration. Thus, each block consists of sub-blocks, each with its own premium rate structure.

Some sub-blocks are profitable, some are not. Some sub-blocks are associated with particular forms of agency organization or contracts or with particular underwriting policies or both and may have a distinctive experience which would need to be recognized. A projection of earnings on a closed block can be extremely time-consuming, requiring a fairly exhaustive actuarial, financial, and functional analysis of past operations and then a complex and sophisticated set of projections.

The projection of earnings on the new-business model office can be handled in a fashion similar to a new-company projection with one major difference—the various parameters entering into the projection (production estimates, plan and age distribution of business, lapse rates, etc.) can be generally estimated with a far greater degree of success.

Expenses can be projected using the results of periodic budget studies and/or a functional analysis. In the former case, one expense estimate is made for both the new and existing business. In the latter case, separate projections are made and then combined.

The model office approach is essentially an asset share approach and, as such, enjoys its advantages and suffers from its shortcomings. Theoretically, we may probe many years into the future. Practically, this is (a) difficult because of the immense work required (even with electronic equipment) to deal with the multitude of parameters and (b) virtually unattainable because the actuary is not omniscient and can only project and extrapolate. The model office approach does show what will happen if certain assumptions are met and the effect of changes in various parameters. With discretion and judgment, it is a most powerful tool.

Analysis of Operation Method

In the "analysis of operation" approach, a company's financial experience is reviewed, analyzed, and projected. Essentially, this involves an examination of each line of "page 5" of the Annual Statement and the forming of relationships and patterns.

Thus, for a smaller company:

- a) Renewal premiums can generally be expressed as a percentage of the previous years' total premiums.
- b) Investment income can be related to the previous years' return and the growth of assets.
- c) Claims might be expressed as a function of expected claims or "tabular cost." It should be noted, however, that the "analysis of increase in reserves" produces a cost of mortality based on net amount at risk rather than expected gross death claims.
- d) The increase in reserves can sometimes be expressed as separate percentages of first-year and renewal premiums. Alternatively, average reserve per \$1,000 factors can be applied to blocks of projected in-force business. Sometimes, the "tabular cost" is estimated and the reserves calculated as a by-product.
- e) Commissions can be expressed as a percentage of premiums, treating first-year, single, and renewal premiums separately.
- f) Taxes can be expressed as a percentage of premiums.
- g) Expenses are broken down in accordance with Exhibit 5 and various relationships formed. The key item is salaries, which must be carefully analyzed. This may be done in bulk or individually, depending on the size of the company. The other items are then individually examined or related to salaries, sales, insurance in force, premium income, or combinations thereof. These items will generally come directly from the company budget for the ensuing year.
- h) First-year premiums are difficult to estimate. Although they are more than offset by directly allocated items (new-business compensation, underwriting costs, sales expenses, first-year claims), an error in their estimation could be embarrassing where net level reserves are set up.

The "analysis of operation" is a fairly crude method and can generally be used only for probing into the immediate future. It is essentially an extrapolation technique, and therefore where required has the accompanying advantage of producing fast results. By its very nature, however, it does not help the analyst to understand what is happening.

The "analysis of operation" approach is often an extension of the annual budget forecast. Since the estimation of expenses is generally the most time-consuming task, it is a most natural appendage.

Over-all Method

In the over-all approach, a company's gain from operations as set forth by the Annual Statement is broken into (a) first-year cost, (b) renewal profit, (c) one-time nonrecurring gains or losses, (d) corporate items, and thereafter projected.

First-year cost—sometimes called new-business strain—is the cost of putting new business on the books and is equal to first-year home office expense plus first-year sales cost plus first-year mortality cost plus the loss of interest in the first year plus first-year reserves minus first-year premiums. In essence, the first-year cost (or first-year investment in new business) is equal to the total first-year cost in excess of first-year premiums. The loss of interest on the surplus investment in new business should not be ignored.

Renewal profit is the net profit accruing to the company from the renewing business. It is the earnings the company would realize if there were no investment in new business. Thus, a \$200,000 gain from operations may mask a \$500,000 gain from renewal operations and a \$300,000 investment in new business. The sum of "first-year cost" plus "renewal profit" should equal the company's financial experience (after adjusting for one-time and corporate items).

One-time nonrecurring items would include past-service pension contributions, capital gains or losses, costs inherent in a conversion to a new mortality table, correction of a previous financial statement, and reserve strengthening.

Corporate items would include interest on the company's capital and surplus, cash and stock dividends, and the sale of stock, whether through a financial underwriting, the picking-up of stock options, or some other means. The granting of stock options generally results in a dilution of a stockholder's interest and, as such, should be indicated in a projection.

Proceeding with the over-all approach, we would:

1. Segregate the company's financial operation into the above four classifications. Actually, "one-time" and "corporate" items are fairly self-evident, so that the prime task is the division between new-business cost and renewal profit.

The income and disbursement items might be broken into the following categories:

- a) Nonexpense items directly or readily chargeable to first year and renewal. These would include premiums, commissions, premium taxes, investment income, policyholder benefits, increase in reserves.
- b) Expense items directly or readily chargeable to first year and renewal. These would include medical fees, inspection reports, auditing fees,

agency expenses, and so forth. In addition, there would be salaries and overhead which can be directly allocated: agency, underwriting, issue, and policyholder service.

- c) Items not readily allocable. These would include a whole mass of items (generally expenses) which cannot be readily allocated. These expenses are generally allocated in proportion to first-year and renewal premiums, number of policies of insurance in force, salaries, or combinations thereof. This split is a very fluid one. It must be tempered with common sense and checked to see that it reproduces the company's expenses.

One oversimplified approach would be to:

- i) allocate all clerical and supervisory salaries which can be readily segregated;
 - ii) break down all the remaining (nonexecutive) salaries in proportion to first-year and renewal premiums;
 - iii) allocate all expense items which can be readily segregated; and
 - iv) break down in proportion to i and ii above executive salaries and general expenses which cannot be directly allocated.
2. While it is possible to project both "first-year cost" and "renewal profit" in total, it is generally more feasible to relate these items to some base which can then itself be projected. Face amount and premiums are the two most common denominators. The relating of "first-year costs" and "renewal profit" to a base generally gives a more meaningful result and is particularly useful for long-range projections. Thus, a progression of historical first-year costs takes on an added dimension when it is related to the amount of new business written. The same consideration applies to renewal profit.

Generally, it makes little difference whether our denominator is gross or net with respect to reinsurance. This may not be true of some of the newer companies, where a significant portion of the business is reinsured (particularly with respect to premiums under a coinsurance agreement).

Dividing first-year cost by new-business production gives us first-year surplus drain per \$1,000. Dividing renewal profit by the business (in force beginning of year less one half of the previous year's production) producing this profit gives us the renewal profit per \$1,000. A historical series of "new-business strain per \$1,000" and "renewal profit per \$1,000" factors can be prepared. These factors are then extrapolated into the future. Thus, projecting future production and in force, one-time and corporate items enables us to estimate future profits (or losses). Alternatively, first-year costs and renewal profits can be related to premiums.

3. Alternatively in 1, c, we can try to work with the large mass of nonallocable expenses directly. Thus we would project:
- a) Preliminary first-year cost per unit (before nonallocable items are taken into account). These preliminary first-year cost figures will be lower than the true first-year costs.
 - b) Preliminary renewal profits per unit (before nonallocable items are taken

into account). These preliminary renewal profit figures will be higher than the true renewal profits.

c) Nonallocable items.

d) Proceeding along this tack, we would:

- i) define and determine nonallocable expenses for the past years and separate them out;
- ii) divide such adjusted company earnings between first year and renewal;
- iii) develop a historical series of unit-adjusted renewal earnings and project the series forward;
- iv) apply the unit-projected adjusted renewal earnings to projected old business treated as a closed block;
- v) develop a model office projection of earnings on future business before allowance for nonallocable expenses; and
- vi) make a separate en masse projection of nonallocable expenses for the combined closed and future block, after studying the historical trend of the major components of nonallocable expense in relation to total in force and total new production as well as relevant aspects of company history.

The over-all method is relatively simple and can be worked out rather quickly. It has the advantage and disadvantage of lumping together, into one average figure, the results of all kinds and types of business. A minimum number of assumptions are made. It can analyze a company's past and present operations. A historical series of renewal profits per \$1,000, or per dollar of renewal premium, can be developed without too much trouble. As a budgetary operation it is excellent and, as such, will indicate how much is available for acquisition. As a method of prediction, it is attractive but dangerous—by its very nature, it hides many cross-currents. It does not indicate the future value of current business. It does not explain why the results are as they are or what would happen if certain factors were changed.

SUMMARY

Life insurance management, in line with management generally, must make full use of all the equipment at its disposal. One of its most valuable tools is the projection of operations. It is an approach which seeks to force greater realism and thus to "substitute facts for appearances and demonstrations for impressions."

Three broad methods (model office, analysis of operation, and over-all) were described and a detailed example of a model office for a new company was presented. This paper will only succeed in its purpose if it will elicit comments, criticism, and contributions on a tool that, while in increasing use, has received far too little discussion.

APPENDIX

PROJECTION OF A NEW LIFE INSURANCE COMPANY

TABLE 1
CALENDAR-YEAR ASSUMPTIONS

Calendar Year*	Production† (1)	Earned Interest Rate (2)	General Expenses (3)	Per Cent of Production Reinsured (4a)	Amount of New Reinsurance (4b)	Premium Tax Rate (5)
1.....	\$25,000,000	3.50%	\$325,000	25%	\$6,250,000	2.25%
2.....	30,000,000	3.75	375,000	23	6,900,000	2.25
3.....	35,000,000	4.00	425,000	21	7,350,000	2.25
4.....	40,000,000	4.00	475,000	20	8,000,000	2.25
5.....	45,000,000	4.00	525,000	19	8,550,000	2.25

* It is assumed that the company commences operation on January 1.

† Production is assumed to be uniformly distributed throughout the year.

TABLE 2
POLICY-YEAR ASSUMPTIONS

Policy Year	Mortality Rate per \$1,000* (1)	Lapse Rates† (2)	Reinsurance (VRT) Premium‡ per \$1,000 (3)	Net Amount at Risk§ (4)	Reinsurance Cost per \$1,000 (5)
1.....	\$0.79	25.0%	\$1.50	\$1,000.00	\$0.71
2.....	0.99	15.0	2.03	995.55	1.04
3.....	1.25	12.5	2.50	982.90	1.23
4.....	1.44	11.0	2.78	968.00	1.30
5.....	1.63	10.0	3.05	952.85	1.35

* Issue age 35, 1955-60 Select Basic Table—Male and Female Lives Combined (*TSA, 1962 Reports*, p. 46).

† Lapse: Rates are derived from Linton's tables, B and C.

‡ Reinsurance premiums are based on the rates of several large reinsurers.

§ \$1,000 — Table 5, B, col. (5).

|| [Col. (3) — col. (1)] × col. (4).

TABLE 3
SERVICE TABLES PER MILLION OF ISSUE

YEAR	BY POLICY YEAR				BY CALENDAR YEAR			
	In Force Beginning of Year (1)	Deaths* (2)	Lapses† (3)	Reinsur- ance Cost‡ per Million Ceded (4)	In Force Begin- ning of Year (5)	Deaths§ (6)	Lapses (7)	Surrender Cost# per Million of Issue (8)
1	\$1,000,000	\$790	\$250,000	\$710.00		\$395	\$ 0	\$ 0
2	749,210	742	112,382	779.18	\$999,605	766	250,000	0
3	636,086	795	79,511	782.39	748,839	769	112,382	500.10
4	555,780	800	61,136	722.51	635,688	798	79,511	1,359.64
5	493,844	805	49,384	666.69	555,379	803	61,136	1,956.35
6	443,655				493,440			

* Table 2, col. (1) × col. (1) of this table. Deaths are assumed to occur throughout the year.

† Table 2, col. (2) × col. (1) of this table. Lapses are assumed to occur at the end of the year.

‡ Table 2, col. (5) × col. (1) of this table.

§ $\frac{1}{2}$ [col. (2)-1 + col. (2)].

|| Col. (3)-1.

Table 5, B, col. (5)-1 × col. (7) of this table.

TABLE 4
PLAN ASSUMPTIONS

Plan (1)	Plan Distri- bution (2)	Cash-Value Basis	Reserve Basis	Average Size (5)	Premi- um* per \$1,000 (6)	Policy Fee per \$1,000† (7)	Weighted Premium‡ per \$1,000 (8)
		(1958 CSO, 3½ Per Cent) (3)	(4)				
Whole Life	50%	Minimum	Graded§	\$10,000	\$17.25	\$1.00	\$ 9.13
20-Pay Life	10	Minimum	CRVM	6,000	26.10	1.67	2.78
RI at 65—Male.	15	Minimum	CRVM	5,000	36.85	2.00	5.83
5-Year R+C Term	25		CRVM	25,000	5.30	0.40	1.43
Total weighted average							\$19.17

* Typical nonparticipating premiums of the newer life insurance companies.

† \$10 policy fee ÷ col. (5).

‡ Col. (2) × [col. (6) + col. (7)].

§ CRVM in the first-year grading to the net level reserve in the twentieth year.

TABLE 5
VALUES PER PLAN

Policy Year	Whole Life (1)	20-Pay Life (2)	RI at 65— Male (3)	5-Year R+C Term (4)	Weighted Average per \$1,000 (5)
A. Commissions*					
1.....	90.0%	85.0%	75.0%	70.0%	\$15.95†
2.....	12.5	12.5	7.5	7.5	2.03†
3.....	7.5	7.5	7.5	7.5	1.44†
4.....	7.5	7.5	7.5	7.5	1.44†
5.....	7.5	7.5	7.5	7.5	1.44†
B. Cash Values per \$1,000‡					
1.....	\$ 0	\$ 0	\$ 0		\$ 0
2.....	0	10	23		4.45
3.....	11	32	56		17.10
4.....	26	55	90		32.00
5.....	41	79	125		47.15
C. Mean Reserves per \$1,000‡					
1.....	\$ 1	\$ 1	\$ 9	\$1	\$ 2.20
2.....	15	23	40	2	16.30
3.....	29	45	72	2	30.30
4.....	44	67	105	2	44.95
5.....	58	90	139	2	59.35

* All business is assumed to be sold through the general agency system. Where appropriate the sales costs should be broken down into commissions, override, expense reimbursement, development allowance, and so forth.

† Commission dollars. Table 4, col. (8), weighted by commission percentages for each plan.

‡ See Table 4 for bases.

TABLE 6
ITEMS BASED ON PREMIUM PAYING IN FORCE

POLICY YEAR	CALENDAR YEAR				
	1	2	3	4	5
A. Premium Paying In Force (In Force Beginning of Policy Year;* 000 Omitted)					
1.....	\$ 25,000	\$ 18,730	\$ 15,902	\$ 13,895	\$ 12,346
2.....		30,000	22,476	19,083	16,673
3.....			35,000	26,222	22,263
4.....				40,000	29,968
5.....					45,000
Total....	\$ 25,000	\$ 48,730	\$ 73,378	\$ 99,200	\$ 126,250
B. Premiums Paid†					
1.....	\$479,250	\$359,054	\$ 304,841	\$ 266,367	\$ 236,673
2.....		575,100	430,865	365,821	319,621
3.....			670,950	502,676	426,782
4.....				766,800	574,487
5.....					862,650
Total....	\$479,250	\$934,154	\$1,406,656	\$1,901,664	\$2,420,213
Premium taxes.....	10,783	21,018	31,650	42,787	54,455
C. Commissions‡					
1.....	\$398,750	\$ 38,022	\$ 22,899	\$ 20,009	\$ 17,778
2.....		478,500	45,626	27,480	24,009
3.....			558,250	53,231	32,059
4.....				638,000	60,835
5.....					717,750
Total....	\$398,750	\$516,522	\$ 626,775	\$ 738,720	\$ 852,431

* Each item of Table 1, col. (1) × Table 3, col. (1).

† Average weighted premium of \$19.17 from Table 4, col. (8), applied to Table 6, A; premiums are assumed to be paid annually.

‡ Table 5, A, col. (5), applied to horizontal elements of Table 6, A.

TABLE 7
ITEMS BASED ON SERVICE TABLES

POLICY YEAR	CALENDAR YEAR				
	1	2	3	4	5
A. Death Claims*					
1.....	\$ 9,875	\$ 19,150	\$ 19,225	\$ 19,950	\$ 20,075
2.....		11,850	22,980	23,070	23,940
3.....			13,825	26,810	26,915
4.....				15,800	30,640
5.....					17,775
Total....	\$ 9,875	\$ 31,000	\$ 56,030	\$ 85,630	\$ 119,345
B. Reinsurance Costs†					
1.....	\$ 4,438	\$ 4,870	\$ 4,890	\$ 4,516	\$ 4,167
2.....		4,899	5,376	5,398	4,985
3.....			5,219	5,727	5,751
4.....				5,680	6,233
5.....					6,071
Total....	\$ 4,438	\$ 9,769	\$ 15,485	\$ 21,321	\$ 27,207
Adjusted reinsurance cost‡.....	2,219	7,104	12,627	18,403	24,264
C. Surrender Benefits§					
1.....	\$ 0	\$ 0	\$ 12,503	\$ 33,991	\$ 48,909
2.....		0	0	15,003	40,789
3.....			0	0	17,504
4.....				0	0
5.....					0
Total....	\$ 0	\$ 0	\$ 12,503	\$ 48,994	\$ 107,202

* Table 1, col. (1), applied to Table 3, col. (6).

† Table 1, col. (4b) applied to Table 3, col. (4).

‡ Mean of t th and $(t-1)$ th calendar-year reinsurance costs adjusts for increase in unearned premium reserve.

§ Table 1, col. (1), applied to Table 3, col. (8).

TABLE 8
RESERVES

YEAR	CALENDAR YEAR				
	1	2	3	4	5
A. Insurance in Force at End of Calendar Year* (000 Omitted)					
1.....	\$24,990	\$ 18,721	\$ 15,892	\$ 13,884	\$ 12,336
2.....		29,988	22,465	19,071	16,661
3.....			34,986	26,209	22,249
4.....				39,984	29,954
5.....					44,982
Total....	\$24,990	\$ 48,709	\$ 73,343	\$ 99,148	\$ 126,182
B. Reserves†					
1.....	\$54,978	\$305,152	\$481,528	\$ 624,086	\$ 732,142
2.....		65,974	366,180	577,851	748,912
3.....			76,969	427,207	674,145
4.....				87,965	488,250
5.....					98,960
Total....	\$54,978	\$371,126	\$924,677	\$1,717,109	\$2,742,409
Adjusted for immediate payment of claims‡.....	55,940	377,621	940,859	1,747,158	2,790,401
Increase in re- serves.....	55,940	321,681	563,238	806,299	1,043,243

* Each item of Table 1, col. (1) × Table 3, col. (5)₄₁.

† Table 5, C, col. (5), applied to horizontal elements of Table 8, A.

‡ 1.0175 × each total.

TABLE 9—SUMMARY OF OPERATIONS

ITEM No.	ITEM	SOURCE	CALENDAR YEAR				
			1	2	3	4	5
1.....	Insurance operations:	Table 6, B	\$ 479,250	\$ 575,100	\$ 670,950	\$ 766,800	\$ 862,650
2.....	Premiums:	Table 6, B	0	359,054	735,706	1,134,864	1,557,563
3.....	First-year	$\frac{1}{2}[(1)+(2)+(12)-(13)]$	—	—	—	—	—
4.....	Renewal	$+(15)-(22)-1$	4,679	10,511	7,121	7,278	32,162
5.....	Investment income from ins. oper.						
6.....	Total income	Items (1) to (3)	\$ 474,571	\$ 923,643	\$ 1,399,535	\$ 1,908,942	\$ 2,452,375
7.....	Disbursements:	Table 7, A	\$ 9,875	\$ 31,000	\$ 56,030	\$ 85,630	\$ 119,345
8.....	Death claims	Table 7, B	2,219	7,104	12,627	18,403	24,264
9.....	Reinsurance costs	Table 7, C	0	0	12,503	48,994	107,202
10.....	Surrender benefits	Table 6, C	398,750	478,500	558,250	638,000	717,750
11.....	Commissions:	Table 6, C	0	38,022	68,525	100,720	134,681
12.....	First-year	Table 1	325,000	375,000	425,000	475,000	525,000
13.....	Renewal	Table 6, B	10,783	21,018	31,650	42,787	54,455
14.....	General expenses	Table 8, B	55,940	321,681	563,238	806,299	1,043,243
15.....	Premium taxes	Items (5) to (12)	\$ 802,567	\$ 1,272,325	\$ 1,727,823	\$ 2,215,833	\$ 2,725,940
16.....	Increase in reserves	(4) — (13)	\$ —	\$ —	\$ —	\$ —	\$ —
17.....	Total disbursements	(15)-(14)	\$ —	\$ —	\$ —	\$ —	\$ —
18.....	Gain		\$ —	\$ —	\$ —	\$ —	\$ —
19.....	Insurance operation earned surplus		\$ —	\$ —	\$ —	\$ —	\$ —
20.....	Capital funds:	\$2,500,000	\$ 0	\$ 0	\$ 0	\$ 0	\$ 0
21.....	Contributed	$\frac{1}{2}X(18)-1$	87,500	97,031	107,381	111,676	116,144
22.....	Interest	(18)-1+(17)	2,587,500	2,684,531	2,791,912	2,903,588	3,019,732
23.....	Accumulated capital funds	(15)+(18)	2,259,504	2,007,853	1,786,946	1,591,731	1,434,310
24.....	Annual statement capital and surplus	Table 1	25,000	30,000	35,000	40,000	45,000
25.....	Issue (000 omitted)	Table 8, A	24,990	48,709	73,343	99,148	126,182
26.....	In force (000 omitted)	Table 8, B	55,940	377,621	940,859	1,747,158	2,790,401
27.....	Reserves						

DISCUSSION OF PRECEDING PAPER

ELLIS D. FLINN:

I would like to thank Mr. Gold for writing this paper. The problem of projecting company earnings is one which is certainly encountered frequently by actuaries today, yet one which has received scant attention in actuarial literature. Insurance company management is coming more and more to the realization that a forecast of probable future earnings—the more reliable the better—is essential to sound planning. Third-generation computers have made it possible to refine the projection process by making it practicable to introduce many more factors into the calculation. And, although in the past the work involved in a careful projection was so time-consuming that the effect of varying one or more assumptions was left to the judgment of the management team, it is now possible to run off “high” and “low” forecasts, thereby reducing considerably the judgment area. The availability of electronic computers, together with the increasing pressure of competition, is probably responsible for the increasing interest in projections.

The purposes for which a projection can be used are well set out by Mr. Gold. But the mere making of a projection showing a satisfactory profit position does not guarantee the success of a company. Management must understand the assumptions on which the projection was based. In a sense a projection is a budget of such items as production level, overhead expense, mortality and lapse rates, and so forth, and, like any other budget, management must exercise control to make it work out.

Of the three methods outlined I believe that the model office method is far superior, especially for longer term projections. My remaining comments are about this method.

The sample projection in the Appendix was presented as a simplified projection and, therefore, did not take all things into account. In actual practice, additional factors would be introduced, among which, it seems to me, should be those discussed in the following paragraphs.

1. Each plan should be projected separately. This makes it possible to take account more accurately of the different volume and rates of mortality, lapses, expenses, and amount of reinsurance ceded that affect each plan. Reinsurance, for example, would tend to be understated when plans are grouped, since reinsurance will tend to be mainly on the whole life and term plans, where the amount at risk is greatest.

2. Premium mode should be taken into account. The effect of taking

premium mode into account is to reduce the total premium collected, since not all policies will be paid until the end of the policy year. Especially when results are adjusted to a calendar-year basis, we have found that recognition of the mode can make a significant difference, and it is the calendar-year basis which is of greatest usefulness to management.

3. In spite of the many pitfalls of a long-term projection, a projection should be carried out far enough for a company to see whether early losses will be offset by later projected profits and to aid in tax planning to take full advantage of loss carry-overs.

4. For a new company, I prefer, for competitive reasons, to use the assumed ultimate expense rates in the asset share calculation and from a budget forecast to estimate the excess expenses before these ultimate rates are achieved. This should enable the company to recognize and control expenses in excess of the competitive level (another form of surplus strain) and to reduce this excess each year. In addition, the company should set a time limit on the period in which it expects to operate in excess of ultimate costs.

In Mr. Gold's review of the discussion I would ask him to explain the rationale behind a decreasing proportion being reinsured each year. With capital and surplus falling, it seems unlikely that the retention limit would be raised.

ABRAHAM HAZELCORN:

Mr. Gold is to be commended on this contribution to the actuarial literature. Having made the projections for new insurance companies described by Mr. Gold, we at Lybrand, Ross Bros. & Montgomery cannot emphasize too strongly some of the warnings included in Mr. Gold's paper. He has reflected the requirements for submission of financial projections of at least two important insurance departments.

In our first assignment involving a report including financial projection, we were told that the main purpose of the ten-year projection is to "inform ignorant management" of the operating results of a life insurance company. This point cannot be overstressed. As Mr. Gold has stated, management of new companies is composed of optimists. Putting aside those who think in terms of stock manipulation, many a knowledgeable businessman who becomes a director of a life company does not appreciate the basic progress in the operating results of a well-managed life company.

It is important to note that the projection should not be viewed as merely the satisfaction of a state requirement. While it is not possible to know the complexion of business which is to be sold and expenses of a company not yet licensed, the projection can have future use. In our

transmittal letter to the superintendent or commissioner of insurance, we state that it is our intention to use the submission as a basis for future planning. It should be possible to explain the major deviations after the first year of operation from the results of the required insurance department projections. In addition, complaints will arise if these actuarial assignments are viewed merely as a *pro forma* satisfaction of a step in licensing.

The description of the new company's plan of operation and the interpretation of financial projections are important parts of the submission. An insurance department is concerned with the caliber of a prospective insurer even if it has no foreseeable financing problem. It therefore may be properly interested in the insurance talents and the personal integrity of the individual promoters and management personnel.

While the Appendix is illustrative only, I would like to make some comments on the assumptions. The idea of separating interest on the capital-and-surplus account from the insurance operation is a good one. In the extreme case a company could be misleading its stockholders because the interest on a huge initial capital and surplus may exceed the operating loss. Talk of "operating in the black" the first year would probably be meaningless. Some jurisdictions require a division between capital and surplus. The arithmetic of Table 9 would not change, but the division is important to ascertain the timing of the need for additional financing.

The production assumption in Table 4 contains 25 per cent term insurance. Most new companies experience a much higher percentage. We have shown a projection based on several ages at issue instead of one age. If that one age is 35, there may be a distortion since we have discovered very low margins in the premium at 35. For a mortality rate it may be necessary to assume a percentage of female lives and a percentage of substandard lives.

An excellent source of data for companies organizing in New York is the *New York Insurance Report*. Portions of the annual statements of companies authorized to do business in New York are displayed. Pages 2, 3, and 4 of the NAIC blank are shown along with Exhibits 5 and 8, Schedule G, the Policy Exhibit, and part of Schedule M if the company is a mutual. If the background data of other recently formed companies are known, the general expenses can be modified appropriately and use can be made of the information for projections.

E. J. MOORHEAD:

The aim of this discussion is to supplement Mr. Gold's observations about the usefulness of projections.

How many actuaries can let their projections speak to them and then act with confidence on what the projections tell them? Very few, I fear.

We simply have not mastered the art of devising factors, particularly expense factors, that are genuinely true to life. For example, when we say that a particular category of first-year expense can be expressed as, say, \$30 per policy, we are saying that that expense will double if the number of policies doubles and will be cut in half if the number of policies is reduced by half. The significant but elusive fact is that the relationship between the quantity of items handled and the cost of handling is much more complex than that. Now that we are in an era of computer elasticity, let us not cling to approaches that belong to the much more restricted era of desk calculators.

A question on which more work is greatly needed is that of how to handle what is often loosely called "overhead expense" and sometimes is misleadingly labeled "fixed expense." One recent explorer in this particular jungle is Professor James S. Hekimian, of Massachusetts Institute of Technology, in his book *Management Control in Life Insurance Branch Offices* and in his paper entitled "A Profit Calculation for Agency Management" (*Journal of Risk and Insurance* [March, 1966]).

My personal belief is that Hekimian's analysis is in some respects impractical and that he has carried his theory of variable versus fixed costs beyond the boundary of what is reasonable. I would be interested to hear from actuaries who have studied what the Professor has done and who either agree or disagree with the opinion here expressed.

If the Society is to have workshops at future meetings—and I hope that it will—may I suggest one worthy topic: "How Can We Create Projections and Asset Shares That Will Reliably Tell Us What We Need To Know?"

WILLIAM J. NOVEMBER:

As usual, the author has done a service in focusing attention on a problem of substantial current interest, particularly as it relates to new companies. Methods for making projections are very much in a stage of development, and we can all profit from pooling our thoughts on the subject.

As I see the problem, the form of the projection and the effort put into it depend very much on the purpose served and the information available for the projection. This might have been emphasized by the author before jumping into a description of a new-company projection, and, if he had done so, I believe that the presentation might have taken a different turn.

The situation for a company in the process of formation is quite different from that of a company in existence for a few years and alto-

gether different from that of a well-established company. Each of these types forms a separate category, really, and the projection procedure generated by the actuary will be related to the situation with which he is dealing.

For the companies in the process of formation, a projection of the financial results that might be anticipated is highly important to prospective investors and to supervisory authorities who are faced with the responsibility for granting licenses to new companies. The New York Insurance Department deserves credit for making a financial projection a prerequisite to its consideration of a new license application.

It is unfortunate that in this situation the projection factors are all conjectural, since there is no experience to go on except what has happened in other new companies. The result is hardly a substitution of facts for appearances, but, if a good job is done, it can be a valuable guide to capitalization and budget-planning.

The projection need not be the tool for arriving at decisions concerning such matters as gross premium levels and type of agency organization, as seems suggested by the author. In fact, it is more likely that preliminary studies will be made to enable the entrepreneurs to establish premium levels, the form and scale of agency compensation, the method and extent of reinsurance, and so forth, and that these studies will then provide the means for developing an over-all projection. If the preliminary studies are in the form of profit-margin projections at typical ages for the basic plans of insurance contemplated, the projection can take the form of a synthesis of the results for the various plans and ages by means of a model office assumption and the application of the anticipated levels of future production.

In this process the model office would not be applied to develop average factors for the company as a whole but would be used to bring together the results for the individual policies as determined by the factors deemed appropriate for them. I believe that this is a better approach to developing the over-all picture, aside from the inherent advantage that it has in revealing the *why* of the over-all results.

With modern electronic machinery, the projection is a natural extension of the preliminary studies, which should be undertaken anyway. Some adjustments will be required, such as for the extra "tooling up" costs of the early years and the investment income to be anticipated from the capitalization, but these should present no serious problems. A shift to a calendar-year basis is a complication that has to be planned for if the preliminary studies are on a policy-year basis. This can be avoided by using a calendar-year structure throughout.

Moving now to companies in their early years of operation, we see that projections serve the important purpose of keeping such companies posted on their future capitalization needs. Careful planning of additional capital is necessary, for obvious reasons, so projections of operations are worth a bit of trouble. Fortunately, experience factors will have begun to emerge and production levels will be more than a guess. The whole problem can be tackled with more assurance as to the significance of the results. The basic approach is likely to be that suitable for companies in the process of formation but with a refinement of factors in light of some actual experience.

An interesting purpose that can be served at this stage of a company's development is to provide a basis for a decision on electing the level premium reserve basis under Section 818(c) of the federal income tax law. Whenever such an election is made, the adjustment of the reserve on outstanding business from a preliminary term to a net level basis is lost as a charge against operating results. The sooner the election is made, therefore, the smaller will be the loss. On the other hand, if operating losses of the early years are of such size as to leave in doubt the company's capacity to take full advantage of them through the income tax law's loss-carry-forward provisions, there is no point to adding to the losses. An analysis of what the future holds in the way of operating losses then becomes good tax management.

As has already been mentioned, the well-established companies are in an altogether different situation regarding projections of operations. Their purposes will not be as vital as those for new companies. Projections will normally be wanted in connection with setting company objectives and with planning future operations. The information available for the projections is, of course, much more plentiful.

A company that has been in business for some time is likely to have blocks of business on different premium and actuarial bases. The model office method for projecting the results on business already in force can be quite cumbersome, as the author has brought out. Yet this may be the preferred method for future business. Hence the use of two different methods might be the logical procedure. The purpose of the study should dictate the amount of effort that goes into the job, and I suspect that it will not be as great, relatively, as it is in the case of projections made for new companies.

MEL STEIN:

Mr. Gold is to be congratulated for having written a badly needed nontechnical survey paper on this extremely important topic.

Mr. Gold describes three methods of making projections for an existing

company. I will now briefly outline a fourth approach, which is somewhat related to Mr. Gold's model office approach. A detailed calendar-year gross premium valuation is made for business in force and for current sales. This is based upon two separate model offices. At the same time a budget of company expenses is projected.

The following results will then be projected for each of the individual years covered by the projection:

1. Renewal book profits from insurance in force during the year.
2. First-year investment in new business.
3. The amounts of the major elements that determine the book profits.
4. The amount of insurance in force.
5. The value of insurance in force, determined as the present value of the future book profits expected from this insurance. These values will, of course, be greatly influenced by the yield rates used to discount the book profits.
6. The portion of the company's total expenses not absorbed by the insurance issued and in force during the year. This can be a substantial portion of expenses in the case of a small, young company.
7. The annual statement profit, which is the sum of
 - a) book profits from insurance issued and in force during the year,
 - b) earnings on noninvested surplus and capital,
 - c) expenses not absorbed by the insurance issued and in force (negative), and
 - d) miscellaneous sources of income such as those mentioned by Mr. Gold.
8. The real profit of the company, which is the sum of
 - a) the annual statement profit, and
 - b) the increase in the value of the insurance in force during the year.
9. The value of the agency force at the end of the year. As there is more than one approach that may be taken in accomplishing this, I will only say that there is no one correct value. However, if a sound valuation procedure is consistently followed every year, valuable information is made available to management and stockholders.
10. The going-concern profit of the company during the year. This is the sum of
 - a) the annual statement profit,
 - b) the increase in the value of insurance in force, and
 - c) the increase in the value of the agency force.

I would again like to stress that there is no one correct value to be placed on a company's insurance in force and agency force. However, a consistent approach using the same discount rate provides invaluable information regarding a company's true performance.

Projections for a new company are interdependent with the company's gross-premium structure. The following procedure is recommended for making projections and determining gross premiums for a new company.

1. Based on the proposed market, desired general competitiveness of gross premiums, and sales vehicle, a ten-year projection is made of sales and insurance in force. It is understood that low, most likely, and high sales projections may be made.
2. A ten-year projection of expenses is then made for each sales projection.
3. Gross-premium calculations will then be made using different ultimate expense levels. Each set of gross-premium rates will be based on different amounts of expense assumptions and will be used to project the amount of expense implicitly provided for in the gross-premium structure. This, in turn, will show management the decreasing amounts of projected expense which will not be absorbed by the gross-premium structure under the company projections. This unabsorbed expense is really an additional investment by the stockholders.

Thus, the final choice of premiums will be determined by the weight management places on (a) competitiveness of premiums, (b) ultimate profitability of premiums, (c) additional investment of stockholders, and (d) the period when these additional investments will no longer be required.

No comment is made on Mr. Gold's numerical example as it obviously was meant to be a crude illustration rather than a refined technical model of a projection. I would again like to commend Mr. Gold for his straightforward and timely paper on this important subject.

IRWIN T. VANDERHOOF:

I think that the members of the Society owe a continually increasing debt to Mr. Gold for his series of papers on the business aspects of insurance. Budgets and projections of operations are standard tools of management in most businesses, but sometimes we may become too concerned with our mathematical profundities and ignore this. Except for the model office approach, there is little that I can find in the literature that indicates any handling of projections of operations. It seems to me very good for the business that Mr. Gold has remedied this lack.

My comments cover an observation on the logical relationship among the three types of projections discussed, an opinion about the usefulness of different types of projections, some other references in this general area that I have come across, and the discussion of the experience of Standard Security Life, with its use of projections since its inception in 1959.

Logically, it seems to me that these different mathematical approaches toward projections are not so distinct and different as they might appear. After all, our conventional model office approach simply represents many policies by a relatively smaller number of policy age groupings. The

analysis of operations approach simply combines all the commission, all the premium items, and so forth, that might appear in a model office into large aggregates of the type that appear in the annual statement form. If we simply did a summation of the items in our model office, we should end up with an analysis of operations approach and should logically get the same answer. Similarly, if the analysis of operations items are grouped, we obtain the so-called over-all method as a result of the grouping. From this, it seems to me to follow that, if the underlying data and assumptions are equally good, there is no clear-cut choice, on the basis of the final result, of one method over the other. They all start with a model office and end with a statement of profit or loss. The only question is how the aggregates are summed before or after the projection is done. In the model office the summation is after the projection of the various cells; in the other approaches the summation is done before the projection.

This logically leads to a second point which is perhaps more important, and that is the appropriate basis for choice among the different methods. It is obvious that the model office approach involves very considerable amounts of work, as compared to the other two. This is not a final argument, however, if the model office approach has advantages under some circumstances. I believe that it does, depending on the question one is asking. If you are asking a question about the effect on future profits of a change in the mix of business, a change in the premium rates on important plans, or a change in underwriting standards, and so forth, then the model office approach is the only one that can give meaningful answers. On the other hand, if you are asking questions about the profitability depending upon adherence to specific budget limitations, then the analysis of operation approach would seem quite adequate. In the analysis of operation approach, all the data are presented that should be necessary to determine after the fact why it is that the projection did not work out. This negative control or after-the-fact analysis is probably most important from a management point of view, in that it allows management to evaluate its performance with respect to predetermined goals and to establish a reason for failure to meet the goals or to look for the reasons that allowed the goals to be surpassed. The over-all approach does not seem to be usable for management-control functions but is perhaps suitable for the type of broad projection from the outside necessary for a security analyst.

In James Walter's *The Investment Process*, published by Harvard Business School, there is a discussion of cash-flow projections which make rather neat distinctions between the accounting tautologies and behavioral equations involved in a projection of any kind of operation. These

projections are done primarily from a cash flow point of view and include a sort of Monte Carlo approach toward the simulation of operations under various conditions. It would seem that adding on the possible variations in some of the parameters within limits, say, death claims, might have some advantages in predicting the possible necessity for additional financing for a small new company. This approach has not, to the best of my knowledge, been used. One technique which fits within the general area of analysis of operations, for the life insurance business, is included in the Reynolds Griffith book, *The Valuation of Life Insurance Stock*. Mr. Griffith bases his approach toward values of life insurance stock on discounting the projected profits of a block of life insurance business in force. His approach, therefore, does involve long-range projections of business, and he has developed some computer programs which he feels are adequate for that purpose. I know of no other literature appropriate to this subject and would be interested in hearing of any from any other members.

The last area that I would like to cover is the experience of Standard Security in the use of projections. This may be of interest, in that it covers a period from the inception of the company through \$400,000,000 in force. Our first attempt at projections used the conventional model office approach. This was nice because it gave us a feeling of some confidence, in that we were using a generally accepted method and could, from the model office, tie ourselves into projections of the entire operation. The amount of work involved was, of course, very substantial and would prevent our doing this type of projection very often. Unfortunately, for a new company, the problem is that, in our experience, it is just not possible to tell what kind of business you will be selling and in what volume. These things are, at best, educated guesses, and they cannot be expected to work out. For a new company, however, it seems to me that this is about the only approach one can take.

Determined that we should avoid doing that much work again, if at all possible, and imbued with the belief that the directors and management of the company should have a clear idea as to how insurance profits arise, we worked out factors which were intended to represent the strain on surplus on new issues and the profit on renewal business, which Mr. Gold refers to as the over-all approach. The approach is very attractive, in that it reduces everything to simplest terms. If the sales are so much, and the renewal business is so much at the beginning of the year, then the profits for the year fall out rather automatically. At that point, we even attempted to develop several different figures from this over-all approach, including the statutory profit on an over-all basis, the cash flow on an over-all basis, and the adjusted earnings on an over-all basis. These

factors were constructed from a model office approach and were helpful, in that it was very easy to explain the meaning of these items to management and to the directors, and it made them feel very knowledgeable about the life insurance business. Unfortunately, the use of these factors was misunderstood by everyone who had access to them, and, when projections based upon this approach seemed not to work out, there was nothing within the approach which would give any indication as to the reasons for failure. Our experience would be, then, that the over-all approach, while perhaps useful from the point of view of a security analyst since the first-year drain on surplus is related to the figure which analysts use for adjusting earnings, is relatively less useful in the planning and control functions which are the functions of management.

Since about 1961, we have been using a version of the analysis of operations approach. Projections are normally done in March or April for the next three years. The results have worked out reasonably well over this period of time and have worked out very well if we consider the function of projections to be primarily for management planning and control rather than an all-out attempt to probe the future. The analysis of operations approach does give an indication as to what the effect on surplus will be on a year-by-year basis and allows co-ordination of these results with the planned expenditures of the company. Further, if the results differ from those projected, there is enough breakdown to indicate the area in which the difference developed, so that the process of rectification of this area in the future can be intelligently undertaken.

DONALD B. WARREN:

In view of Mr. Gold's statement that very little has appeared in the actuarial literature about financial projections, I am delighted to note the close affinity between his paper and my paper entitled "Financial Projections for New Life Insurance Companies," which I delivered in 1964 before the Seventeenth International Congress of Actuaries in London.

In my paper I developed a rather detailed method of projection because (1) it seemed desirable that the individual items therein should attempt to tie in as closely as possible with similar items in forthcoming annual statements, (2) I wanted to illustrate the effect of starting a new company in the middle of a calendar year (which would be the usual situation), and (3) I wanted the rate of production to build up gradually during a given year rather than starting and continuing at the same high level. I also tried to demonstrate the effects, particularly in the first calendar year, of sales of fractional premium business.

For these reasons I first made complete separate calendar-year projec-

tions for representative plans, ages, and modes of payment and then synthesized those projections into the summary of operations. The use of computer techniques permits rather complicated and detailed calculations to be made for a relatively small expenditure of time and money. Mr. Gold, on the other hand, has developed one artificial plan of insurance which is supposed to represent satisfactorily variations in premiums, commissions, cash values, reserves, lapse rates, and size.

I think that the most vulnerable part of Mr. Gold's procedure is the use of annual premium projections throughout. In the first calendar year of a new company's operation, this assumption can produce substantial variations between projected and actual premium income and commissions. It can also overstate substantially the expected insurance in force at the first year-end, since it allows for no lapses. It can also have similar effects in subsequent years if production increases rapidly. Admittedly, projected versus actual operating losses (particularly in the first calendar year) will be little affected by mode-of-payment assumptions, since commissions, claims, and reserve increases will probably just about be equal to whatever premiums are collected and the operating loss will closely approximate the amount of general insurance expenses. Such expenses are pretty much determined before a company gets into production and are relatively independent of any but extreme variations in production. It is relatively easy to explain variations between projections and actual annual statement results to a group of entrepreneurs if those variations are proportional to production; but it is much more difficult to explain them if they are due to assuming annual premium collections when actual sales are perhaps 50 per cent fractional. The closer the projection is to actual results, the more faith the new company will have in actuarial techniques during those formative years when it most needs to be guided by those techniques.

I have a couple of specific comments. First, there is the handling of reinsurance in Table 2 of the Appendix. The calculation of the net amount at risk is based on cash values, whereas I think most reinsurance is still based on reserves. This, of course, is a minor matter. Second, since social security tax rates are trending ever higher, it would appear that commission rates (at least in the first year) should be adjusted upward to allow for the employer's social security contribution.

I think that the method of handling investment income in Table 9 is worth discussing. In line 3 (investment income from insurance operations), Mr. Gold charges interest on the accumulated losses from all previous years against the operations of each emerging year. This may well be good theory, but it seems to obscure each individual year's actual

operating results. I think that this might be difficult to explain to non-technical entrepreneurs. I would prefer to show the gain (or loss) from what I consider to be the true insurance operations each year in line 14 and then insert a separate line, 14a, for the investment deduction for interest on accumulated losses; this would, of course, change the entry in line 3.

(AUTHOR'S REVIEW OF DISCUSSION)

MELVIN L. GOLD:

I would like to thank Messrs. Flinn, Hazelcorn, Moorhead, November, Stein, Vanderhoof, and Warren for their comments and contributions.

As pointed out, the sample projection was for illustrative purposes only. An expanded projection will take into account other parameters, particularly age distribution and mode of payment. As Mr. Warren and Mr. Flinn indicated, the use of modal premiums allows for a closer tie-in between the projection and the annual statement.

The suggestion that each plan be individually projected and then synthesized at the end has considerable merit. While it may result in some additional work, it allows for far greater flexibility. The actuary is then in a better position to show the effect of changes in the parameters.

With regard to Mr. Flinn's question, we have found that with time a decreasing proportion of a new company's business is reinsured. This occurs partly because the initial submission generally contains many large semicaptive cases; also retention limits tend to be raised as a company's in force grows and it gains confidence in its underwriting.

Mr. Vanderhoof's observations on the relationship among the three projection approaches were particularly incisive, as were his comments on the practical aspects of which approach to follow. I was disappointed that Messrs. November, Stein, and Vanderhoof were the only members to comment on the more complex problem of making projections for existing companies.

I am sorry that I was not aware of Mr. Warren's contribution to the Seventeenth International Congress. He is to be commended for his contribution to the actuarial literature. As his title suggests, however, it was concerned only with new companies.

I would like to take this opportunity to thank Messrs. Herbert L. DePrenger, Roland F. Dorman, Manuel Gelles, George B. Kyle, Irving Rosenthal, James B. Ross, and C. David Silletto for reviewing the paper prior to its submission. Their suggestions were most helpful.

Ga-1951 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970—ACTUARIAL NOTE

G. B. SAKSENA

A CONSIDERABLE amount of time and effort is needed in the preparation of a mortality table from the experience of assured lives. Actuaries are thus led to various devices to keep pace with the improvement in mortality with lapse of time; for example, rating-down of ages, using a conservative rate of interest, and, quite recently, the use of projection scales.

The latest available mortality table of annuitants (Ga-1951) was prepared by Mr. Ray M. Peterson. His tables were published in *TSA*, Volume IV, on pages 246–307. Mr. Peterson also provided in his paper two projection scales which could be used to determine rates of mortality at any future date after 1951.

Recently Mr. Gordon J. Munro prepared the (static) Ga-1951 Mortality Table projected to 1960 (*TSA*, XII, 353–68), using the Projection Scale C as given by Mr. Peterson. The author has now prepared the (static) Ga-1951 Male Mortality Table using Projection Scale C to 1970 in the hope that this table can be used for the next few years. The author has not prepared a corresponding female mortality table, as the male table could be used for females by rating down the age by five years, as in the case of the 1937 Standard Annuity Table.

The values of q_x were first derived by using the formula $q_x^{1970} = q_x^{1951} \cdot (1 - 0.01r_x)^{19}$, where r_x was taken from Table 1 of Mr. Munro's paper. The radix for age five was chosen 9999.9999 (the same as that used by Mr. Peterson for the Ga-1951 Mortality Table) to prepare the Ga-1951 Mortality Table Projection C to 1970. Monetary functions were computed at $3\frac{1}{2}$ per cent in order to facilitate comparisons with Mr. Munro's tables.

The values of q_x as computed above were found to be extremely irregular. It was, therefore, decided to "graduate" the projected table by a Gompertz formula similar to that used in the 1937 Standard Annuity Table (*TASA*, XXXIX, 8). The constants used were:

$$\log_{10} c = .042 ,$$

$$10^4 \text{colog}_{10} p_x = A_x + 10^{.042(x-18.86)} ,$$

where

$$\begin{aligned} A_x &= 0.000014032 (x^2 - 14x + 177)(x - 35)^2 \text{ for ages 5-35 ;} \\ &= 0 \text{ for ages 35-105 ;} \\ &= 152.1 \frac{(x - 105)^2}{110 - x} \text{ for ages 105-110 .} \end{aligned}$$

For the Seniority Table, the following formula was used:

$$w - x = \frac{\log_{10} [1 + (1/c)^{x-y}]}{\log_{10} c}.$$

Table 1 gives the “graduated” and the “ungraduated” values of q_x ; Table 2 gives the “graduated” values of l_x , d_x , q_x , together with the values of D_x , N_x , and $\ddot{a}_x$ based on “graduated” table; and Table 3 provides the Table of Uniform Seniority based on “graduated” table.

TABLE 1

GA-51 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970
UNGRADUATED AND GRADUATED RATES OF MORTALITY

AGE x	VALUES OF q_x		AGE x	VALUES OF q_x		AGE x	VALUES OF q_x	
	Ungradu- ated	Graduated		Ungradu- ated	Graduated		Ungradu- ated	Graduated
5...	0.000,440	0.000,444	41...	0.001,726	0.001,957	76...	0.057,193	0.056,182
6...	.000,409	.000,417	42...	.001,929	.002,156	77...	0.063,679	0.061,708
7...	.000,389	.000,397	43...	.002,180	.002,374	78...	0.070,984	0.067,756
8...	.000,379	.000,384	44...	.002,478	.002,615	79...	0.079,077	0.074,374
9...	.000,375	.000,377	45...	.002,819	.002,880	80...	0.087,783	0.081,609
10...	.000,376	.000,374	46...	.003,201	.003,165	81...	0.096,960	0.089,514
11...	.000,383	.000,375	47...	.003,621	.003,494	82...	0.106,580	0.098,143
12...	.000,391	.000,380	48...	.004,079	.003,848	83...	0.116,600	0.107,553
13...	.000,398	.000,387	49...	.004,573	.004,238	84...	0.127,022	0.117,805
14...	.000,407	.000,396	50...	.005,099	.004,667	85...	0.137,825	0.128,961
15...	.000,417	.000,407	51...	.005,659	.005,140	86...	0.149,077	0.141,088
16...	.000,428	.000,418	52...	.006,251	.005,661	87...	0.160,882	0.154,250
17...	.000,441	.000,431	53...	.006,875	.006,234	88...	0.173,336	0.168,515
18...	.000,454	.000,444	54...	.007,530	.006,864	89...	0.186,540	0.183,951
19...	.000,469	.000,458	55...	.008,217	.007,559	90...	0.200,594	0.200,622
20...	.000,485	.000,473	56...	.008,934	.008,323	91...	0.212,555	0.218,592
21...	.000,504	.000,488	57...	.009,684	.009,164	92...	0.225,161	0.237,920
22...	.000,524	.000,505	58...	.010,474	.010,090	93...	0.238,524	0.258,657
23...	.000,546	.000,522	59...	.011,322	.011,109	94...	0.252,765	0.280,848
24...	.000,570	.000,541	60...	.012,248	.012,230	95...	0.268,025	0.304,523
25...	.000,597	.000,563	61...	.013,281	.013,463	96...	0.284,455	0.329,702
26...	.000,627	.000,587	62...	.014,451	.014,820	97...	0.302,223	0.356,383
27...	.000,660	.000,615	63...	.015,802	.016,313	98...	0.321,515	0.384,546
28...	.000,697	.000,647	64...	.017,376	.017,954	99...	0.342,526	0.414,144
29...	.000,736	.000,685	65...	.019,227	.019,759	100...	0.365,462	0.445,103
30...	.000,780	.000,729	66...	.021,412	.021,744	101...	0.390,538	0.477,316
31...	.000,830	.000,781	67...	.023,711	.023,925	102...	0.417,979	0.510,639
32...	.000,883	.000,842	68...	.025,974	.026,322	103...	0.450,096	0.544,891
33...	.000,943	.000,914	69...	.028,302	.028,955	104...	0.489,201	0.579,854
34...	.001,009	.000,998	70...	.030,948	.031,848	105...	0.537,605	0.615,266
35...	.001,082	.001,096	71...	.034,332	.035,025	106...	0.597,619	0.653,873
36...	.001,161	.001,207	72...	.038,109	.038,512	107...	0.671,554	0.700,526
37...	.001,250	.001,330	73...	.042,212	.042,338	108...	0.761,722	0.761,723
38...	.001,347	.001,465	74...	.046,705	.046,536	109...	0.870,434	0.860,084
39...	.001,456	.001,613	75...	0.051,575	0.051,138	110...	1.000,000	1.000,000
40...	0.001,575	0.001,777						

TABLE 2

Ga-51 MALE MORTALITY TABLE PROJECTED WITH SCALE C TO 1970
 COMMUTATION COLUMNS AT 3½ PER CENT INTEREST
 BASED ON "GRADUATED" TABLE

x	l_x	d_x	q_x	D_x	N_x	δ_x
5.....	9,999.9999	4.4400	0.000,444	8,419.7316	224,011.2535	26.6055
6.....	9,995.5599	4.1681	.000,417	8,131.3944	215,591.5219	26.5135
7.....	9,991.3918	3.9666	.000,397	7,853.1436	207,460.1275	26.4175
8.....	9,987.4252	3.8352	.000,384	7,584.5661	199,606.9839	26.3175
9.....	9,983.5900	3.7638	.000,377	7,325.2692	192,022.4178	26.2137
10.....	9,979.8262	3.7325	.000,374	7,074.8866	184,697.1486	26.1060
11.....	9,976.0937	3.7410	.000,375	6,833.0826	177,622.2620	25.9945
12.....	9,972.3527	3.7895	.000,380	6,599.5365	170,789.1794	25.8790
13.....	9,968.5632	3.8578	.000,387	6,373.9407	164,189.6429	25.7595
14.....	9,964.7054	3.9460	.000,396	6,156.0135	157,815.7022	25.6360
15.....	9,960.7594	4.0540	.000,407	5,945.4838	151,659.6887	25.5084
16.....	9,956.7054	4.1619	.000,418	5,742.0909	145,714.2049	25.3765
17.....	9,952.5435	4.2895	.000,431	5,545.5949	139,972.1140	25.2402
18.....	9,948.2540	4.4170	.000,444	5,355.7534	134,426.5191	25.0995
19.....	9,943.8370	4.5543	.000,458	5,172.3434	129,070.7657	24.9540
20.....	9,939.2827	4.7013	.000,473	4,995.1444	123,898.4223	24.8038
21.....	9,934.5814	4.8481	.000,488	4,823.9437	118,903.2779	24.6486
22.....	9,929.7333	5.0145	.000,505	4,658.5406	114,079.3342	24.4882
23.....	9,924.7188	5.1807	.000,522	4,498.7324	109,420.7936	24.3226
24.....	9,919.5381	5.3665	.000,541	4,344.3325	104,922.0612	24.1515
25.....	9,914.1716	5.5817	.000,563	4,195.1519	100,577.7287	23.9748
26.....	9,908.5899	5.8163	.000,587	4,051.0048	96,382.5768	23.7923
27.....	9,902.7736	6.0902	.000,615	3,911.7168	92,331.5720	23.6038
28.....	9,896.6834	6.4032	.000,647	3,777.1122	88,419.8552	23.4094
29.....	9,890.2802	6.7748	.000,685	3,647.0226	84,642.7430	23.2087
30.....	9,883.5054	7.2051	.000,729	3,521.2796	80,995.7204	23.0018
31.....	9,876.3003	7.7134	.000,781	3,399.7223	77,474.4408	22.7885
32.....	9,868.5869	8.3094	.000,842	3,282.1904	74,074.7185	22.5687
33.....	9,860.2775	9.0123	.000,914	3,168.5283	70,792.5281	22.3424
34.....	9,851.2652	9.8316	.000,998	3,058.5819	67,623.9998	22.1096
35.....	9,841.4336	10.7862	.001,096	2,952.2024	64,565.4179	21.8703
36.....	9,830.6474	11.8656	.001,207	2,849.2432	61,613.2155	21.6244
37.....	9,818.7818	13.0590	.001,330	2,749.5693	58,763.9723	21.3721
38.....	9,805.7228	14.3654	.001,465	2,653.0554	56,014.4030	21.1132
39.....	9,791.3574	15.7935	.001,613	2,559.5833	53,361.3476	20.8477
40.....	9,775.5639	17.3712	.001,777	2,469.0383	50,801.7643	20.5755
41.....	9,758.1927	19.0968	.001,957	2,381.3051	48,332.7260	20.2967
42.....	9,739.0959	20.9975	.002,156	2,296.2753	45,951.4209	20.0113
43.....	9,718.0984	23.0708	.002,374	2,213.8401	43,655.1456	19.7192
44.....	9,695.0276	25.3525	.002,615	2,133.8980	41,441.3055	19.4205
45.....	9,669.6751	27.8487	.002,880	2,056.3458	39,307.4075	19.1152
46.....	9,641.8264	30.5164	.003,165	1,981.0855	37,251.0617	18.8034
47.....	9,611.3100	33.5819	.003,494	1,908.0342	35,269.9762	18.4850
48.....	9,577.7281	36.8551	.003,848	1,837.0700	33,361.9420	18.1604
49.....	9,540.8730	40.4342	.004,238	1,768.1169	31,524.8720	17.8296
50.....	9,500.4388	44.3385	.004,667	1,701.0856	29,756.7551	17.4928
51.....	9,456.1003	48.6044	.005,140	1,635.8905	28,055.6695	17.1501
52.....	9,407.4959	53.2558	.005,661	1,572.4464	26,419.7790	16.8017
53.....	9,354.2401	58.3143	.006,234	1,510.6713	24,847.3326	16.4479
54.....	9,295.9258	63.8072	.006,864	1,450.4867	23,336.6613	16.0888
55.....	9,232.1186	69.7856	.007,559	1,391.8170	21,886.1746	15.7249
56.....	9,162.3330	76.2581	.008,323	1,334.5858	20,494.3576	15.3563
57.....	9,086.0749	83.2648	.009,164	1,278.7227	19,159.7718	14.9835
58.....	9,002.8101	90.8384	.010,090	1,224.1589	17,881.0491	14.6068
59.....	8,911.9717	99.0031	.011,109	1,170.8282	16,656.8902	14.2266
60.....	8,812.9686	107.7826	.012,230	1,118.6681	15,486.0620	13.8433
61.....	8,705.1860	117.1979	.013,463	1,067.6200	14,367.3939	13.4574
62.....	8,587.9881	127.2740	.014,820	1,017.6296	13,299.7739	13.0694
63.....	8,460.7141	138.0196	.016,313	968.6458	12,282.1443	12.6797
64.....	8,322.6945	149.4257	.017,954	920.6225	11,313.4985	12.2890
65.....	8,173.2688	161.4956	.019,759	873.5204	10,392.8760	11.8977
66.....	8,011.7732	174.2080	.021,744	827.3048	9,519.3556	11.5065
67.....	7,837.5652	187.5137	.023,925	781.9477	8,692.0508	11.1159
68.....	7,650.0515	201.3647	.026,322	737.4296	7,910.1031	10.7266
69.....	7,448.6868	215.6767	.028,955	693.7381	7,172.6735	10.3392
70.....	7,233.0101	230.3569	.031,848	650.8705	6,478.9354	9.9543
71.....	7,002.6532	245.2679	.035,025	608.8324	5,828.0649	9.5725
72.....	6,757.3853	260.2404	.038,512	567.6407	5,219.2325	9.1946
73.....	6,497.1449	275.0761	.042,338	527.3234	4,651.5918	8.8211
74.....	6,222.0688	289.5502	.046,536	487.9203	4,124.2684	8.4528
75.....	5,932.5186	303.3771	.051,138	449.4826	3,636.3481	8.0901
76.....	5,629.1415	316.2564	.056,182	412.0743	3,186.8655	7.7337
77.....	5,312.8851	327.8475	.061,708	375.7712	2,774.7912	7.3843
78.....	4,985.0376	337.7662	.067,756	340.6600	2,399.0200	7.0423
79.....	4,647.2714	345.6362	.074,374	306.8389	2,058.3600	6.7083
80.....	4,301.6352	351.0521	.081,609	274.4136	1,751.5211	6.3828
81.....	3,950.5831	353.6325	.089,514	243.4966	1,477.1075	6.0662
82.....	3,596.9506	353.0155	0.098,143	214.2031	1,233.6109	5.7591

TABLE 2—Continued

x	l_x	d_x	q_x	D_x	N_x	$\bar{a}_x$
83.....	3,243.9351	348.8950	0.107,553	186.6479	1,019.4078	5.4617
84.....	2,895.0401	341.0502	0.117,805	160.9404	832.7599	5.1743
85.....	2,553.9899	329.3651	0.128,961	137.1796	671.8195	4.8974
86.....	2,224.6248	313.8679	0.141,088	115.4481	534.6399	4.6310
87.....	1,910.7569	294.7343	0.154,250	95.8065	419.1918	4.3754
88.....	1,616.0226	272.3240	0.168,515	78.2883	323.3853	4.1307
89.....	1,343.6986	247.1747	0.183,951	62.8942	245.0970	3.8970
90.....	1,096.5239	219.9868	0.200,622	49.5891	182.2028	3.6743
91.....	876.5371	191.6040	0.218,592	38.3000	132.6137	3.4625
92.....	684.9331	162.9593	0.237,920	28.9159	94.3137	3.2617
93.....	521.9738	135.0122	0.258,657	21.2910	65.3978	3.0716
94.....	386.9616	108.6774	0.280,848	15.2502	44.1068	2.8922
95.....	278.2842	84.7439	0.304,523	10.5963	28.8566	2.7233
96.....	193.5403	63.8106	0.329,702	7.1203	18.2603	2.5645
97.....	129.7297	46.2335	0.356,383	4.6113	11.1400	2.4158
98.....	83.4962	32.1081	0.384,546	2.8676	6.5287	2.2767
99.....	51.3881	21.2821	0.414,144	1.7052	3.6611	2.1470
100.....	30.106,027	13.400,283	0.445,103	0.965,202,56	1.955,897,02	2.0264
101.....	16.705,744	7.973,919	0.477,316	0.517,476,33	0.990,694,46	1.9145
102.....	8.731,825	4.458,810	0.510,639	0.261,330,05	0.473,218,13	1.8108
103.....	4.273,015	2.328,327	0.544,891	0.123,560,14	0.211,888,08	1.7149
104.....	1.944,688	1.127,635	0.579,854	0.054,331,73	0.088,327,94	1.6257
105.....	0.817,053	0.502,705	0.615,266	0.022,055,33	0.033,996,21	1.5414
106.....	0.314,348	0.205,544	0.653,873	0.008,198,49	0.011,940,88	1.4565
107.....	0.108,804	0.076,220	0.700,526	0.002,741,75	0.003,742,39	1.3650
108.....	0.032,584	0.024,820	0.761,723	0.000,793,32	0.001,000,64	1.2613
109.....	0.007,764	0.006,678	0.860,084	0.000,182,64	0.000,207,32	1.1351
110.....	0.001,086	0.001,086	1.000,000	0.000,024,68	0.000,024,68	1.0000

TABLE 3

GA-51 MALE MORTALITY TABLE PROJECTED
WITH SCALE C TO 1970

TABLE OF UNIFORM SENIORITY
BASED ON "GRADUATED" TABLE

Difference of Ages (Years)	Addition to Older Age (Years)	Difference of Ages (Years)	Addition to Older Age (Years)
0.....	7.1674	21.....	1.2749
1.....	6.6795	22.....	1.1638
2.....	6.2157	23.....	1.0618
3.....	5.7758	24.....	0.9684
4.....	5.3596	25.....	0.8828
5.....	4.9667	26.....	0.8045
6.....	4.5966	27.....	0.7329
7.....	4.2487	28.....	0.6675
8.....	3.9225	29.....	0.6077
9.....	3.6171	30.....	0.5532
10.....	3.3319	31.....	0.5034
11.....	3.0659	32.....	0.4580
12.....	2.8184	33.....	0.4166
13.....	2.5885	34.....	0.3789
14.....	2.3752	35.....	0.3446
15.....	2.1777	36.....	0.3133
16.....	1.9951	37.....	0.2848
17.....	1.8265	38.....	0.2589
18.....	1.6710	39.....	0.2353
19.....	1.5278	40.....	0.2138
20.....	1.3960		

DISCUSSION OF PRECEDING PAPER

WILLIAM H. CROSSON:

It is quite useful to have presented before the Society of Actuaries the tables derived by projecting the *Ga*-1951 table to various years. Mr. Munro projected the male table to 1960 (*TSA*, XII, 353), and Mr. Saksena projects the male table to 1970.

Mr. Saksena has gone beyond Mr. Munro by graduating the resulting mortality rates. A Makeham graduation of the *Ga*-1951 table was considered and was rejected, and a modified Whittaker-Henderson Type B graduation was used instead. Now, after projecting nineteen years, we find that we can use a Gompertz graduation. This is certainly an unexpected, but fortunate, development. I hope that Mr. Saksena can accommodate us by describing how he arrived at the particular graduation in the note and by presenting a discussion of the closeness of fit of the graduated to the ungraduated projected mortality rates.

Since the table has been regraduated, the name of the table should suggest this fact, to distinguish the table from the corresponding ungraduated table. I would suggest "The *Ga*-1951 Male Mortality Table Projected with Scale C to 1970 (Regraduated)."

(AUTHOR'S REVIEW OF DISCUSSION)

G. B. SAKSENA:

I must thank Mr. William H. Crosson for his remarks regarding my actuarial note.

The values of q_x and u_x were plotted on several types of graph papers in order to determine whether or not a mathematical curve could be fitted to the projected table, and after several attempts it was found that a Gompertz curve could be satisfactorily fitted between the ages of 35 and 105 (the most essential range of the table). Mr. Crosson is surprised that a Gompertz curve could be fitted to the projected table, whereas the *Ga*-51 table could not be graduated by the Makeham or the Gompertz Law. On this point the author has the following comments.

A few years ago the author requested and obtained from Mr. Peterson the crude data from which he constructed the *Ga*-51 table with a view to fitting a Gompertz or a Makeham curve to the *Ga*-51 table. Because so many adjustments had been made to the crude data before arriving at the final *Ga*-51 table, the author found it extremely difficult to graduate it applying the various criteria of good graduation described in Mr. Miller's

monograph on graduation and still preserve some resemblance to the published table. He had to abandon the project. In the author's opinion the "kink" in the rates of mortality at the higher ages is not an essential feature of the crude data. It is simply due to the paucity of the exposures and deaths at ages above 95 (see *TSA*, IV, 290-91). We have always been taught that rates of mortality progress smoothly from age to age, and this is exactly what the author has tried to provide by fitting the Gompertz curve from age 35 to age 105. As a matter of fact, if the rates of mortality for the males and females were compared in the *Ga-51* table, the male mortality rates are lower than the female rates from ages 103 to 110, primarily due to the fact that both mortality tables have been forced to terminate at age 110. This feature could not be attributed to the crude data available to Mr. Peterson when the total number of deaths for males over age 95 were 16 and for females 0. As a test, the author used the rates of mortality at ages above 90 as given by Mr. Peterson, instead of the graduated rates, to study the effect on $\ddot{a}_x$, and the maximum difference at age 5 is 0.0015. The author agrees that there should be some kind of evidence to show the quality of the graduation, and would have done some form of comparison to prove this point. Unfortunately, it is not feasible to make any mathematical comparisons due to the fact that this is a projected table so that it is not possible to compare the actual with the expected deaths (which would be normally possible for a table compiled from actual exposures and deaths). The comparison that could be made would be a graphical one. Table 1 supplied with the actuarial note provides the ungraduated and the graduated rates of mortality. The author plotted the values of q 's on various types of graph paper, and the changes in the curvature and the slopes of the tangents of the ungraduated rates are exceedingly irregular. The graduated rates of mortality, of course, proceed with mathematical smoothness and cross and recross the ungraduated rates as they should in order to preserve the over-all fidelity of the ungraduated rates. The greatest advantage of using the graduated rates of mortality is the facility with which one could compute joint life functions. If the ungraduated rates were used, this facility would be lost, and computation of joint life functions would be a monumental task. The author was not quite sure which of the two rates of mortality would be more acceptable to actuaries in general, and that is why in Table 1 he provided both rates of mortality. The author prefers to use the graduated rates due to the advantages enumerated above.

INDICES TO THE COST OF VESTED PENSION BENEFITS

DANIEL F. MCGINN

INTRODUCTION

WHEN pension plans are established or modified, the actuary is frequently requested to evaluate the probable cost of various alternative vesting schedules, and he is often required to make his evaluation under circumstances that preclude a refined calculation. In addition, the complexity and time-consuming nature of the calculations usually prevent actuaries from making specific allowance for the cost of a pension plan's vesting provisions in the same manner that they traditionally have computed the effect of salary changes, of disability or death benefits, and so forth.

The primary purpose of this paper is to set forth one method of evaluating vested pension costs and to suggest a simple way for the actuary to make provision in his pension cost calculations for specific vesting schedules.

EVALUATION OF VESTED PENSION COSTS

In order to evaluate the long-term effect on pension plan costs of various vesting schedules and employment turnover characteristics, it was assumed for a typical plan that (1) entry age level annual costs provide a reasonable measure of long-range pension costs and (2) one level unit of pension benefit accrues for each year of service. With these basic assumptions, formulas were developed to derive level annual costs and corresponding accrued liabilities for retirement age 65 under ten different employment turnover assumptions for three basic vesting conditions:

Vesting Condition 1.—100 per cent vesting in accrued pension benefit at age z .

Vesting Condition 2.—50 per cent vesting in accrued pension benefit at age z and 10 per cent additional vesting each year thereafter until full vesting occurs at age $z + 5$.

Vesting Condition 3.—50 per cent vesting in accrued pension benefit at age z and 5 per cent additional vesting each year thereafter until full vesting occurs at age $z + 10$.

The basic costs were derived as follows:

Definition of Symbols

y = Age at entry into employment.

x = Attained age, $x > y$.

z = Age at which initial vesting occurs.

$q_x^{(d)}$ = Probability that a life age x will die before attaining age $x + 1$.

$q_x^{(w)}$ = Probability that a life age x will terminate employment before attaining age $x + 1$ for any reason other than death.

$q_x^{(T)}$ = Probability that a life age x will terminate before attaining age $x + 1$ for any reason, that is,

$$q_x^{(T)} = q_x^{(d)} + q_x^{(w)}.$$

${}_np_x$ = Probability that a life age x will survive to age $x + n$.

$${}_np_x = \prod_{t=0}^{n-1} (1 - q_{x+t}^{(d)}).$$

$q_x^{(ws)}$ = Probability that a life age x will terminate employment before attaining age $x + 1$ and survive to age $x + 1$.

$$q_x^{(ws)} = q_x^{(w)}(1 - \frac{1}{2}q_x^{(d)}).$$

NOTE.—The mortality rate from a standard mortality table is used in this paper both as a probability of death in the service table and as an annual rate of mortality operating independently of other causes.

${}_np_x^{(T)}$ = Probability that a life age x will still be employed at age $x + n$.

$${}_np_x^{(T)} = \prod_{t=0}^{n-1} (1 - q_{x+t}^{(T)}).$$

$(NC)_y^v$ = Level annual cost which will fund from entry age to retirement age 65 the pension benefit—with provision for a specified vesting schedule—of one unit per annum, payable monthly, for each year of service.

$(AL)_{y,x}^v$ = Accrued liability at attained age x for the pension benefits—less the value of any vested termination occurring before age x —provided by the level annual cost, $(NC)_y^v$.

FORMULAS

VESTING CONDITION 1

1. Entry Age Level Annual Cost

$$(NC)_y^v = \frac{(65 - y) \cdot \ddot{a}_{65}^{(12)} \cdot v^{65-y} \cdot {}_{65-y}p_y^{(T)}}{\sum_{k=y}^{64} v^{k-y} \cdot {}_{k-y}p_y^{(T)}} \times \left[1 + \sum_{k=y}^{64} \frac{k-y}{65-y} \cdot \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}}{{}_{65-k}p_k^{(T)}} \right].$$

2. *Accrued Liability*

If $x \leq z$, then

$$(AL)_{y,x}^v = (NC)_y^v \sum_{k=y}^{x-1} \frac{(1+i)^{x-k}}{x-k} \frac{1+i}{p_k^{(T)}} = [(AL)_{y,x-1}^v + (NC)_y^v] \frac{1+i}{p_{x-1}^{(T)}}.$$

If $x > z$,

$$(AL)_{y,x}^v = [(AL)_{y,x-1}^v + (NC)_y^v - (x-y-1) \bar{a}_{65}^{(12)} \cdot v^{66-x} \cdot q_{x-1}^{(ws)} \cdot {}_{65-x}p_x] \frac{1+i}{p_{x-1}^{(T)}}.$$

VESTING CONDITIONS 2 AND 3

1. *Entry Age Level Annual Cost*

$$(NC)_y^v = \frac{(65-y) \bar{a}_{65}^{(12)} \cdot v^{65-y} \cdot {}_{65-y}p_y^{(T)}}{\sum_{k=y}^{64} v^{k-y} {}_{k-y}p_y^{(T)}} \left[1 + 0.5 \sum_{k=z}^{64} \frac{k-y}{65-y} \cdot \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}}{65-k} \frac{1}{p_k^{(T)}} + r \cdot \sum_{s=1}^m \sum_{k=z+s}^{64} \frac{k-y}{65-y} \cdot \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}}{65-k} \frac{1}{p_k^{(T)}} \right],$$

where $r = 0.10$ and $m = 5$ for Vesting Condition 2; $r = 0.05$ and $m = 10$ for Vesting Condition 3.

2. *Accrued Liability*

If $x \leq z$, the formula is identical with that for Vesting Condition 1 for $x \leq z$. If $z < x \leq z + m$,

$$(AL)_{y,x}^v = \{ (AL)_{y,x-1}^v + (NC)_y^v - [0.5 + r(x-z-1)] \times (x-y-1) \bar{a}_{65}^{(12)} v^{66-x} \cdot q_{x-1}^{(ws)} \cdot {}_{65-x}p_x \} \frac{1+i}{p_{x-1}^{(T)}},$$

where $r = 0.10$ and $m = 5$ for Vesting Condition 2; $r = 0.05$ and $m = 10$ for Vesting Condition 3.

If $x > z + m$, the formula is identical with that for Vesting Condition 1 for $x > z$.

DERIVATION OF INDICES OF VESTED PENSION COSTS

Having established the basic formulas for the entry age level annual costs and accrued liabilities, cost indices were developed from the following ratios for each of the stated vesting conditions:

Level Annual Cost Index

$$(I)_y^{NC} = \frac{(NC)_y^v}{(NC)_y},$$

where $(NC)_y$ represents the level annual cost of pension benefits with "no vesting" before age 65.

Accrued Liability Cost Index

$$(I)_{y,x}^{AL} = \frac{(AL)_{y,x}^v}{(AL)_{y,x}},$$

where $(AL)_{y,x}$ represents the "no vesting" accrued liability at attained age x for the pension benefits funded by $(NC)_y$.

The formulas for each index and comments on their characteristics are included in Appendix A.

CALCULATION OF COST INDICES

Using the above-defined relationship, calculations were made by the IBM 1620 computer to generate the indices shown in Appendix B for each of the employment turnover tables included in Appendix C. Mortality experience rates were assumed to follow the 1966 Group Annuity Table (Male 1951 Group Annuity Table with mortality improvement projected according to Scale C to 1966), and interest was assumed to average $3\frac{3}{4}$ per cent annually throughout life.

By appropriate choice of the indices shown in Appendix B, numerous sets of indices can be fixed for many typical types of vesting schedules. To illustrate how the tables can be used, tables of indices have been constructed for the following four vesting schedules, using the ten turnover tables included in Appendix B.

Vesting Schedule	Vesting Condition
A.....	100 per cent full and immediate vesting.
B.....	100 per cent vesting after fifteen years of service (no vesting before fifteen years of service).
C.....	50 per cent vesting after five years of service and 5 per cent additional vesting for each of the following ten years of service.
D.....	100 per cent vesting upon attainment of age 45 and ten years of service.

TABLE 2
ACCRUED LIABILITY COST INDICES: $(I)_{x,z}^{AL} = (AL)_{y,z}^v / (AL)_{y,z}$
MALE LIVES
VESTING SCHEDULE A

ENTRY AGE y	VESTING AGE z	ATTAINED AGE x	EMPLOYMENT TURNOVER TABLES									
			I	II	III	IV	V	VI	VII	VIII	IX	X
20	20	25	1.1153	1.1817	1.1757	1.4727	1.4609	2.7786	2.7600	2.7667	2.1537	1.9910
		30	1.0411	1.1183	1.1163	1.3002	1.2980	2.0043	2.0173	2.0107	1.4200	1.7073
		35	0.9906	1.0707	1.0698	1.1797	1.1796	1.5640	1.5765	1.5704	1.1432	1.4649
		40	0.9655	1.0343	1.0339	1.0924	1.0929	1.2933	1.3026	1.2981	1.0261	1.2745
		45	0.9633	1.0056	1.0054	1.0264	1.0270	1.1126	1.1191	1.1160	0.9710	1.1314
		50	0.9750	0.9861	0.9861	0.9821	0.9825	0.9997	1.0040	1.0019	0.9459	1.0321
		55	0.9861	0.9825	0.9825	0.9707	0.9710	0.9618	0.9643	0.9631	0.9684	0.9852
		60	0.9941	0.9926	0.9926	0.9874	0.9875	0.9833	0.9844	0.9839	0.9862	0.9850
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999
30	30	35	1.0082	1.0667	1.0609	1.1508	1.1393	1.3925	1.3711	1.3775	1.1008	1.3477
		40	0.9828	1.0330	1.0309	1.0759	1.0723	1.1990	1.1962	1.1969	1.0077	1.2058
		45	0.9779	1.0062	1.0053	1.0184	1.0170	1.0622	1.0630	1.0626	0.9641	1.0935
		50	0.9856	0.9879	0.9874	0.9796	0.9791	0.9748	0.9761	0.9756	0.9454	1.0133
		55	0.9925	0.9845	0.9842	0.9711	0.9708	0.9508	0.9517	0.9513	0.9705	0.9771
		60	0.9970	0.9937	0.9936	0.9881	0.9880	0.9793	0.9796	0.9795	0.9878	0.9822
		65	1.0000	0.9999	0.9999	1.0000	0.9999	0.9999	1.0000	0.9999	0.9999	1.0000
40	40	45	0.9958	1.0110	1.0081	1.0238	1.0180	1.0546	1.0438	1.0483	0.9969	1.0792
		50	0.9969	0.9933	0.9923	0.9875	0.9856	0.9791	0.9759	0.9772	0.9709	1.0108
		55	0.9986	0.9890	0.9885	0.9785	0.9776	0.9590	0.9576	0.9582	0.9871	0.9797
		60	0.9995	0.9959	0.9958	0.9920	0.9917	0.9845	0.9839	0.9841	0.9952	0.9848
		65	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
50	50	55	1.0000	0.9971	0.9959	0.9942	0.9919	0.9884	0.9841	0.9843	1.0000	0.9972
		60	1.0000	0.9992	0.9990	0.9985	0.9980	0.9971	0.9960	0.9960	1.0000	0.9936
		65	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999

VESTING SCHEDULE B

ENTRY AGE y	VESTING AGE z	ATTAINED AGE x	EMPLOYMENT TURNOVER TABLES									
			I	II	III	IV	V	VI	VII	VIII	IX	X
20	35	25	1.0591	1.1330	1.1330	1.3012	1.3012	1.7899	1.7899	1.7899	1.3147	1.6266
		30	1.0591	1.1330	1.1330	1.3012	1.3012	1.7899	1.7899	1.7899	1.3147	1.6266
		35	1.0591	1.1330	1.1330	1.3012	1.3012	1.7899	1.7899	1.7899	1.3147	1.6266
		40	1.0084	1.0723	1.0725	1.1663	1.1671	1.4233	1.4262	1.4248	1.1375	1.3673
		45	0.9908	1.0293	1.0296	1.0726	1.0735	1.1916	1.1944	1.1931	1.0449	1.1871
		50	0.9920	1.0006	1.0007	1.0104	1.0111	1.0479	1.0500	1.0490	0.9936	1.0655
		55	0.9955	0.9905	0.9906	0.9867	0.9871	0.9893	0.9906	0.9900	0.9962	1.0038
		60	0.9981	0.9960	0.9960	0.9943	0.9944	0.9953	0.9959	0.9956	0.9983	0.9929
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
30	45	35	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
		40	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
		45	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
		50	0.9992	1.0060	1.0061	1.0148	1.0151	1.0392	1.0406	1.0401	0.9908	1.0659
		55	0.9995	0.9939	0.9940	0.9897	0.9899	0.9858	0.9868	0.9865	0.9950	1.0049
		60	0.9998	0.9975	0.9975	0.9957	0.9958	0.9940	0.9944	0.9943	0.9979	0.9937
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
40	55	45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182
		50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
50	65	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

TABLE 2—Continued
VESTING SCHEDULE C

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLES									
			I	II	III	IV	V	VI	VII	VIII	IX	X
20	25	25	1.1346	1.2017	1.2017	1.5055	1.5055	2.7449	2.7449	2.7449	2.1094	1.9938
		30	1.0733	1.1477	1.1482	1.3625	1.3651	2.1370	2.1528	2.1455	1.5504	1.7706
		35	1.0180	1.0953	1.0958	1.2326	1.2354	1.6844	1.6985	1.6920	1.2569	1.5281
		40	0.9827	1.0493	1.0498	1.1246	1.1270	1.3626	1.3732	1.3683	1.0999	1.3108
		45	0.9743	1.0150	1.0154	1.0466	1.0484	1.1547	1.1622	1.1587	1.0200	1.1532
		50	0.9818	0.9918	0.9921	0.9944	0.9956	1.0254	1.0303	1.0280	0.9775	1.0452
		55	0.9899	0.9857	0.9858	0.9776	0.9784	0.9765	0.9793	0.9780	0.9868	0.9924
		60	0.9957	0.9939	0.9940	0.9904	0.9907	0.9897	0.9910	0.9904	0.9942	0.9881
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		35	1.0245	1.0774	1.0774	1.1704	1.1704	1.4195	1.4195	1.4195	1.1484	1.3644
30	35	40	1.0001	1.0483	1.0485	1.1065	1.1075	1.2592	1.2643	1.2626	1.0619	1.2489
		45	0.9895	1.0187	1.0189	1.0435	1.0447	1.1131	1.1183	1.1166	1.0034	1.1315
		50	0.9921	0.9948	0.9950	0.9936	0.9945	1.0034	1.0075	1.0061	0.9681	1.0343
		55	0.9959	0.9880	0.9882	0.9785	0.9790	0.9664	0.9688	0.9679	0.9827	0.9881
		60	0.9983	0.9951	0.9952	0.9911	0.9913	0.9858	0.9868	0.9865	0.9929	0.9868
		65	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		45	1.0006	1.0171	1.0171	1.0353	1.0353	1.0754	1.0754	1.0754	1.0143	1.0939
		50	0.9994	1.0012	1.0013	1.0032	1.0034	1.0096	1.0104	1.0100	0.9887	1.0354
		55	0.9997	0.9938	0.9939	0.9882	0.9883	0.9784	0.9789	0.9787	0.9950	0.9963
		60	0.9999	0.9977	0.9977	0.9956	0.9956	0.9918	0.9920	0.9919	0.9981	0.9910
40	45	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0070
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9986
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0070
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9986
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0070
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9986
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0070

VESTING SCHEDULE D

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLES									
			I	II	III	IV	V	VI	VII	VIII	IX	X
20	45	25	1.0030	1.0473	1.0473	1.0985	1.0985	1.2139	1.2139	1.2139	1.0531	1.2243
		30	1.0030	1.0473	1.0473	1.0985	1.0985	1.2139	1.2139	1.2139	1.0531	1.2243
		35	1.0030	1.0473	1.0473	1.0985	1.0985	1.2139	1.2139	1.2139	1.0531	1.2243
		40	1.0030	1.0473	1.0473	1.0985	1.0985	1.2139	1.2139	1.2139	1.0531	1.2243
		45	1.0030	1.0473	1.0473	1.0985	1.0985	1.2139	1.2139	1.2139	1.0531	1.2243
		50	0.9995	1.0115	1.0115	1.0263	1.0264	1.0615	1.0619	1.0617	0.9989	1.0878
		55	0.9997	0.9966	0.9966	0.9956	0.9957	0.9971	0.9974	0.9972	0.9993	1.0162
		60	0.9999	0.9985	0.9986	0.9981	0.9981	0.9987	0.9988	0.9988	0.9997	0.9982
		65	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		35	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
30	45	40	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
		45	1.0024	1.0392	1.0392	1.0815	1.0815	1.1767	1.1767	1.1767	1.0427	1.1889
		50	0.9992	1.0060	1.0061	1.0148	1.0151	1.0392	1.0406	1.0401	0.9908	1.0659
		55	0.9995	0.9939	0.9940	0.9897	0.9899	0.9858	0.9868	0.9865	0.9950	1.0049
		60	0.9998	0.9975	0.9975	0.9957	0.9958	0.9940	0.9944	0.9943	0.9979	0.9937
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		45	1.0000	1.0101	1.0101	1.0205	1.0205	1.0424	1.0424	1.0424	1.0000	1.0661
		50	1.0000	1.0101	1.0101	1.0205	1.0205	1.0424	1.0424	1.0424	1.0000	1.0661
		55	1.0000	0.9963	0.9963	0.9931	0.9932	0.9878	0.9881	0.9880	1.0000	1.0043
		60	1.0000	0.9986	0.9986	0.9974	0.9974	0.9954	0.9955	0.9954	1.0000	0.9940
40	50	65	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0009

The preceding illustrative entry age level annual cost indices demonstrate that various turnover rates produce a wide range of variation in the relative costs of a given vesting schedule. Of course, it is important to recognize that, for a specific vesting schedule, an increase in the cost index, $(I)_y^{NC}$, resulting from an increase in the level of turnover rates, does not mean that the actual dollar cost of vesting increases in the same manner. This is true because a higher level of turnover rates increases the relative cost of a vesting schedule, but, at the same time, decreases the relative cost of the retirement pension. For example, for Vesting Schedule A, the entry age 20 level annual cost index for Turnover Table I is 1.1850, while the corresponding index for Table V is 1.6788. From Appendix D the "no vesting" level annual costs for these same turnover tables are 32.6048 and 19.1523, respectively. Using these factors, the entry age 20 level annual cost of vesting for Vesting Schedule A is 6.0319 ($= 0.1850 \times 32.6048$) for Table I and 13.0006 ($= 0.6788 \times 19.1523$) for Table V. Thus, while the relative value of the vested pension benefits increases 266.9 per cent [$= (0.6788 - 0.1850)/0.1850$] because of the change in turnover tables, the actual dollar increase in the cost of vested pension benefits is 115.5 per cent [$= (13.0006 - 6.0319)/6.0319$].

The tables of accrued liability cost indices, $(I)_{y,z}^{AL}$, vary even more widely than the entry age level annual cost indices. Naturally, when the entry age level annual cost for a specific vesting schedule generates a level annual cost index greater than 1.0000, the accrued liability cost index has to be less than 1.0000 at those durations where the tabular turnover rates are small or nonexistent. For example, using the same entry age 20 annual cost indices in the above example, the accrued liability cost index for Table I is first below 1.0000 at about duration 10 while the corresponding index for Table V decreases below 1.0000 between duration 20 and 25. In other words, considerably more time is required for the Table V accrued liability cost index to become less than 1.0000 than is required for the Table I index, because the Table V turnover rates are considerably higher and the turnover rates continue to higher attained ages.

EFFECT ON INDICES OF CHANGES IN MORTALITY AND INTEREST ASSUMPTIONS

By examination of the formulas in Appendix A, it can be observed that level annual cost indices, $(I)_y^{NC}$, are not affected by changes in the interest assumption. These indices are a function only of the mortality and turnover rate assumptions for a particular vesting age. However, the accrued liability cost indices are influenced by the interest assumption since the factor $[\ddot{a}_{x:65-x}^{(T)} \div \ddot{s}_{y:x-y}^{(T)}]$ is involved for all $x > z$. It also should be noted

that the accrued liability cost indices are the same as the level annual cost indices for all $x \leq z$ for any given combination of mortality and turnover assumptions.

In order to evaluate the effect on the cost indices of changes in interest and mortality assumptions, indices were calculated for numerous turnover tables and entry age/vesting age combinations on both the 1966 Group Annuity Table and the 1937 Standard Annuity Table at $2\frac{3}{4}$ per cent interest per annum. A simple comparison of the resulting indices illustrates the effect on the cost indices of a change in the mortality factor, while a comparison of the results for the 1966 Group Annuity Table at $2\frac{3}{4}$ per cent per annum with the indices shown in Appendix B indicates the effect of a change of 1 per cent in the interest factor.

TABLE 3
LEVEL ANNUAL COST INDICES—TURNOVER TABLE VI

ENTRY AGE	VESTING AGE	VESTING CONDITION 1		VESTING CONDITION 2		VESTING CONDITION 3	
		1937 S.A.	1966 GAT	1937 S.A.	1966 GAT	1937 S.A.	1966 GAT
20	20	3.8481	3.8353	3.6960	3.6841	3.4728	3.4620
	30	2.3374	2.3325	2.1637	2.1595	2.0595	2.0558
	40	1.4466	1.4453	1.3732	1.3722	1.3279	1.3271
30	30	1.6160	1.6140	1.5944	1.5925	1.5537	1.5520
	40	1.3384	1.3375	1.2887	1.2879	1.2542	1.2536
	50	1.0570	1.0570	1.0367	1.0366	1.0326	1.0326
40	40	1.1437	1.1434	1.1365	1.1362	1.1216	1.1213
	50	1.0425	1.0424	1.0276	1.0276	1.0244	1.0244
50	50	1.0084	1.0085	1.0063	1.0064	1.0053	1.0053

NOTE.—Although the above indices were calculated using a $2\frac{3}{4}$ per cent per annum interest factor, the indices generated for the 1966 Group Annuity Table are identical with the indices included in Appendix B, which were based on a $3\frac{1}{2}$ per cent interest assumption.

Effect of a Change in Mortality Assumption

In general, the mortality factor has little bearing on the cost indices because the cost of a vesting schedule is most significant at the younger ages, where employment turnover rates are usually substantial and where mortality rates are very low.

Tables 3 and 4 compare for several entry age/vesting age combinations the results of calculations for a moderately heavy turnover table, Turnover Table VI. This simple comparison illustrates that differences in pre-retirement mortality levels have virtually no effect on level annual cost indices and little effect on accrued liability cost indices. In other words, even though level annual costs and accrued liabilities differ significantly with changes in mortality assumptions, the cost indices are

practically independent of mortality—whether male or female, the Standard Annuity Table or the 1951 Group Annuity Table (with or without provision for mortality improvement).

*Effect of a Change in the Interest Assumption on the
Accrued Liability Indices*

Although a change of 1 per cent per annum in the interest assumption has more significance than a change from the 1966 Group Annuity Table to the 1937 Standard Annuity Table, even a combination of the most

TABLE 4
ACCRUED LIABILITY COST INDICES—TURNOVER TABLE VI

ENTRY AGE	VESTING AGE	DURA- TION	VESTING CONDITION 1		VESTING CONDITION 2		VESTING CONDITION 3	
			1966	1937	1966	1937	1966	1937
			GAT 2½ Per Cent	S.A. 2½ Per Cent	GAT 2½ Per Cent	S.A. 2½ Per Cent	GAT 2½ Per Cent	S.A. 2½ Per Cent
20	20	5	2.7337	2.7488	2.8345	2.8483	2.7618	2.7741
		10	1.9492	1.9641	1.9878	2.0017	2.0444	2.0569
		15	1.5142	1.5278	1.5346	1.5474	1.5646	1.5762
		20	1.2525	1.2642	1.2647	1.2758	1.2826	1.2927
		25	1.0809	1.0906	1.0886	1.0977	1.0999	1.1082
		30	0.9769	0.9841	0.9815	0.9885	0.9886	0.9950
		35	0.9470	0.9520	0.9498	0.9546	0.9539	0.9584
		40	0.9763	0.9789	0.9775	0.9800	0.9794	0.9817
	40	45	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		5	1.4453	1.4465	1.3722	1.3732	1.3270	1.3279
		10	1.4453	1.4465	1.3722	1.3732	1.3270	1.3279
		15	1.4453	1.4465	1.3722	1.3732	1.3270	1.3279
		20	1.4453	1.4465	1.3722	1.3732	1.3270	1.3279
		25	1.2021	1.2035	1.2058	1.2069	1.1831	1.1840
		30	1.0529	1.0540	1.0552	1.0562	1.0566	1.0575
		35	0.9916	0.9924	0.9930	0.9937	0.9938	0.9944
		40	0.9962	0.9966	0.9968	0.9972	0.9972	0.9975
		45	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

liberal vesting schedule (100 per cent full and immediate vesting) and youngest entry ages generates indices that differ by less than 5 per cent. As the difference between entry age and vesting age increases, the differences between indices diminish rapidly. Also, as the turnover rates decrease, the accrued liability cost indices converge. Table 5 shows the indices developed for entry age 20 and vesting ages 20 and 40 for Turnover Table VI.

When one recognizes the degree of error "built into" any employment turnover assumption, it is reasonable to conclude that a single set of accrued liability cost indices could be used for actuarial valuations with widely different interest bases.

As a practical matter, most larger plans provide vesting only after a minimum period of five or ten years of participation, and this fact alone minimizes the differences between accrued liability cost indices generated by different interest rate assumptions. In most instances the differences between indices will be within 1-2 per cent.

POSSIBLE APPLICATIONS OF VESTING COST INDICES

Because of the complexities of calculating the costs related to specific vesting schedules, few actuaries have been able to do more than make

TABLE 5
ACCRUED LIABILITY COST INDICES—TURNOVER TABLE VI
1966 GROUP ANNUITY TABLE

ENTRY AGE	VESTING AGE	DURA- TION	VESTING CONDITION 1		VESTING CONDITION 2		VESTING CONDITION 3	
			2½ Per Cent	3½ Per Cent	2½ Per Cent	3½ Per Cent	2½ Per Cent	3½ Per Cent
20	20	5	2.7337	2.7786	2.8345	2.8692	2.7618	2.7904
		10	1.9492	2.0043	1.9878	2.0374	2.0444	2.0858
		15	1.5142	1.5640	1.5346	1.5807	1.5646	1.6052
		20	1.2525	1.2933	1.2647	1.3029	1.2826	1.3170
		25	1.0809	1.1126	1.0886	1.1184	1.0999	1.1270
		30	0.9769	0.9997	0.9815	1.0033	1.9886	1.0085
		35	0.9470	0.9618	0.9498	0.9638	0.9539	0.9668
		40	0.9763	0.9833	0.9775	0.9842	0.9794	0.9855
		45	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
	40	5	1.4453	1.4453	1.3722	1.3722	1.3270	1.3270
		10	1.4453	1.4453	1.3722	1.3722	1.3270	1.3270
		15	1.4453	1.4453	1.3722	1.3722	1.3270	1.3270
		20	1.4453	1.4453	1.3722	1.3722	1.3270	1.3270
		25	1.2021	1.2049	1.2058	1.2077	1.1831	1.1847
		30	1.0529	1.0560	1.0552	1.0577	1.0566	1.0588
		35	0.9916	0.9940	0.9930	0.9949	0.9938	0.9955
		40	0.9962	0.9973	0.9968	0.9978	0.9972	0.9980
		45	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

rough approximations of the cost of vesting. Actuaries often make provision for a specific vesting schedule by using a turnover table which is somewhat more conservative than the anticipated turnover experience. Sometimes turnover tables are truncated at an age close to the average age at which the actuary anticipates benefits will become 100 per cent vested, or the turnover rates are graded to correspond roughly to the incidence of a graduated vesting schedule.

The author believes that the vesting cost indices in Appendix B can help the pension actuary obtain a reasonably accurate measure of the cost of vested pension benefits and eliminate the need for rough approxima-

tions. As stated previously, the indices of vesting costs relate to three basic vesting conditions and the specific levels of employment turnover. However, by appropriate choice of entry age, turnover table, vesting condition, and vesting age, vesting cost indices can be obtained that will be representative of the indices for male and female lives that would be generated by most combinations of mortality, interest, and turnover assumptions in use by pension actuaries today.

Illustration of the Use of Cost Indices

To develop a simple example of how the indices can be used, assume the following situation:

1. A plan provides a level unit of pension benefit for each year of service.
2. The entry age normal actuarial cost method is used.
3. Turnover Table IV from Appendix C applies.
4. Mortality experience rates follow the 1966 Group Annuity Table.
5. Interest averages $3\frac{3}{4}$ per cent annually.
6. All employees are males.
7. Expenses average 5 per cent of the level annual costs.
8. 50 per cent vesting in accrued pension credits occurs upon completion of five years of service and 10 per cent additional vesting accrues for each of the following five years of service. (Vesting Condition 2—Vesting age equals Entry age + 5).
9. Employee data are:

Entry Age	Attained Age	Number of Employees
20.....	25	30
25.....	30	25
30.....	40	20
35.....	45	15
40.....	50	10

Using the entry age level annual costs and accrued liabilities for Turnover Table IV from Appendix D, the normal cost and accrued liabilities—without provision for vesting—can be calculated. By multiplying each of the resulting costs by the appropriate index from Appendix B, the costs of the plan with the specified vesting schedule can be derived (for example, see Tables 6 and 7). These calculations show that the particular vesting schedule/turnover table combination increases the “no vesting” level annual cost by 22.8 per cent and increases the related accrued liability cost by about 14.0 per cent. Obviously, such simple assumptions will never exist in actual practice, and so the indices could rarely be taken directly

from the table without modification. As a practical matter, the vesting cost indices can be applied quite easily. For example, the actuary can make his cost calculations without provision for vesting; summarize his calculation results into five-year entry age groupings (i.e., ages 18-22, 23-27, etc.); and apply the level annual cost indices for the appropriate vesting condition and turnover table to the summarized level annual costs of each entry age grouping.

TABLE 6
LEVEL ANNUAL COST CALCULATION

Entry Age y (1)	"No Vesting" Level Annual Cost/Employee (2)	No. Employees (3)	"No Vesting" Level Annual Cost [=(2)×(3)] (4)	$(I)_y^{NC}$ Entry Age Level Annual Cost Index (5)	"With Vesting" Level Annual Cost [=(4)×(5)] (6)
20.	19.7587	30	592.761	1.5414	913.682
25.	29.6850	25	742.125	1.3204	979.902
30.	40.6931	20	813.862	1.1860	965.240
35.	52.4397	15	786.595	1.0990	864.468
40.	64.8032	10	648.032	1.0421	675.314
		100	3,583.375		4,398.606

TABLE 7
ACCRUED LIABILITY COST CALCULATION

Entry Age y (1)	Attained Age x (2)	"No Vesting" Accrued Liability/ Employee (3)	No. Employees (4)	"No Vesting" Accrued Liability [=(3)×(4)] (5)	$(I)_{x,y}^{AL}$ Accrued Liability Cost Index (6)	"With Vesting" Accrued Liability [=(5)×(6)] (7)
20.	25	151.0471	30	4,531.413	1.5413	6,984.267
25.	30	209.0651	25	5,226.627	1.3203	6,900.716
30.	40	656.5156	20	13,130.312	1.1110	14,587.777
35.	45	791.7739	15	11,876.608	1.0441	12,400.366
40.	50	931.2956	10	9,312.956	1.0044	9,353.933
			100	44,077.916		50,227.059

In the same manner, the accrued liability costs—without provision for vesting—can be summarized into similar attained age groupings for each entry age grouping to develop the corresponding accrued liabilities.

Actually, there are numerous modifications of the indices shown in Appendix B that may be made to obtain measures of the cost of different types of vesting schedules. For example, assume that a vesting schedule

provides for 10 per cent vesting in accrued pension credits for each completed year of service. The level annual cost index for this vesting schedule can be obtained by modifying indices for Conditions 1 and 3. That is, when vesting age equals the entry age, the modified index equals

$$1 + 2 \cdot \{[(I)_y^{NC, \text{ Vesting Condition 3}} - 1] - 0.50[(I)_y^{NC, \text{ Vesting Condition 1}} - 1]\}.$$

For Turnover Table V and entry age 20, the modified index would be

$$\begin{aligned} &= 1 + 2 [(1.6226 - 1) - 0.5 (1.6788 - 1)] ; \\ &= 1 + 2 [0.6226 - 0.5 (0.6788)] ; \\ &= 1 + 2 [0.6226 - 0.3394] ; \\ &= 1.5664. \end{aligned}$$

COMMENTS ON THE USE OF COST INDICES

An examination of the assumptions used in this paper to derive the indices of pension costs will demonstrate that they have certain limitations:

1. The cost indices included in this paper are applicable directly only to the entry age normal actuarial cost method.
2. Cost indices have been developed only for certain employment turnover rate assumptions, and so the indices may not be appropriate for some actuarial valuations. It is hoped that the variety of turnover tables will allow the "mixing" of entry age cost indices so that reasonable approximations may be obtained to the cost of various vesting schedules.
3. Cost indices have been developed only for retirement age 65 and only for three basic vesting conditions.
4. The cost indices are based on the assumption that a level unit of pension credit is earned for each year of service, but this assumption may not be valid in many situations; for example, where a flat pension benefit is provided; where a limitation is imposed on the number of years of service recognized for pension benefit accruals; where the level of pension benefit credits has changed over the years since a plan's inception; and so forth.

Despite these (and perhaps other) limitations, an inspection of the formulas in Appendix A will indicate that indices can be generated quite easily for any particular combination of retirement age, vesting condition, and employment turnover table. In fact, numerous modifications can be made to these formulas so that indices could be developed for virtually any combination of factors and actuarial cost methods.

CONCLUSION

It is the hope of the author that this paper presents a different view of the problem of "pricing" pension plan vesting schedules and that the con-

cept of cost indices may be explored by pension actuaries who need practical tools in their everyday work. The method of evaluating vested pension costs that is demonstrated in this paper represents one approach to a very complicated and important financial aspect of pension plans. If the concept of cost indices provides a useful insight into the development of practical methods of calculating costs of pension vesting schedules, this paper will have served its purpose.

APPENDIX A

FORMULAS FOR INDICES OF VESTED PENSION COSTS

The level annual cost of pension benefits for retirement age 65 with "no vesting" before age 65 is defined by the relationship:

$$(NC)_y = \frac{(65 - y) \ddot{a}_{65}^{(12)} \cdot v^{65-y} \cdot {}_{65-y}p_y^{(T)}}{\sum_{k=y}^{64} v^{k-y} \cdot {}_{k-y}p_y^{(T)}},$$

and the corresponding accrued liability at age x is equal to

$$(NC)_y \cdot \sum_{k=y}^{x-1} \frac{(1+i)^{x-k}}{{}_{x-k}p_k^{(T)}}.$$

By defining a new relationship,

$$f_y^{z,x} = \sum_{k=x}^{x-1} \frac{k-y}{65-y} \cdot \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}}{{}_{65-k}p_k^{(T)}},$$

the level annual cost indices, $(I)_y^{NC}$, and the accrued liability cost indices, $(I)_{y,x}^{AL}$, can be expressed in the following manner.

A. Level Annual Cost Indices, $(I)^{NC}$

1. Vesting Condition 1:

$$(I)_y^{NC} = 1 + f_y^{z,65};$$

2. Vesting Conditions 2 and 3:

$$(I)_y^{NC} = 1 + 0.5 f_y^{z,65} + r \sum_{s=1}^m f_y^{z+s,65},$$

where $r = 0.10$ and $m = 5$ for Vesting Condition 2; $r = 0.05$ and $m = 10$ for Vesting Condition 3.

B. Accrued Liability Cost Indices, $(I)_{y,x}^{AL}$

By definition,

$$\ddot{a}_{y:\overline{x-y}|}^{(T)} = \sum_{k=y}^{x-1} v^{k-y} \cdot {}_{k-y}p_u^{(T)}.$$

Thus, the accrued liability indices can be expressed as follows:

1. For Vesting Condition 1:

$$x \leq z,$$

$$(I)_{y,x}^{AL} = 1 + f_y^{z,65};$$

$$x > z,$$

$$(I)_{y,x}^{AL} = 1 + f_y^{z,65} - f_y^{z,x} \cdot \frac{\ddot{a}_{x:\overline{65-x}|}^{(T)}}{\ddot{s}_{y:\overline{x-y}|}^{(T)}}.$$

NOTE.—Appendix E sets forth the derivation of $(I)_{y,x}^{AL}$ for $x > z$ for Vesting Condition 1. The derivation of $(I)_{y,x}^{AL}$ for $x > z$ for Vesting Conditions 2 and 3 can be similarly derived.

2. For Vesting Conditions 2 and 3:

$$x \leq z,$$

$$(I)_{y,x}^{AL} = 1 + 0.5f_y^{z,65} + r \sum_{s=1}^m f_y^{z+s,65};$$

$$z < x \leq z + m,$$

$$(I)_{y,x}^{AL} = 1 + [0.5 + (x - z - 1)r]f_y^{z,65} + r \sum_{s=x-z}^m f_y^{z+s,65} \\ - \left(0.5f_y^{z,x} + r \sum_{s=1}^{x-z-1} f_y^{z+s,x} \right) \frac{\ddot{a}_{x:\overline{65-x}|}^{(T)}}{\ddot{s}_{y:\overline{x-y}|}^{(T)}};$$

$$x > z + m,$$

$$(I)_{y,x}^{AL} = 1 + f_y^{z,65} - \left(0.5f_y^{z,x} + r \sum_{s=1}^m f_y^{z+s,x} \right) \frac{\ddot{a}_{x:\overline{65-x}|}^{(T)}}{\ddot{s}_{y:\overline{x-y}|}^{(T)}};$$

where $r = 0.10$ and $m = 5$ for Vesting Condition 2; $r = 0.05$ and $m = 10$ for Vesting Condition 3.

Comments on the Characteristics of the Indices

1. By a slight change in the formula for $f_y^{z,x}$, it becomes obvious that special commutation columns can be developed for any given mortality and turnover table combination so that cost indices can be easily developed for any retirement age. For example,

$$f_y^{z,x} = \frac{1}{65-y} \cdot \left[\sum_{k=z}^{x-1} \frac{k \cdot q_k^{(ws)} \cdot {}_{64-k}p_{k+1}^{(T)}}{{}_{65-k}p_k^{(T)}} - y \cdot \sum_{k=z}^{x-1} \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}^{(T)}}{{}_{65-k}p_k^{(T)}} \right].$$

If retirement age Q replaces age 65, then the formula for $f_y^{z,x}$ becomes

$$\frac{1}{Q-y} ({}_kW_{z,x}^Q - y \cdot W_{z,x}^Q),$$

where

$${}_kW_{z,x}^Q = \sum_{k=z}^{x-1} \frac{k \cdot q_k^{(ws)} \cdot {}_{Q-k-1}p_{k+1}^{(T)}}{{}_Q - k p_k^{(T)}},$$

and

$$W_{z,x}^Q = \sum_{k=z}^{x-1} \frac{q_k^{(ws)} \cdot {}_{Q-k-1}p_{k+1}^{(T)}}{{}_Q - k p_k^{(T)}}.$$

By developing ${}_kW_{z,Q}^Q$ and $W_{z,Q}^Q$ for all z greater than the lowest possible entry age y , useful commutation columns are derived so that calculation of indices can be easily performed.

2. The formulas for Vesting Conditions 2 and 3 can be generalized merely by changing 0.5 to r' and by redefining r and m appropriately to recognize different levels of initial vesting and periods of graded vesting.

3. An analysis of the formulas for the accrued liability indices will indicate that, as x approaches 65, $f_y^{x,65}$ and the ratio $\ddot{a}_{x:65-x}^{(T)} \div \ddot{s}_{y:x-y}^{(T)}$ approach zero so that the accrued liability index must be 1 at retirement age 65.

APPENDIX B

TABLE B1

ENTRY AGE LEVEL ANNUAL COST INDICES— $(I)_{y}^{nc} = (NC)_{y}^{nc} / (NC)_{y}$
MALE LIVES

ENTRY AGE y	VESTING AGE z	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV			EMPLOYMENT TURNOVER TABLE V		
		VESTING CONDITIONS			VESTING CONDITIONS			VESTING CONDITIONS			VESTING CONDITIONS			VESTING CONDITIONS		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
20	20	1.1850	1.1805	1.1698	1.2465	1.2423	1.2330	1.2498	1.2447	1.2350	1.6634	1.6465	1.6134	1.6788	1.6571	1.6226
	25	1.1633	1.1481	1.1347	1.2269	1.2138	1.2017	1.2269	1.2138	1.2017	1.5855	1.5414	1.5055	1.5855	1.5414	1.5055
	30	1.1113	1.0952	1.0842	1.1819	1.1672	1.1554	1.1819	1.1672	1.1554	1.4374	1.3956	1.3652	1.4374	1.3956	1.3652
	35	1.0591	1.0470	1.0405	1.1330	1.1192	1.1086	1.1330	1.1192	1.1086	1.3012	1.2667	1.2419	1.3012	1.2667	1.2419
	40	1.0214	1.0151	1.0130	1.0876	1.0753	1.0664	1.0876	1.0753	1.0663	1.1892	1.1611	1.1419	1.1892	1.1611	1.1419
	45	1.0031	1.0018	1.0017	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655	1.0985	1.0773	1.0655
25	50	1.0000	1.0000	1.0000	1.0156	1.0100	1.0089	1.0156	1.0100	1.0089	1.0316	1.0203	1.0180	1.0316	1.0203	1.0180
	25	1.0976	1.0943	1.0871	1.1598	1.1570	1.1505	1.1627	1.1590	1.1522	1.3891	1.3800	1.3609	1.3997	1.3872	1.3671
	30	1.0821	1.0721	1.0640	1.1464	1.1372	1.1285	1.1464	1.1372	1.1285	1.3463	1.3204	1.2979	1.3463	1.3204	1.2979
	35	1.0483	1.0390	1.0335	1.1147	1.1039	1.0950	1.1147	1.1039	1.0950	1.2582	1.2314	1.2106	1.2582	1.2314	1.2106
	40	1.0186	1.0132	1.0114	1.0787	1.0681	1.0601	1.0787	1.0681	1.0601	1.1697	1.1456	1.1284	1.1697	1.1456	1.1284
	45	1.0028	1.0016	1.0015	1.0438	1.0347	1.0294	1.0438	1.0347	1.0295	1.0911	1.0718	1.0608	1.0911	1.0718	1.0608
30	50	1.0000	1.0000	1.0000	1.0147	1.0095	1.0084	1.0147	1.0095	1.0084	1.0298	1.0192	1.0171	1.0298	1.0192	1.0171
	30	1.0445	1.0423	1.0380	1.1007	1.0987	1.0939	1.1033	1.1005	1.0955	1.2292	1.2237	1.2114	1.2371	1.2292	1.2161
	35	1.0344	1.0286	1.0245	1.0910	1.0842	1.0775	1.0910	1.0842	1.0775	1.2030	1.1860	1.1704	1.2030	1.1860	1.1704
	40	1.0150	1.0108	1.0092	1.0673	1.0589	1.0521	1.0673	1.0589	1.0521	1.1447	1.1256	1.1110	1.1447	1.1256	1.1110
	45	1.0024	1.0014	1.0013	1.0392	1.0313	1.0264	1.0392	1.0313	1.0264	1.0816	1.0647	1.0548	1.0816	1.0647	1.0548
	50	1.0000	1.0000	1.0000	1.0136	1.0088	1.0078	1.0136	1.0088	1.0078	1.0276	1.0178	1.0158	1.0276	1.0178	1.0158

TABLE B1—Continued

ENTRY Age <i>y</i>	VESTING Age <i>z</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV			EMPLOYMENT TURNOVER TABLE V		
		Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
35	35	1.0159	1.0147	1.0125	1.0595	1.0580	1.0542	1.0614	1.0593	1.0553	1.1293	1.1255	1.1168	1.1340	1.1290	1.1197
	40	1.0102	1.0075	1.0064	1.0521	1.0467	1.0414	1.0521	1.0467	1.0414	1.1113	1.0990	1.0878	1.1113	1.0990	1.0878
	45	1.0019	1.0011	1.0010	1.0331	1.0268	1.0226	1.0331	1.0268	1.0226	1.0688	1.0553	1.0467	1.0688	1.0553	1.0467
	50	1.0000	1.0000	1.0000	1.0122	1.0079	1.0070	1.0122	1.0079	1.0070	1.0247	1.0160	1.0142	1.0247	1.0160	1.0142
40	40	1.0035	1.0030	1.0024	1.0308	1.0295	1.0265	1.0322	1.0306	1.0274	1.0646	1.0618	1.0553	1.0679	1.0642	1.0574
	45	1.0012	1.0007	1.0007	1.0246	1.0204	1.0171	1.0246	1.0204	1.0171	1.0511	1.0421	1.0354	1.0511	1.0421	1.0354
	50	1.0000	1.0000	1.0000	1.0102	1.0066	1.0059	1.0102	1.0066	1.0059	1.0206	1.0134	1.0119	1.0206	1.0134	1.0119
	45	1.0002	1.0001	1.0001	1.0119	1.0109	1.0090	1.0126	1.0114	1.0094	1.0244	1.0222	1.0184	1.0259	1.0234	1.0193
50	50	1.0000	1.0000	1.0000	1.0071	1.0047	1.0041	1.0071	1.0047	1.0041	1.0144	1.0095	1.0084	1.0144	1.0095	1.0084
	50	1.0000	1.0000	1.0000	1.0021	1.0015	1.0013	1.0029	1.0022	1.0018	1.0041	1.0031	1.0026	1.0058	1.0044	1.0036

TABLE B1—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII			EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
		Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
20	20	3.8353	3.6841	3.4620	4.0203	3.8082	3.5703	3.9309	3.7486	3.5182	3.6708	3.3877	3.0999	2.2295	2.2084	2.1588
	25	3.1744	2.9095	2.7450	3.1744	2.9095	2.7450	3.1744	2.9095	2.7450	2.5246	2.2391	2.1094	2.1287	2.0588	1.9939
	30	2.3325	2.1595	2.0558	2.3326	2.1595	2.0558	2.3326	2.1595	2.0558	1.6726	1.5537	1.4961	1.8868	1.8080	1.7477
	35	1.7900	1.6807	1.6126	1.7900	1.6807	1.6126	1.7900	1.6807	1.6126	1.3148	1.2596	1.2301	1.6267	1.5574	1.5083
	40	1.4453	1.3722	1.3271	1.4453	1.3722	1.3271	1.4453	1.3722	1.3271	1.1455	1.1159	1.0989	1.4020	1.3468	1.3096
	45	1.2139	1.1662	1.1411	1.2139	1.1662	1.1411	1.2139	1.1662	1.1411	1.0532	1.0359	1.0312	1.2244	1.1835	1.1595
25	50	1.0650	1.0417	1.0371	1.0650	1.0417	1.0371	1.0650	1.0417	1.0371	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
	25	2.2440	2.1937	2.1090	2.3169	2.2428	2.1518	2.2946	2.2286	2.1391	1.6957	1.6475	1.5836	1.7816	1.7664	1.7314
	30	2.0182	1.9115	1.8354	2.0182	1.9115	1.8354	2.0182	1.9115	1.8354	1.4895	1.4166	1.3747	1.7090	1.6600	1.6154
	35	1.6699	1.5851	1.5282	1.6699	1.5851	1.5282	1.6699	1.5851	1.5282	1.2617	1.2190	1.1943	1.5404	1.4866	1.4454
	40	1.3981	1.3353	1.2949	1.3981	1.3353	1.2949	1.3981	1.3353	1.2949	1.1285	1.1031	1.0880	1.3628	1.3154	1.2820
	45	1.1976	1.1542	1.1309	1.1976	1.1542	1.1309	1.1976	1.1542	1.1309	1.0486	1.0329	1.0286	1.2089	1.1717	1.1493
30	50	1.0615	1.0395	1.0351	1.0615	1.0395	1.0351	1.0615	1.0395	1.0351	1.0000	1.0000	1.0000	1.0905	1.0677	1.0581
	30	1.6140	1.5925	1.5520	1.6496	1.6171	1.5732	1.6374	1.6087	1.5659	1.2540	1.2404	1.2187	1.4805	1.4698	1.4454
	35	1.5155	1.4621	1.4195	1.5155	1.4621	1.4195	1.5155	1.4621	1.4195	1.1935	1.1667	1.1484	1.4295	1.3955	1.3645
	40	1.3375	1.2879	1.2536	1.3375	1.2879	1.2536	1.3375	1.2879	1.2536	1.0667	1.0867	1.0738	1.3125	1.2751	1.2467
	45	1.1767	1.1389	1.1177	1.1767	1.1389	1.1177	1.1767	1.1389	1.1177	1.0427	1.0290	1.0252	1.1889	1.1565	1.1361
	50	1.0570	1.0366	1.0326	1.0570	1.0366	1.0326	1.0570	1.0366	1.0326	1.0000	1.0000	1.0000	1.0847	1.0636	1.0546
35	35	1.3096	1.2982	1.2747	1.3251	1.3096	1.2843	1.3172	1.3038	1.2794	1.1025	1.0971	1.0871	1.2816	1.2740	1.2567
	40	1.2566	1.2247	1.1985	1.2566	1.2247	1.1985	1.2566	1.2247	1.1985	1.0777	1.0648	1.0550	1.2454	1.2213	1.1995
	45	1.1489	1.1184	1.1001	1.1489	1.1184	1.1001	1.1489	1.1184	1.1001	1.0349	1.0239	1.0207	1.1623	1.1362	1.1185
	50	1.0509	1.0328	1.0291	1.0509	1.0328	1.0291	1.0509	1.0328	1.0291	1.0000	1.0000	1.0000	1.0770	1.0582	1.0499
	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0198	1.0130	1.0115

TABLE B1—Continued

ENTRY AGE <i>y</i>	VESTING AGE	EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII			EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
		Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
		1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
40	40	1.1434	1.1362	1.1213	1.1520	1.1426	1.1267	1.1483	1.1399	1.1244	1.0370	1.0342	1.0287	1.1515	1.1460	1.1335
	45	1.1098	1.0897	1.0755	1.1098	1.0897	1.0755	1.1098	1.0897	1.0755	1.0239	1.0167	1.0143	1.1251	1.1078	1.0940
	50	1.0424	1.0276	1.0244	1.0424	1.0276	1.0244	1.0424	1.0275	1.0244	1.0000	1.0000	1.0000	1.0662	1.0507	1.0433
	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182	1.0120	1.0106
	60	1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
45	45	1.0513	1.0466	1.0385	1.0549	1.0493	1.0407	1.0538	1.0486	1.0401	1.0075	1.0059	1.0048	1.0692	1.0652	1.0571
	50	1.0297	1.0196	1.0172	1.0297	1.0196	1.0172	1.0297	1.0196	1.0172	1.0000	1.0000	1.0000	1.0500	1.0394	1.0335
	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0159	1.0105	1.0092
	60	1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
	50	1.0085	1.0064	1.0053	1.0119	1.0089	1.0074	1.0116	1.0088	1.0073	1.0000	1.0000	1.0000	1.0230	1.0205	1.0171
55	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0119	1.0081	1.0070
	60	1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0010	1.0005	
	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0040	1.0032	1.0026
	60	1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0000	1.0000		1.0007	1.0004	

TABLE B2
ACCRUED LIABILITY COST INDICES— $(I)_{y,z}^{AL} = (AL)_{y,z}^v / (AL)_{y,z}$
MALE LIVES

ENTRY AGE y	VESTING AGE z	ATTAINED AGE x	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
20	20	25	1.1153	1.1252	1.1247	1.1817	1.1912	1.1912	1.1757	1.1872	1.1877	1.4727	1.4971	1.4910
		30	1.0411	1.0454	1.0556	1.1183	1.1224	1.1316	1.1163	1.1213	1.1307	1.3002	1.3105	1.3306
		35	0.9906	0.9931	0.9989	1.0707	1.0729	1.0781	1.0698	1.0726	1.0779	1.1797	1.1853	1.1964
		40	0.9655	0.9671	0.9707	1.0343	1.0357	1.0388	1.0359	1.0356	1.0388	1.0924	1.0959	1.1026
		45	0.9633	0.9643	0.9667	1.0056	1.0065	1.0085	1.0054	1.0065	1.0085	1.0264	1.0286	1.0328
		50	0.9750	0.9757	0.9771	0.9861	0.9867	0.9879	0.9861	0.9867	0.9879	0.9821	0.9834	0.9860
		55	0.9861	0.9864	0.9872	0.9825	0.9828	0.9835	0.9825	0.9828	0.9835	0.9707	0.9714	0.9729
20	25	60	0.9941	0.9942	0.9946	0.9926	0.9927	0.9930	0.9926	0.9927	0.9930	0.9874	0.9877	0.9883
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.1632	1.1481	1.1346	1.2268	1.2138	1.2017	1.2268	1.2138	1.2017	1.5855	1.5413	1.5055
		30	1.0618	1.0762	1.0733	1.1376	1.1504	1.1477	1.1385	1.1511	1.1482	1.3475	1.3743	1.3625
		35	1.0025	1.0107	1.0180	1.0814	1.0886	1.0953	1.0822	1.0893	1.0958	1.2058	1.2206	1.2326
		40	0.9729	0.9781	0.9827	1.0409	1.0452	1.0493	1.0415	1.0458	1.0498	1.1083	1.1173	1.1246
		45	0.9681	0.9714	0.9743	1.0097	1.0125	1.0150	1.0102	1.0129	1.0154	1.0364	1.0420	1.0466
		50	0.9780	0.9800	0.9818	0.9886	0.9903	0.9918	0.9889	0.9906	0.9921	0.9882	0.9916	0.9944
		55	0.9877	0.9888	0.9899	0.9839	0.9848	0.9857	0.9841	0.9850	0.9858	0.9741	0.9761	0.9776
		60	0.9948	0.9953	0.9957	0.9932	0.9936	0.9939	0.9932	0.9936	0.9940	0.9889	0.9897	0.9904
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
20	30	25	1.1112	1.0952	1.0841	1.1818	1.1671	1.1554	1.1818	1.1671	1.1554	1.4374	1.3956	1.3651
		30	1.1112	1.0952	1.0841	1.1818	1.1671	1.1554	1.1818	1.1671	1.1554	1.4374	1.3956	1.3651
		35	1.0307	1.0394	1.0361	1.1062	1.1142	1.1100	1.1066	1.1145	1.1102	1.2555	1.2695	1.2566
		40	0.9907	0.9961	0.9999	1.0559	1.0609	1.0648	1.0564	1.0612	1.1470	1.1385	1.1470	1.1532
		45	0.9794	0.9829	0.9853	1.0191	1.0222	1.0247	1.0195	1.0225	1.0249	1.0553	1.0606	1.0645
		50	0.9850	0.9871	0.9886	0.9944	0.9962	0.9977	0.9946	0.9968	0.9979	1.0034	1.0034	1.0054
		55	0.9916	0.9928	0.9936	0.9871	0.9881	0.9890	0.9872	0.9883	0.9891	0.9806	0.9825	0.9838
20	35	60	0.9964	0.9969	0.9973	0.9945	0.9950	0.9953	0.9946	0.9950	0.9954	0.9917	0.9925	0.9930
		65	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999
		25	1.0591	1.0470	1.0405	1.1330	1.1192	1.1085	1.1330	1.1192	1.1085	1.3012	1.2666	1.2418
		30	1.0591	1.0470	1.0405	1.1330	1.1192	1.1085	1.1330	1.1192	1.1085	1.3012	1.2666	1.2418
		35	1.0591	1.0470	1.0405	1.1330	1.1192	1.1085	1.1330	1.1192	1.1085	1.3012	1.2666	1.2418
		40	1.0084	1.0125	1.0106	1.0723	1.0769	1.0723	1.0725	1.0771	1.0724	1.1663	1.1733	1.1615
		45	0.9908	0.9934	0.9949	1.0293	1.0322	1.0345	1.0296	1.0324	1.0346	1.0726	1.0770	1.0802
20	40	50	0.9920	0.9936	0.9945	1.0006	1.0023	1.0037	1.0007	1.0025	1.0038	1.0104	1.0131	1.0151
		55	0.9955	0.9964	0.9969	0.9905	0.9915	0.9923	0.9906	0.9916	0.9924	0.9867	0.9882	0.9893
		60	0.9981	0.9985	0.9987	0.9960	0.9964	0.9967	0.9960	0.9964	0.9967	0.9943	0.9949	0.9954
		65	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.0213	1.0150	1.0130	1.0875	1.0752	1.0663	1.0875	1.0752	1.0663	1.1891	1.1611	1.1418
		30	1.0213	1.0150	1.0130	1.0875	1.0752	1.0663	1.0875	1.0752	1.0663	1.1891	1.1611	1.1418
		35	1.0213	1.0150	1.0130	1.0875	1.0752	1.0663	1.0875	1.0752	1.0663	1.1891	1.1611	1.1418
20	40	40	1.0213	1.0150	1.0130	1.0875	1.0752	1.0663	1.0875	1.0752	1.0663	1.1891	1.1611	1.1418
		45	0.9990	1.0004	1.0001	1.0389	1.0414	1.0373	1.0390	1.0415	1.0373	1.0869	1.0905	1.0810
		50	0.9971	0.9979	0.9982	1.0064	1.0079	1.0091	1.0064	1.0080	1.0091	1.0192	1.0214	1.0229
		55	0.9983	0.9988	0.9990	0.9938	0.9946	0.9953	0.9938	0.9947	0.9953	0.9916	0.9928	0.9937
		60	0.9993	0.9995	0.9995	0.9973	0.9977	0.9980	0.9974	0.9977	0.9980	0.9964	0.9969	0.9973
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2--Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
20	45	25	1.0030	1.0017	1.0016	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655
		30	1.0030	1.0017	1.0016	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655
		35	1.0030	1.0017	1.0016	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655
		40	1.0030	1.0017	1.0016	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655
		45	1.0030	1.0017	1.0016	1.0473	1.0373	1.0316	1.0473	1.0373	1.0316	1.0985	1.0773	1.0655
		50	0.9995	0.9997	0.9997	1.0115	1.0127	1.0103	1.0115	1.0128	1.0104	1.0263	1.0279	1.0228
		55	0.9997	0.9998	0.9998	0.9966	0.9973	0.9977	0.9966	0.9973	0.9977	0.9956	0.9965	0.9971
		60	0.9999	0.9999	0.9999	0.9985	0.9988	0.9990	0.9986	0.9988	0.9990	0.9981	0.9985	0.9987
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999
		25	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
20	50	30	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
		35	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
		40	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
		45	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
		50	1.0000	1.0000	1.0000	1.0155	1.0100	1.0088	1.0155	1.0100	1.0088	1.0315	1.0202	1.0180
		55	1.0000	1.0000	1.0000	0.9988	0.9992	0.9993	0.9989	0.9992	0.9993	0.9986	0.9991	0.9992
		60	1.0000	1.0000	1.0000	0.9995	0.9997	0.9997	0.9995	0.9997	0.9997	0.9993	0.9996	0.9996
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.0443	1.0522	1.0527	1.1132	1.1202	1.1204	1.1072	1.1162	1.1169	1.2708	1.2869	1.2848
		30	0.9966	1.0000	1.0076	1.0679	1.0708	1.0775	1.0657	1.0695	1.0765	1.1616	1.1684	1.1825
25	25	35	0.9722	0.9741	0.9783	1.0330	1.0346	1.0383	1.0320	1.0379	1.0379	1.0831	1.0853	1.0930
		40	0.9694	0.9731	0.9731	1.0054	1.0064	1.0085	1.0050	1.0062	1.0084	1.0206	1.0228	1.0273
		45	0.9796	0.9803	0.9818	0.9866	0.9872	0.9884	0.9864	0.9871	0.9884	0.9796	0.9809	0.9835
		50	0.9796	0.9803	0.9818	0.9866	0.9872	0.9884	0.9864	0.9871	0.9884	0.9796	0.9809	0.9835
		55	0.9889	0.9893	0.9901	0.9832	0.9835	0.9842	0.9831	0.9835	0.9842	0.9700	0.9707	0.9722
		60	0.9954	0.9956	0.9959	0.9930	0.9931	0.9934	0.9930	0.9931	0.9934	0.9873	0.9876	0.9883
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999
		25	1.0443	1.0522	1.0527	1.1132	1.1202	1.1204	1.1072	1.1162	1.1169	1.2708	1.2869	1.2848
		30	0.9966	1.0000	1.0076	1.0679	1.0708	1.0775	1.0657	1.0695	1.0765	1.1616	1.1684	1.1825
		35	0.9722	0.9741	0.9783	1.0330	1.0346	1.0383	1.0320	1.0379	1.0379	1.0831	1.0853	1.0930
		40	0.9694	0.9731	0.9731	1.0054	1.0064	1.0085	1.0050	1.0062	1.0084	1.0206	1.0228	1.0273
		45	0.9796	0.9803	0.9818	0.9866	0.9872	0.9884	0.9864	0.9871	0.9884	0.9796	0.9809	0.9835
		50	0.9796	0.9803	0.9818	0.9866	0.9872	0.9884	0.9864	0.9871	0.9884	0.9796	0.9809	0.9835
		55	0.9889	0.9893	0.9901	0.9832	0.9835	0.9842	0.9831	0.9835	0.9842	0.9700	0.9707	0.9722
		60	0.9954	0.9956	0.9959	0.9930	0.9931	0.9934	0.9930	0.9931	0.9934	0.9873	0.9876	0.9883
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
25	30	30	1.0820	1.0721	1.0639	1.1463	1.1372	1.1284	1.1463	1.1372	1.1284	1.3463	1.3203	1.2979
		35	1.0129	1.0233	1.0222	1.0918	1.0913	1.0893	1.0824	1.0917	1.0897	1.1932	1.2123	1.2056
		40	0.9813	0.9871	0.9919	1.0406	1.0457	1.0506	1.0411	1.0462	1.0510	1.0988	1.1092	1.1182
		45	0.9748	0.9783	0.9812	1.0099	1.0129	1.0158	1.0103	1.0133	1.0161	1.0307	1.0369	1.0422
		50	0.9828	0.9849	0.9866	0.9892	0.9910	0.9927	0.9895	0.9913	0.9929	0.9856	0.9892	0.9923
		55	0.9907	0.9918	0.9927	0.9846	0.9856	0.9865	0.9848	0.9857	0.9866	0.9733	0.9753	0.9770
		60	0.9961	0.9966	0.9970	0.9936	0.9940	0.9944	0.9937	0.9941	0.9944	0.9887	0.9896	0.9903
25	35	65	0.9999	1.0000	1.0000	0.9999	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.0483	1.0389	1.0335	1.1146	1.1039	1.0949	1.1146	1.1039	1.0949	1.2582	1.2313	1.2106
		35	1.0483	1.0389	1.0335	1.1146	1.1039	1.0949	1.1146	1.1039	1.0949	1.2582	1.2313	1.2106
		40	1.0011	1.0066	1.0055	1.0584	1.0645	1.0612	1.0587	1.0647	1.0614	1.1342	1.1449	1.1364
		45	0.9867	0.9900	0.9919	1.0203	1.0239	1.0268	1.0206	1.0241	1.0270	1.0516	1.0579	1.0628
		50	0.9899	0.9918	0.9930	0.9954	0.9974	0.9992	0.9956	0.9976	0.9993	0.9979	1.0016	1.0045
		55	0.9945	0.9956	0.9962	0.9880	0.9891	0.9900	0.9881	0.9892	0.9901	0.9801	0.9822	0.9838
25	40	60	0.9977	0.9977	0.9981	0.9950	0.9954	0.9958	0.9950	0.9955	0.9959	0.9916	0.9925	0.9931
		65	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	1.0000	0.9999	0.9999	1.0000	0.9999
		30	1.0185	1.0131	1.0113	1.0787	1.0681	1.0601	1.0787	1.0681	1.0601	1.1697	1.1455	1.1283
		35	1.0185	1.0131	1.0113	1.0787	1.0681	1.0601	1.0787	1.0681	1.0601	1.1697	1.1455	1.1283
		40	1.0185	1.0131	1.0113	1.0787	1.0681	1.0601	1.0787	1.0681	1.0601	1.1697	1.1455	1.1283
		45	0.9972	0.9991	0.9989	1.0322	1.0357	1.0323	1.0323	1.0358	1.0324	1.0782	1.0725	1.0703
		50	0.9961	0.9972	0.9976	1.0023	1.0044	1.0059	1.0024	1.0045	1.0060	1.0102	1.0136	1.0160
25	40	55	0.9979	0.9985	0.9987	0.9917	0.9928	0.9937	0.9918	0.9929	0.9937	0.9869	0.9888	0.9901
		60	0.9991	0.9993	0.9994	0.9965	0.9970	0.9973	0.9966	0.9970	0.9974	0.9945	0.9952	0.9958
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2--Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
25	45	30	1.0027	1.0016	1.0015	1.0437	1.0346	1.0293	1.0437	1.0346	1.0293	1.0910	1.0718	1.0608
		35	1.0027	1.0016	1.0015	1.0437	1.0346	1.0293	1.0437	1.0346	1.0293	1.0910	1.0718	1.0608
		40	1.0027	1.0016	1.0015	1.0437	1.0346	1.0293	1.0437	1.0346	1.0293	1.0910	1.0718	1.0608
		45	1.0027	1.0016	1.0015	1.0437	1.0346	1.0293	1.0437	1.0346	1.0293	1.0910	1.0718	1.0608
		50	0.9994	0.9996	0.9996	1.0091	1.0108	1.0087	1.0091	1.0109	1.0088	1.0212	1.0239	1.0194
		55	0.9996	0.9998	0.9998	0.9954	0.9963	0.9969	0.9954	0.9964	0.9969	0.9929	0.9944	0.9953
		60	0.9998	0.9999	0.9999	0.9980	0.9984	0.9987	0.9981	0.9985	0.9987	0.9970	0.9976	0.9980
25	50	65	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.0000	1.0000	1.0000	1.0147	1.0094	1.0084	1.0147	1.0094	1.0084	1.0298	1.0191	1.0170
		35	1.0000	1.0000	1.0000	1.0147	1.0094	1.0084	1.0147	1.0094	1.0084	1.0298	1.0191	1.0170
		40	1.0000	1.0000	1.0000	1.0147	1.0094	1.0084	1.0147	1.0094	1.0084	1.0298	1.0191	1.0170
		45	1.0000	1.0000	1.0000	1.0147	1.0094	1.0084	1.0147	1.0094	1.0084	1.0298	1.0191	1.0170
		50	1.0000	1.0000	1.0000	1.0147	1.0094	1.0084	1.0147	1.0094	1.0084	1.0298	1.0191	1.0170
		55	1.0000	1.0000	1.0000	0.9984	0.9990	0.9991	0.9984	0.9990	0.9991	0.9977	0.9985	0.9986
30	30	60	1.0000	1.0000	1.0000	0.9993	0.9995	0.9996	0.9993	0.9995	0.9996	0.9990	0.9993	0.9994
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		35	1.0082	1.0138	1.0146	1.0667	1.0718	1.0719	1.0609	1.0678	1.0685	1.1508	1.1618	1.1609
		40	0.9828	0.9851	0.9899	1.0330	1.0351	1.0400	1.0309	1.0337	1.0388	1.0759	1.0804	1.0904
		45	0.9779	0.9792	0.9817	1.0062	1.0073	1.0099	1.0053	1.0067	1.0094	1.0184	1.0208	1.0260
		50	0.9856	0.9863	0.9877	0.9879	0.9885	0.9899	0.9874	0.9883	0.9897	0.9796	0.9810	0.9839
		55	0.9925	0.9929	0.9936	0.9845	0.9848	0.9855	0.9842	0.9847	0.9854	0.9711	0.9718	0.9733
30	60	60	0.9970	0.9971	0.9974	0.9937	0.9938	0.9941	0.9936	0.9938	0.9941	0.9881	0.9884	0.9890
		65	1.0000	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	1.0000	0.9999	1.0000	0.9999	1.0000

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
30	35	35	1.0344	1.0285	1.0245	1.0910	1.0842	1.0774	1.0910	1.0842	1.0774	1.2029	1.1860	1.1704
		40	0.9937	1.0001	1.0001	1.0429	1.0499	1.0483	1.0433	1.0502	1.0485	1.0972	1.1110	1.1065
		45	0.9838	0.9872	0.9895	1.0114	1.0151	1.0187	1.0118	1.0154	1.0189	1.0296	1.0369	1.0435
		50	0.9889	0.9908	0.9921	0.9908	0.9928	0.9948	0.9910	0.9930	0.9950	0.9859	0.9899	0.9936
		55	0.9942	0.9952	0.9959	0.9860	0.9870	0.9880	0.9861	0.9871	0.9882	0.9744	0.9765	0.9785
		60	0.9976	0.9980	0.9983	0.9943	0.9947	0.9951	0.9943	0.9948	0.9952	0.9895	0.9903	0.9911
30	40	65	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000
		35	1.0149	1.0107	1.0092	1.0672	1.0589	1.0521	1.0672	1.0589	1.0521	1.1446	1.1256	1.1109
		40	1.0149	1.0107	1.0092	1.0672	1.0589	1.0521	1.0672	1.0589	1.0521	1.1446	1.1256	1.1109
		45	0.9951	0.9975	0.9976	1.0241	1.0286	1.0262	1.0243	1.0288	1.0263	1.0545	1.0627	1.0569
		50	0.9951	0.9965	0.9970	0.9978	1.0002	1.0022	0.9979	1.0004	1.0024	0.9998	1.0043	1.0078
		55	0.9974	0.9982	0.9984	0.9896	0.9909	0.9919	0.9897	0.9910	0.9920	0.9817	0.9841	0.9860
30	45	60	0.9989	0.9992	0.9993	0.9958	0.9963	0.9967	0.9958	0.9963	0.9967	0.9925	0.9935	0.9942
		65	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	0.9999	0.9999	1.0000	1.0000
		35	1.0024	1.0014	1.0013	1.0392	1.0312	1.0264	1.0392	1.0312	1.0264	1.0815	1.0647	1.0547
		40	1.0024	1.0014	1.0013	1.0392	1.0312	1.0264	1.0392	1.0312	1.0264	1.0815	1.0647	1.0547
		45	1.0024	1.0014	1.0013	1.0392	1.0312	1.0264	1.0392	1.0312	1.0264	1.0815	1.0647	1.0547
		50	0.9992	0.9995	0.9995	1.0060	1.0084	1.0067	1.0061	1.0084	1.0067	1.0148	1.0188	1.0151
30	45	55	0.9995	0.9997	0.9997	0.9939	0.9951	0.9959	0.9940	0.9952	0.9959	0.9957	0.9918	0.9931
		60	0.9998	0.9999	0.9999	0.9975	0.9980	0.9983	0.9975	0.9980	0.9983	0.9957	0.9966	0.9971
		65	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
30	50	35	1.0000	1.0000	1.0000	1.0136	1.0088	1.0078	1.0136	1.0088	1.0078	1.0276	1.0178	1.0158
		40	1.0000	1.0000	1.0000	1.0136	1.0088	1.0078	1.0136	1.0088	1.0078	1.0276	1.0178	1.0158
		45	1.0000	1.0000	1.0000	1.0136	1.0088	1.0078	1.0136	1.0088	1.0078	1.0276	1.0178	1.0158
		50	1.0000	1.0000	1.0000	1.0136	1.0088	1.0078	1.0136	1.0088	1.0078	1.0276	1.0178	1.0158
		55	1.0000	1.0000	1.0000	0.9979	0.9986	0.9987	0.9979	0.9986	0.9988	0.9965	0.9977	0.9980
		60	1.0000	1.0000	1.0000	0.9991	0.9994	0.9995	0.9991	0.9994	0.9995	0.9985	0.9990	0.9991
35	35	65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		40	0.9953	0.9985	0.9993	1.0340	1.0377	1.0376	1.0299	1.0347	1.0351	1.0740	1.0819	1.0811
		45	0.9874	0.9887	0.9909	1.0080	1.0095	1.0131	1.0065	1.0084	1.0122	1.0193	1.0224	1.0296
		50	0.9918	0.9925	0.9936	0.9900	0.9908	0.9925	0.9894	0.9903	0.9922	0.9822	0.9837	0.9873
		55	0.9960	0.9963	0.9969	0.9863	0.9867	0.9876	0.9860	0.9865	0.9874	0.9738	0.9746	0.9763
		60	0.9984	0.9985	0.9988	0.9946	0.9948	0.9951	0.9945	0.9947	0.9951	0.9896	0.9899	0.9906
35	40	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999
		40	1.0102	1.0075	1.0063	1.0520	1.0466	1.0414	1.0520	1.0466	1.0414	1.1113	1.0990	1.0877
		45	0.9933	0.9961	0.9964	1.0151	1.0202	1.0190	1.0153	1.0204	1.0191	1.0340	1.0441	1.0410
		50	0.9948	0.9961	0.9967	0.9935	0.9961	0.9985	0.9937	0.9962	0.9986	0.9895	0.9945	0.9990
		55	0.9974	0.9981	0.9984	0.9880	0.9893	0.9905	0.9881	0.9893	0.9905	0.9774	0.9799	0.9822
		60	0.9990	0.9992	0.9993	0.9953	0.9958	0.9963	0.9953	0.9958	0.9963	0.9911	0.9920	0.9929
35	40	65	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
35	45	40	1.0019	1.0011	1.0010	1.0331	1.0267	1.0225	1.0331	1.0267	1.0225	1.0688	1.0552	1.0466
		45	1.0019	1.0011	1.0010	1.0331	1.0267	1.0225	1.0331	1.0267	1.0225	1.0688	1.0552	1.0466
		50	0.9990	0.9994	0.9994	1.0024	1.0053	1.0041	1.0024	1.0054	1.0042	1.0067	1.0122	1.0096
		55	0.9995	0.9997	0.9997	0.9924	0.9938	0.9948	0.9924	0.9939	0.9948	0.9860	0.9888	0.9905
		60	0.9998	0.9998	0.9998	0.9970	0.9976	0.9979	0.9970	0.9976	0.9980	0.9945	0.9955	0.9962
35	50	65	1.0000	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		40	1.0000	1.0000	1.0000	1.0121	1.0078	1.0069	1.0121	1.0078	1.0069	1.0247	1.0159	1.0141
		45	1.0000	1.0000	1.0000	1.0121	1.0078	1.0069	1.0121	1.0078	1.0069	1.0247	1.0159	1.0141
		50	1.0000	1.0000	1.0000	1.0121	1.0078	1.0069	1.0121	1.0078	1.0069	1.0247	1.0159	1.0141
		55	1.0000	1.0000	1.0000	0.9972	0.9981	0.9983	0.9972	0.9982	0.9984	0.9950	0.9967	0.9971
40	40	60	1.0000	1.0000	1.0000	0.9989	0.9992	0.9993	0.9989	0.9992	0.9993	0.9980	0.9987	0.9888
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		45	0.9958	0.9970	0.9975	1.0110	1.0139	1.0137	1.0081	1.0117	1.0119	1.0238	1.0295	1.0289
		50	0.9969	0.9973	0.9978	0.9933	0.9943	0.9968	0.9923	0.9936	0.9962	0.9875	0.9897	0.9945
		55	0.9986	0.9988	0.9990	0.9890	0.9894	0.9905	0.9885	0.9891	0.9902	0.9785	0.9794	0.9816
40	45	60	0.9995	0.9995	0.9996	0.9959	0.9961	0.9965	0.9958	0.9960	0.9964	0.9920	0.9923	0.9931
		65	0.9999	0.9999	0.9999	1.0000	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000
		45	1.0246	1.0204	1.0171	1.0246	1.0204	1.0171	1.0246	1.0204	1.0171	1.0510	1.0420	1.0353
		50	0.9983	1.0017	1.0012	0.9983	1.0017	1.0012	0.9984	1.0218	1.0013	0.9977	1.0044	1.0032
		55	0.9912	0.9927	0.9938	0.9912	0.9927	0.9938	0.9912	0.9927	0.9938	0.9830	0.9860	0.9882
40	60	60	0.9967	0.9973	0.9977	0.9967	0.9973	0.9977	0.9967	0.9973	0.9977	0.9937	0.9948	0.9956
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE I			EMPLOYMENT TURNOVER TABLE II			EMPLOYMENT TURNOVER TABLE III			EMPLOYMENT TURNOVER TABLE IV		
			Vesting Conditions			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3	1	2	3
40	50	45	1.0000	1.0000	1.0000	1.0101	1.0066	1.0058	1.0101	1.0066	1.0058	1.0205	1.0134	1.0118
		50	1.0000	1.0000	1.0000	1.0101	1.0066	1.0058	1.0101	1.0066	1.0058	1.0205	1.0134	1.0118
		55	1.0000	1.0000	1.0000	0.9963	0.9976	0.9979	0.9963	0.9976	0.9979	0.9931	0.9955	0.9960
		60	1.0000	1.0000	1.0000	0.9986	0.9991	0.9992	0.9986	0.9991	0.9992	0.9974	0.9983	0.9985
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
45	45	50	0.9995	0.9997	0.9997	0.9981	1.0000	1.0001	0.9968	0.9991	0.9993	0.9966	1.0004	1.0005
		55	0.9998	0.9999	0.9999	0.9926	0.9932	0.9944	0.9922	0.9929	0.9941	0.9854	0.9867	0.9890
		60	0.9999	0.9999	0.9999	0.9976	0.9978	0.9981	0.9974	0.9977	0.9981	0.9952	0.9956	0.9963
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
		50	1.0000	1.0000	1.0000	1.0071	1.0047	1.0041	1.0071	1.0047	1.0041	1.0144	1.0095	1.0083
45	50	55	1.0000	1.0000	1.0000	0.9956	0.9970	0.9974	0.9956	0.9970	0.9974	0.9913	0.9943	0.9949
		60	1.0000	1.0000	1.0000	0.9985	0.9990	0.9991	0.9985	0.9990	0.9991	0.9971	0.9981	0.9983
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		55	1.0000	1.0000	1.0000	0.9971	0.9978	0.9981	0.9959	0.9969	0.9974	0.9942	0.9956	0.9963
		60	1.0000	1.0000	1.0000	0.9992	0.9994	0.9995	0.9990	0.9992	0.9993	0.9985	0.9989	0.9991
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
50	50	55	1.0000	1.0000	1.0000	0.9971	0.9978	0.9981	0.9959	0.9969	0.9974	0.9942	0.9956	0.9963
		60	1.0000	1.0000	1.0000	0.9992	0.9994	0.9995	0.9990	0.9992	0.9993	0.9985	0.9989	0.9991
		65	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	20	25	1.4600	1.4892	1.4839	2.7786	2.8692	2.7904	2.7600	2.8639	2.7831
		30	1.2980	1.3105	1.3304	2.0043	2.0374	2.0858	2.0173	2.0569	2.1014
		35	1.1796	1.1866	1.1977	1.5640	1.5807	1.6052	1.5765	1.5968	1.6196
		40	1.0929	1.0972	1.1040	1.2933	1.3029	1.3170	1.3026	1.3143	1.3275
		45	1.0270	1.0297	1.0340	1.1126	1.1184	1.1270	1.1191	1.1263	1.1343
		50	0.9825	0.9842	0.9868	0.9997	1.0033	1.0085	1.0040	1.0084	1.0133
		55	0.9710	0.9719	0.9734	0.9618	0.9638	0.9668	0.9643	0.9668	0.9696
20	25	60	0.9875	0.9879	0.9885	0.9833	0.9842	0.9855	0.9844	0.9855	0.9867
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999
		25	1.5855	1.5413	1.5055	3.1744	2.9094	2.7449	3.1744	2.9094	2.7449
		30	1.3518	1.3773	1.3651	2.1487	2.2065	2.1370	2.1753	2.2248	2.1528
		35	1.2097	1.2239	1.2354	1.6370	1.6663	1.6844	1.6574	1.6828	1.6985
		40	1.1113	1.1199	1.1270	1.3353	1.3521	1.3626	1.3495	1.3641	1.3732
		45	1.0385	1.0439	1.0484	1.1381	1.1484	1.1547	1.1477	1.1566	1.1622
20	30	50	0.9896	0.9929	0.9956	1.0153	1.0215	1.0254	1.0215	1.0269	1.0303
		55	0.9749	0.9768	0.9784	0.9707	0.9742	0.9765	0.9743	0.9774	0.9793
		60	0.9892	0.9900	0.9907	0.9872	0.9887	0.9897	0.9887	0.9901	0.9910
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.4374	1.3956	1.3651	2.3325	2.1595	2.0557	2.3325	2.1595	2.0557
		30	1.4374	1.3956	1.3651	2.3325	2.1595	2.0557	2.3325	2.1595	2.0557
		35	1.2573	1.2708	1.2577	1.7300	1.7491	1.7380	1.7380	1.7546	1.7046
20	30	40	1.1404	1.1486	1.1545	1.3888	1.3998	1.4064	1.3961	1.4057	1.4114
		45	1.0567	1.0619	1.0656	1.1706	1.1773	1.1813	1.1761	1.1819	1.1854
		50	1.0008	1.0040	1.0063	1.0351	1.0392	1.0416	1.0388	1.0424	1.0445
		55	0.9813	0.9844	0.9844	0.9820	0.9843	0.9857	0.9842	0.9863	0.9875
		60	0.9919	0.9927	0.9933	0.9921	0.9931	0.9937	0.9931	0.9940	0.9945
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	35	25	1.3012	1.2666	1.2418	1.7899	1.6807	1.6126	1.7899	1.6807	1.6126
		30	1.3012	1.2666	1.2418	1.7899	1.6807	1.6126	1.7899	1.6807	1.6126
		35	1.3012	1.2666	1.2418	1.7899	1.6807	1.6126	1.7899	1.6807	1.6126
		40	1.1671	1.1739	1.1620	1.4233	1.4303	1.3957	1.4262	1.4322	1.3974
		45	1.0735	1.0778	1.0808	1.1916	1.1958	1.1985	1.1944	1.1981	1.2004
		50	1.0111	1.0137	1.0156	1.0479	1.0501	1.0523	1.0500	1.0523	1.0537
		55	0.9871	0.9886	0.9896	0.9893	0.9908	0.9917	0.9906	0.9919	0.9927
20	40	60	0.9944	0.9951	0.9955	0.9953	0.9959	0.9963	0.9959	0.9964	0.9968
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.1891	1.1611	1.1418	1.4453	1.3722	1.3270	1.4453	1.3722	1.3270
		30	1.1891	1.1611	1.1418	1.4453	1.3722	1.3270	1.4453	1.3722	1.3270
		35	1.1891	1.1611	1.1418	1.4453	1.3722	1.3270	1.4453	1.3722	1.3270
		40	1.1891	1.1611	1.1418	1.4453	1.3722	1.3270	1.4453	1.3722	1.3270
		45	1.0873	1.0908	1.0812	1.2049	1.2077	1.1847	1.2061	1.2085	1.1854
20	45	50	1.0196	1.0217	1.0232	1.0560	1.0577	1.0588	1.0571	1.0586	1.0596
		55	0.9919	0.9931	0.9939	0.9940	0.9949	0.9955	0.9947	0.9956	0.9961
		60	0.9965	0.9970	0.9973	0.9973	0.9978	0.9980	0.9976	0.9980	0.9983
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.0985	1.0773	1.0655	1.2139	1.1661	1.1411	1.2139	1.1661	1.1411
		30	1.0985	1.0773	1.0655	1.2139	1.1661	1.1411	1.2139	1.1661	1.1411
		35	1.0985	1.0773	1.0655	1.2139	1.1661	1.1411	1.2139	1.1661	1.1411
20	45	40	1.0985	1.0773	1.0655	1.2139	1.1661	1.1411	1.2139	1.1661	1.1411
		45	1.0985	1.0773	1.0655	1.2139	1.1661	1.1411	1.2139	1.1661	1.1411
		50	1.0264	1.0280	1.0229	1.0615	1.0626	1.0512	1.0619	1.0629	1.0515
		55	0.9957	0.9966	0.9971	0.9971	0.9977	0.9980	0.9974	0.9980	0.9983
		60	0.9981	0.9985	0.9987	0.9987	0.9990	0.9991	0.9988	0.9991	0.9992
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	50	25	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		30	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		35	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		40	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		45	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		50	1.0315	1.0202	1.0180	1.0650	1.0416	1.0370	1.0650	1.0416	1.0370
		55	0.9986	0.9991	0.9992	0.9991	0.9994	0.9994	0.9992	0.9995	0.9995
25	25	60	0.9994	0.9996	0.9996	0.9996	0.9997	0.9997	0.9996	0.9997	0.9998
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.2585	1.2791	1.2778	1.8109	1.8571	1.8324	1.7891	1.8459	1.8214
		35	1.1584	1.1673	1.1814	1.4622	1.4804	1.5110	1.4645	1.4880	1.5169
		40	1.0807	1.0856	1.0933	1.2374	1.2469	1.2630	1.2425	1.2551	1.2705
		45	1.0205	1.0233	1.0279	1.0822	1.0878	1.0971	1.0868	1.0941	1.1032
		50	0.9796	0.9813	0.9840	0.9840	0.9873	0.9928	0.9873	0.9917	0.9971
25	30	55	0.9701	0.9710	0.9725	0.9542	0.9561	0.9592	0.9562	0.9586	0.9617
		60	0.9874	0.9877	0.9884	0.9803	0.9811	0.9824	0.9811	0.9821	0.9834
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.3463	1.3203	1.2979	2.0181	1.9114	1.8353	2.0181	1.9114	1.8353
		35	1.1961	1.2144	1.2073	1.5438	1.5825	1.5523	1.5593	1.5932	1.5615
		40	1.1014	1.1114	1.1201	1.2803	1.3006	1.3150	1.2931	1.3112	1.3241
		45	1.0327	1.0386	1.0437	1.1071	1.1189	1.1273	1.1164	1.1269	1.1345
25	50	50	0.9869	0.9904	0.9934	0.9988	1.0058	1.0107	1.0050	1.0113	1.0157
		55	0.9741	0.9760	0.9777	0.9625	0.9664	0.9692	0.9661	0.9697	0.9722
		60	0.9890	0.9899	0.9906	0.9838	0.9855	0.9867	0.9854	0.9869	0.9880
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
25	35	30	1.2582	1.2313	1.2106	1.6698	1.5850	1.5281	1.6698	1.5850	1.5281
		35	1.2582	1.2313	1.2106	1.6698	1.5850	1.5281	1.6698	1.5850	1.5281
		40	1.1354	1.1458	1.1371	1.3665	1.3626	1.3360	1.3521	1.3664	1.3394
		45	1.0528	1.0590	1.0637	1.1455	1.1549	1.1612	1.1509	1.1592	1.1649
		50	0.9988	1.0025	1.0053	1.0216	1.0271	1.0309	1.0255	1.0305	1.0339
		55	0.9806	0.9827	0.9842	0.9753	0.9784	0.9805	0.9777	0.9805	0.9824
		60	0.9918	0.9927	0.9933	0.9893	0.9907	0.9916	0.9903	0.9916	0.9924
25	40	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.1697	1.1455	1.1283	1.3981	1.3353	1.2949	1.3981	1.3353	1.2949
		35	1.1697	1.1455	1.1283	1.3981	1.3353	1.2949	1.3981	1.3353	1.2949
		40	1.1697	1.1455	1.1283	1.3981	1.3353	1.2949	1.3981	1.3353	1.2949
		45	1.0731	1.0786	1.0707	1.1755	1.1824	1.1628	1.1778	1.1840	1.1641
		50	1.0108	1.0141	1.0164	1.0394	1.0435	1.0462	1.0416	1.0453	1.0477
		55	0.9873	0.9891	0.9904	0.9853	0.9876	0.9891	0.9867	0.9888	0.9901
25	45	60	0.9946	0.9954	0.9959	0.9936	0.9946	0.9953	0.9942	0.9951	0.9957
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		30	1.0910	1.0718	1.0608	1.1976	1.1542	1.1308	1.1976	1.1542	1.1308
		35	1.0910	1.0718	1.0608	1.1976	1.1542	1.1308	1.1976	1.1542	1.1308
		40	1.0910	1.0718	1.0608	1.1976	1.1542	1.1308	1.1976	1.1542	1.1308
		45	1.0910	1.0718	1.0608	1.1976	1.1542	1.1308	1.1976	1.1542	1.1308
		50	1.0215	1.0241	1.0195	1.0525	1.0554	1.0452	1.0534	1.0560	1.0457
25	55	55	0.9931	0.9946	0.9954	0.9927	0.9943	0.9951	0.9934	0.9948	0.9956
		60	0.9971	0.9977	0.9980	0.9968	0.9975	0.9979	0.9971	0.9977	0.9981
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
25	50	30	1.0298	1.0191	1.0170	1.0614	1.0394	1.0351	1.0615	1.0394	1.0351
		35	1.0298	1.0191	1.0170	1.0614	1.0394	1.0351	1.0615	1.0394	1.0351
		40	1.0298	1.0191	1.0170	1.0614	1.0394	1.0351	1.0615	1.0394	1.0351
		45	1.0298	1.0191	1.0170	1.0614	1.0394	1.0351	1.0615	1.0394	1.0351
		50	1.0298	1.0191	1.0170	1.0614	1.0394	1.0351	1.0615	1.0394	1.0351
		55	0.9977	0.9985	0.9987	0.9977	0.9985	0.9987	0.9979	0.9986	0.9988
		60	0.9990	0.9993	0.9994	0.9990	0.9993	0.9994	0.9991	0.9994	0.9994
30	30	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		35	1.1393	1.1541	1.1541	1.3925	1.4193	1.4100	1.3711	1.4061	1.3980
		40	1.0723	1.0785	1.0887	1.1990	1.2097	1.2300	1.1962	1.2109	1.2308
		45	1.0170	1.0203	1.0257	1.0622	1.0678	1.0784	1.0630	1.0709	1.0814
		50	0.9791	0.9809	0.9839	0.9748	0.9779	0.9839	0.9761	0.9805	0.9865
		55	0.9708	0.9718	0.9734	0.9508	0.9525	0.9558	0.9517	0.9541	0.9573
		60	0.9880	0.9884	0.9890	0.9793	0.9800	0.9814	0.9796	0.9806	0.9820
30	35	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999
		35	1.2029	1.1860	1.1704	1.5154	1.4621	1.4195	1.5154	1.4621	1.4195
		40	1.0990	1.1123	1.1075	1.2483	1.2750	1.2592	1.2569	1.2810	1.2643
		45	1.0312	1.0382	1.0447	1.0880	1.1019	1.1131	1.0953	1.1081	1.1183
		50	0.9870	0.9909	0.9945	0.9893	0.9972	1.0034	0.9944	1.0017	1.0075
		55	0.9750	0.9771	0.9790	0.9587	0.9630	0.9664	0.9616	0.9656	0.9688
		60	0.9897	0.9905	0.9913	0.9826	0.9844	0.9858	0.9838	0.9855	0.9868
30	35	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
30	40	35	1.1446	1.1256	1.1109	1.3374	1.2879	1.2535	1.3374	1.2879	1.2535
		40	1.1446	1.1256	1.1109	1.3374	1.2879	1.2535	1.3374	1.2879	1.2535
		45	1.0553	1.0632	1.0574	1.1346	1.1476	1.1327	1.1381	1.1500	1.1347
		50	1.0005	1.0049	1.0083	1.0155	1.0229	1.0279	1.0187	1.0254	1.0301
		55	0.9822	0.9845	0.9863	0.9729	0.9769	0.9797	0.9749	0.9785	0.9811
		60	0.9926	0.9936	0.9943	0.9886	0.9903	0.9914	0.9894	0.9909	0.9920
		65	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
30	45	35	1.0815	1.0647	1.0547	1.1767	1.1388	1.1176	1.1767	1.1388	1.1176
		40	1.1815	1.0647	1.0547	1.1767	1.1388	1.1176	1.1767	1.1388	1.1176
		45	1.0815	1.0647	1.0547	1.1767	1.1388	1.1176	1.1767	1.1388	1.1176
		50	1.0151	1.0190	1.0153	1.0392	1.0448	1.0363	1.0406	1.0457	1.0371
		55	0.9899	0.9920	0.9932	0.9858	0.9888	0.9905	0.9868	0.9896	0.9912
		60	0.9958	0.9967	0.9972	0.9940	0.9953	0.9960	0.9944	0.9956	0.9963
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
30	50	35	1.0276	1.0178	1.0158	1.0569	1.0366	1.0325	1.0569	1.0366	1.0325
		40	1.0276	1.0178	1.0158	1.0569	1.0366	1.0325	1.0569	1.0366	1.0325
		45	1.0276	1.0178	1.0158	1.0569	1.0366	1.0325	1.0569	1.0366	1.0325
		50	1.0276	1.0178	1.0158	1.0569	1.0366	1.0325	1.0569	1.0366	1.0325
		55	0.9966	0.9978	0.9980	0.9954	0.9970	0.9973	0.9957	0.9972	0.9975
		60	0.9986	0.9990	0.9991	0.9980	0.9987	0.9989	0.9982	0.9988	0.9989
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
35	35	40	1.0660	1.0760	1.0762	1.1767	1.1938	1.1893	1.1616	1.1831	1.1801
		45	1.0166	1.0206	1.0281	1.0528	1.0596	1.0736	1.0491	1.0578	1.0722
		50	0.9810	0.9830	0.9867	0.9729	0.9763	0.9834	0.9718	0.9763	0.9836
		55	0.9732	0.9742	0.9761	0.9522	0.9540	0.9576	0.9518	0.9541	0.9578
		60	0.9894	0.9898	0.9905	0.9807	0.9814	0.9828	0.9804	0.9813	0.9829
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
35	40	40	1.1113	1.0989	1.0877	1.2565	1.2247	1.1984	1.2565	1.2247	1.1984
		45	1.0348	1.0445	1.0414	1.0845	1.1035	1.0948	1.0878	1.1059	1.0968
		50	0.9901	0.9949	0.9995	0.9889	0.9985	1.0064	0.9915	1.0007	1.0083
		55	0.9778	0.9801	0.9825	0.9604	0.9653	0.9694	0.9619	0.9666	0.9705
		60	0.9912	0.9921	0.9930	0.9840	0.9859	0.9876	0.9845	0.9864	0.9880
		65	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
35	45	40	1.0688	1.0552	1.0466	1.1488	1.1183	1.1000	1.1488	1.1183	1.1000
		45	1.0688	1.0552	1.0466	1.1488	1.1183	1.1000	1.1488	1.1183	1.1000
		50	1.0070	1.0124	1.0098	1.0213	1.0305	1.0243	1.0226	1.0314	1.0250
		55	0.9862	0.9889	0.9906	0.9770	0.9817	0.9845	0.9779	0.9824	0.9851
		60	0.9945	0.9956	0.9963	0.9907	0.9926	0.9937	0.9910	0.9928	0.9939
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000
35	50	40	1.0247	1.0159	1.0141	1.0508	1.0328	1.0291	1.0508	1.0328	1.0291
		45	1.0247	1.0159	1.0141	1.0508	1.0328	1.0291	1.0508	1.0328	1.0291
		50	1.0247	1.0159	1.0141	1.0508	1.0328	1.0291	1.0508	1.0328	1.0291
		55	0.9950	0.9968	0.9971	0.9921	0.9949	0.9955	0.9924	0.9951	0.9956
		60	0.9980	0.9987	0.9988	0.9968	0.9979	0.9981	0.9969	0.9980	0.9982
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
40	40	45	1.0180	1.0252	1.0254	1.0546	1.0663	1.0642	1.0438	1.0585	1.0575
		50	0.9856	0.9883	0.9934	0.9791	0.9836	0.9929	0.9759	0.9816	0.9912
		55	0.9776	0.9788	0.9811	0.9590	0.9611	0.9653	0.9576	0.9602	0.9647
		60	0.9917	0.9921	0.9930	0.9845	0.9852	0.9868	0.9839	0.9849	0.9866
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
40	45	45	1.0510	1.0420	1.0353	1.1098	1.0896	1.0754	1.1098	1.0896	1.0754
		50	0.9980	1.0047	1.0034	1.0001	1.0128	1.0096	1.0015	1.0137	1.0104
		55	0.9832	0.9861	0.9883	0.9686	0.9743	0.9784	0.9694	0.9750	0.9789
		60	0.9937	0.9948	0.9956	0.9881	0.9903	0.9918	0.9883	0.9905	0.9920
		65	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	1.0000	0.9999	0.9999
40	50	45	1.0205	1.0134	1.0118	1.0424	1.0275	1.0243	1.0424	1.0275	1.0243
		50	1.0205	1.0134	1.0118	1.0424	1.0275	1.0243	1.0424	1.0275	1.0243
		55	0.9932	0.9955	0.9961	0.9878	0.9921	0.9930	0.9881	0.9923	0.9932
		60	0.9974	0.9983	0.9985	0.9954	0.9970	0.9973	0.9955	0.9970	0.9974
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
45	45	50	0.9940	0.9985	0.9989	0.9945	1.0021	1.0021	0.9895	0.9985	0.9990
		55	0.9846	0.9861	0.9885	0.9715	0.9741	0.9786	0.9699	0.9730	0.9776
		60	0.9949	0.9954	0.9962	0.9904	0.9913	0.9928	0.9898	0.9909	0.9925
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE V			EMPLOYMENT TURNOVER TABLE VI			EMPLOYMENT TURNOVER TABLE VII		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
45	50	50	1.0144	1.0095	1.0083	1.0296	1.0196	1.0172	1.0296	1.0196	1.0172
		55	0.9914	0.9943	0.9950	0.9835	0.9891	0.9904	0.9837	0.9892	0.9905
		60	0.9971	0.9981	0.9983	0.9944	0.9963	0.9967	0.9945	0.9963	0.9968
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
50	50	55	0.9919	0.9939	0.9949	0.9884	0.9913	0.9927	0.9841	0.9880	0.9900
		60	0.9980	0.9985	0.9987	0.9971	0.9978	0.9982	0.9960	0.9970	0.9975
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2--Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	20	25	2.7667	2.8649	2.7852	2.1537	2.2453	2.1495	1.9910	2.0198	2.0049
		30	2.0107	2.0474	2.0938	1.4200	1.4558	1.4921	1.7073	1.7183	1.7443
		35	1.5704	1.5891	1.6127	1.1432	1.1638	1.1848	1.4649	1.4706	1.4839
		40	1.2981	1.3089	1.3225	0.9261	1.0395	1.0531	1.2745	1.2777	1.2854
		45	1.1160	1.1225	1.1308	0.9710	0.9799	0.9889	1.1314	1.1333	1.1379
		50	1.0019	1.0059	1.0110	0.9459	0.9516	0.9574	1.0321	1.0333	1.0360
		55	0.9631	0.9654	0.9683	0.9684	0.9718	0.9751	0.9852	0.9858	0.9874
20	25	60	0.9839	0.9849	0.9861	0.9862	0.9876	0.9891	0.9850	0.9852	0.9859
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	3.1744	2.9094	2.7449	2.5245	2.2391	2.1094	2.1287	2.0587	1.9938
		30	2.1630	2.2163	2.1455	1.5649	1.6010	1.5504	1.7600	1.7966	1.7706
		35	1.6479	1.6751	1.6920	1.2266	1.2474	1.2569	1.4920	1.5107	1.5281
		40	1.3429	1.3586	1.3683	1.0803	1.0938	1.0999	1.2900	1.3008	1.3108
		45	1.1432	1.1528	1.1587	1.0069	1.0159	1.0200	1.1407	1.1472	1.1532
20	30	50	1.0186	1.0244	1.0280	0.9691	0.9749	0.9775	1.0377	1.0416	1.0452
		55	0.9726	0.9759	0.9780	0.9819	0.9853	0.9868	0.9883	0.9904	0.9924
		60	0.9880	0.9895	0.9904	0.9921	0.9936	0.9942	0.9863	0.9872	0.9881
		65	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	2.3325	2.1595	2.0557	1.6726	1.5537	1.4961	1.8867	1.8079	1.7477
		30	2.3325	2.1595	2.0557	1.6726	1.5537	1.4961	1.8867	1.8079	1.7477
		35	1.7343	1.7520	1.7024	1.2887	1.2974	1.2719	1.5569	1.5780	1.5503
20	30	40	1.3927	1.4030	1.4091	1.1205	1.1262	1.1289	1.3273	1.3394	1.3487
		45	1.1736	1.1798	1.1835	1.0337	1.0374	1.0392	1.1631	1.1704	1.1759
		50	1.0371	1.0409	1.0432	0.9863	0.9887	0.9899	1.0511	1.0555	1.0588
		55	0.9832	0.9854	0.9867	0.9920	0.9934	0.9941	0.9957	0.9982	1.0000
		60	0.9926	0.9936	0.9942	0.9965	0.9971	0.9974	0.9895	0.9905	0.9913
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	35	25	1.7899	1.6807	1.6126	1.3147	1.2596	1.2300	1.6266	1.5574	1.5082
		30	1.7899	1.6807	1.6126	1.3147	1.2596	1.2300	1.6266	1.5574	1.5082
		35	1.7899	1.6807	1.6126	1.3147	1.2596	1.2300	1.6266	1.5574	1.5082
		40	1.4248	1.4313	1.3966	1.1375	1.1401	1.1259	1.3673	1.3780	1.3537
		45	1.1931	1.1970	1.1995	1.0449	1.0466	1.0476	1.1871	1.1935	1.1981
		50	1.0490	1.0514	1.0529	0.9936	0.9947	0.9953	1.0655	1.0693	1.0721
		55	0.9900	0.9914	0.9922	0.9962	0.9969	0.9972	1.0038	1.0059	1.0074
20	40	60	0.9956	0.9962	0.9966	0.9983	0.9986	0.9988	0.9929	0.9938	0.9945
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.4453	1.3722	1.3270	1.1455	1.1158	1.0989	1.4019	1.3467	1.3095
		30	1.4453	1.3722	1.3270	1.1455	1.1158	1.0989	1.4019	1.3467	1.3095
		35	1.4453	1.3722	1.3270	1.1455	1.1158	1.0989	1.4019	1.3467	1.3095
		40	1.2055	1.2082	1.1851	1.0502	1.0512	1.0428	1.2079	1.2130	1.1942
		45	1.0566	1.0582	1.0592	0.9970	0.9976	0.9979	1.0780	1.0810	1.0831
20	45	50	0.9943	0.9953	0.9958	0.9982	0.9986	0.9988	1.0107	1.0124	1.0135
		55	0.9975	0.9979	0.9982	0.9992	0.9993	0.9994	0.9959	0.9966	0.9971
		60	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		25	1.2139	1.1661	1.1411	1.0531	1.0358	1.0312	1.2243	1.1835	1.1595
		30	1.2139	1.1661	1.1411	1.0531	1.0358	1.0312	1.2243	1.1835	1.1595
		35	1.2139	1.1661	1.1411	1.0531	1.0358	1.0312	1.2243	1.1835	1.1595
20	45	40	1.2139	1.1661	1.1411	1.0531	1.0358	1.0312	1.2243	1.1835	1.1595
		45	1.2139	1.1661	1.1411	1.0531	1.0358	1.0312	1.2243	1.1835	1.1595
		50	1.0617	1.0627	1.0514	0.9989	0.9992	0.9993	1.0878	1.0900	1.0786
		55	0.9972	0.9978	0.9982	0.9993	0.9995	0.9996	1.0162	1.0174	1.0182
		60	0.9988	0.9990	0.9992	0.9997	0.9998	0.9998	0.9982	0.9988	0.9991
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
20	50	25	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		30	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		35	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		40	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		45	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		50	1.0650	1.0416	1.0370	1.0000	1.0000	1.0000	1.0950	1.0708	1.0608
		55	0.9991	0.9994	0.9995	1.0000	1.0000	1.0000	1.0202	1.0209	1.0172
		60	0.9996	0.9997	0.9997	1.0000	1.0000	1.0000	0.9999	1.0003	1.0004
		65	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
25	25	30	1.7930	1.8467	1.8228	1.3144	1.3553	1.3422	1.6023	1.6246	1.6156
		35	1.4625	1.4844	1.5141	1.0948	1.1134	1.1379	1.4009	1.4097	1.4300
		40	1.2402	1.2518	1.2676	1.0001	1.0110	1.0255	1.2369	1.2415	1.2520
		45	1.0849	1.0917	1.1009	0.9566	0.9634	0.9725	1.1104	1.1130	1.1190
		50	0.9860	0.9900	0.9955	0.9385	0.9427	0.9484	1.0214	1.0229	1.0264
		55	0.9554	0.9577	0.9607	0.9651	0.9675	0.9707	0.9802	0.9811	0.9830
		60	0.9808	0.9817	0.9831	0.9851	0.9861	0.9875	0.9831	0.9835	0.9843
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
25	30	30	2.0181	1.9114	1.8353	1.4894	1.4166	1.3747	1.7090	1.6600	1.6154
		35	1.5542	1.5897	1.5585	1.1741	1.2021	1.1886	1.4429	1.4712	1.4545
		40	1.2889	1.3077	1.3211	1.0468	1.0633	1.0728	1.2587	1.2735	1.2869
		45	1.1133	1.1243	1.1321	0.9859	0.9962	1.0022	1.1229	1.1313	1.1390
		50	1.0029	1.0094	1.0141	0.9567	0.9631	0.9668	1.0286	1.0335	1.0380
		55	0.9649	0.9686	0.9712	0.9754	0.9791	0.9812	0.9842	0.9869	0.9893
		60	0.9848	0.9864	0.9876	0.9895	0.9910	0.9919	0.9848	0.9859	0.9870
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
25	35	30	1.6698	1.5850	1.5281	1.2617	1.2189	1.1943	1.5403	1.4865	1.4453
		35	1.6698	1.5850	1.5281	1.2617	1.2189	1.1943	1.5403	1.4865	1.4453
		40	1.3502	1.3651	1.3383	1.0984	1.1080	1.0980	1.3094	1.3256	1.3071
		45	1.1491	1.1578	1.1636	1.0183	1.0243	1.0278	1.1519	1.1611	1.1682
		50	1.0242	1.0294	1.0329	0.9768	0.9806	0.9828	1.0455	1.0509	1.0550
		55	0.9769	0.9798	0.9818	0.9868	0.9890	0.9902	0.9934	0.9963	0.9986
		60	0.9900	0.9913	0.9921	0.9943	0.9953	0.9958	0.9887	0.9900	0.9909
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
25	40	30	1.3981	1.3353	1.2949	1.1285	1.1031	1.0879	1.3628	1.3154	1.2820
		35	1.3981	1.3353	1.2949	1.1285	1.1031	1.0879	1.3628	1.3154	1.2820
		40	1.3981	1.3353	1.2949	1.1285	1.1031	1.0879	1.3628	1.3154	1.2820
		45	1.1770	1.1835	1.1637	1.0372	1.0408	1.0340	1.1824	1.1905	1.1745
		50	1.0408	1.0447	1.0472	0.9886	0.9908	0.9922	1.0632	1.0680	1.0713
		55	0.9862	0.9884	0.9898	0.9935	0.9948	0.9955	1.0031	1.0057	1.0075
		60	0.9940	0.9950	0.9956	0.9972	0.9977	0.9981	0.9928	0.9939	0.9947
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
25	45	30	1.1976	1.1542	1.1308	1.0486	1.0328	1.0285	1.2088	1.1717	1.1492
		35	1.1976	1.1542	1.1308	1.0486	1.0328	1.0285	1.2088	1.1717	1.1492
		40	1.1976	1.1542	1.1308	1.0486	1.0328	1.0285	1.2088	1.1717	1.1492
		45	1.1976	1.1542	1.1308	1.0486	1.0328	1.0285	1.2088	1.1717	1.1492
		50	1.0531	1.0558	1.0455	0.9957	0.9970	0.9974	1.0786	1.0823	1.0720
		55	0.9931	0.9946	0.9954	0.9975	0.9983	0.9985	1.0115	1.0135	1.0148
		60	0.9970	0.9977	0.9980	0.9989	0.9992	0.9993	0.9964	0.9972	0.9977
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
		Vesting Conditions			Vesting Conditions			Vesting Conditions		
		1	2	3	1	2	3	1	2	3
25	50	1.0614	1.0394	1.0351	1.0000	1.0000	1.0000	1.0950	1.0676	1.0580
		1.0614	1.0394	1.0351	1.0000	1.0000	1.0000	1.0950	1.0676	1.0580
		1.0614	1.0394	1.0351	1.0000	1.0000	1.0000	1.0950	1.0676	1.0580
		1.0614	1.0394	1.0351	1.0000	1.0000	1.0000	1.0950	1.0676	1.0580
		1.0614	1.0394	1.0351	1.0000	1.0000	1.0000	1.0950	1.0676	1.0580
		0.9978	0.9986	0.9987	1.0000	1.0000	1.0000	1.0180	1.0192	1.0157
		0.9990	0.9994	0.9994	1.0000	1.0000	1.0000	0.9991	0.9996	0.9998
30	30	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
		1.3775	1.4099	1.4015	1.1008	1.1216	1.1210	1.3477	1.3649	1.3597
		1.1969	1.2103	1.2304	1.0077	1.0168	1.0314	1.2058	1.2126	1.2281
		1.0626	1.0697	1.0803	0.9641	0.9692	0.9772	1.0935	1.0971	1.1050
		0.9756	0.9796	0.9856	0.9454	0.9483	0.9530	1.0133	1.0153	1.0197
		0.9513	0.9535	0.9568	0.9705	0.9721	0.9746	0.9771	0.9781	0.9804
		0.9795	0.9804	0.9818	0.9878	0.9884	0.9895	0.9822	0.9827	0.9836
30	35	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999
		1.5154	1.4621	1.4195	1.1934	1.1667	1.1484	1.4294	1.3954	1.3644
		1.2541	1.2790	1.2626	1.0484	1.0664	1.0619	1.2382	1.2598	1.2489
		1.0929	1.1060	1.1166	0.9866	0.9966	1.0034	1.1102	1.1214	1.1315
		0.9927	1.0001	1.0061	0.9584	0.9641	0.9681	1.0225	1.0287	1.0343
		0.9607	0.9647	0.9679	0.9775	0.9806	0.9827	0.9819	0.9852	0.9881
		0.9834	0.9851	0.9865	0.9907	0.9920	0.9929	0.9843	0.9856	0.9868
30	40	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
		1.3374	1.2879	1.2535	1.1067	1.0866	1.0738	1.3125	1.2750	1.2466
		1.3374	1.2879	1.2535	1.1067	1.0866	1.0738	1.3125	1.2750	1.2466
		1.1369	1.1492	1.1340	1.0189	1.0263	1.0217	1.1485	1.1607	1.1485
		1.0176	1.0245	1.0294	0.9770	0.9813	0.9841	1.0436	1.0504	1.0555
		0.9742	0.9780	0.9806	0.9876	0.9899	0.9914	0.9931	0.9967	0.9994
		0.9891	0.9907	0.9918	0.9948	0.9958	0.9964	0.9889	0.9903	0.9915
30	40	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
30	45	35	1.1767	1.1388	1.1176	1.0427	1.0290	1.0251	1.1889	1.1564	1.1361
		40	1.1767	1.1388	1.1176	1.0427	1.0290	1.0251	1.1889	1.1564	1.1361
		45	1.1767	1.1388	1.1176	1.0427	1.0290	1.0251	1.1889	1.1564	1.1361
		50	1.0401	1.0454	1.0368	0.9908	0.9937	0.9945	1.0659	1.0171	1.0630
		55	0.9865	0.9894	0.9910	0.9950	0.9966	0.9970	1.0049	1.0080	1.0099
		60	0.9943	0.9955	0.9962	0.9979	0.9986	0.9987	0.9937	0.9950	0.9958
30	50	65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		35	1.0569	1.0366	1.0325	1.0000	1.0000	1.0000	1.0847	1.0636	1.0545
		40	1.0569	1.0366	1.0325	1.0000	1.0000	1.0000	1.0847	1.0636	1.0545
		45	1.0569	1.0366	1.0325	1.0000	1.0000	1.0000	1.0847	1.0636	1.0545
		50	1.0569	1.0366	1.0325	1.0000	1.0000	1.0000	1.0847	1.0636	1.0545
		55	0.9956	0.9972	0.9975	1.0000	1.0000	1.0000	1.0148	1.0168	1.0137
35	35	60	0.9981	0.9988	0.9989	1.0000	1.0000	1.0000	0.9978	0.9987	0.9990
		65	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
		40	1.1691	1.1886	1.1848	1.0284	1.0392	1.0396	1.1836	1.1966	1.1935
		45	1.0511	1.0589	1.0730	0.9799	0.9844	0.9924	1.0827	1.0878	1.0993
		50	0.9725	0.9764	0.9836	0.9576	0.9598	0.9639	1.0095	1.0120	1.0177
		55	0.9521	0.9541	0.9578	0.9787	0.9798	0.9819	0.9766	0.9779	0.9808
35	40	60	0.9806	0.9814	0.9829	0.9916	0.9920	0.9928	0.9827	0.9832	0.9843
		65	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999
		40	1.2565	1.2247	1.1984	1.0776	1.0648	1.0550	1.2454	1.2212	1.1995
		45	1.0863	1.1048	1.0959	1.0001	1.0106	1.0085	1.1068	1.1229	1.1157
		50	0.9903	0.9997	1.0074	0.9679	0.9732	0.9772	1.0214	1.0293	1.0365
		55	0.9612	0.9660	0.9700	0.9838	0.9865	0.9885	0.9826	0.9866	0.9902
35	60	60	0.9843	0.9862	0.9878	0.9936	0.9947	0.9955	0.9851	0.9867	0.9881
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
35	45	40	1.1488	1.1183	1.1000	1.0349	1.0238	1.0206	1.1623	1.1361	1.1185
		45	1.1488	1.1183	1.1000	1.0349	1.0238	1.0206	1.1623	1.1361	1.1185
		50	1.0220	1.0310	1.0247	0.9855	0.9901	0.9914	1.0488	1.0574	1.0508
		55	0.9775	0.9821	0.9848	0.9927	0.9957	0.9957	0.9963	1.0006	1.0035
		60	0.9909	0.9927	0.9938	0.9971	0.9980	0.9983	0.9905	0.9922	0.9934
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999	0.9999
35	50	40	1.0508	1.0328	1.0291	1.0000	1.0000	1.0000	1.0770	1.0582	1.0498
		45	1.0508	1.0328	1.0291	1.0000	1.0000	1.0000	1.0770	1.0582	1.0498
		50	1.0508	1.0328	1.0291	1.0000	1.0000	1.0000	1.0770	1.0582	1.0498
		55	0.9923	0.9950	0.9955	1.0000	1.0000	1.0000	1.0103	1.0134	1.0108
		60	0.9968	0.9979	0.9982	1.0000	1.0000	1.0000	0.9961	0.9974	0.9979
		65	0.9999	1.0000	0.9999	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
35	55	40	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0198	1.0129	1.0114
		45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0198	1.0129	1.0114
		50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0198	1.0129	1.0114
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0198	1.0129	1.0114
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0003	1.0002
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
40	40	45	1.0483	1.0617	1.0603	1.9969	1.0027	1.0029	1.0792	1.0888	1.0868
		50	0.9772	0.9824	0.9919	0.9709	0.9731	0.9774	1.0108	1.0144	1.0225
		55	0.9582	0.9606	0.9649	0.9871	0.9880	0.9900	0.9797	0.9813	0.9849
		60	0.9841	0.9850	0.9867	0.9952	0.9956	0.9963	0.9848	0.9854	0.9867
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
		45	1.1098	1.0896	1.0754	1.0239	1.0166	1.0143	1.1250	1.1077	1.0939
40	45	50	1.0009	1.0133	1.0100	0.9811	0.9869	0.9887	1.0280	1.0392	1.0354
		55	0.9690	0.9747	0.9787	0.9916	0.9941	0.9950	0.9873	0.9923	0.9963
		60	0.9882	0.9904	0.9919	0.9969	0.9978	0.9981	0.9876	0.9895	0.9910
		65	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	1.0000	0.9999
		45	1.1098	1.0896	1.0754	1.0239	1.0166	1.0143	1.1250	1.1077	1.0939
		50	1.0009	1.0133	1.0100	0.9811	0.9869	0.9887	1.0280	1.0392	1.0354

TABLE B2—Continued

ENTRY AGE <i>y</i>	VESTING AGE <i>z</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
40	50	45	1.0424	1.0275	1.0243	1.0000	1.0000	1.0000	1.0661	1.0506	1.0433
		50	1.0424	1.0275	1.0243	1.0000	1.0000	1.0000	1.0661	1.0506	1.0433
		55	0.9880	0.9922	0.9931	1.0000	1.0000	1.0000	1.0043	1.0088	1.0069
		60	0.9954	0.9970	0.9973	1.0000	1.0000	1.0000	0.9940	0.9957	0.9965
		65	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
40	55	45	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182	1.0119	1.0105
		50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182	1.0119	1.0105
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0182	1.0119	1.0105
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9992	0.9999	0.9998
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	1.0000	0.9999
40	60	45	1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
		50	1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
		55	1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
		60	1.0000	1.0000		1.0000	1.0000		1.0011	1.0006	
		65	1.0000	1.0000		1.0000	1.0000		0.9999	0.9999	
45	45	50	0.9909	0.9993	0.9998	0.9859	0.9889	0.9909	1.0186	1.0252	1.0244
		55	0.9703	0.9732	0.9778	0.9933	0.9963	0.9970	0.9867	0.9889	0.9933
		60	0.9900	0.9910	0.9925	0.9984	0.9988	0.9990	0.9886	0.9893	0.9908
		65	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
		50	1.0296	1.0196	1.0172	1.0000	1.0000	1.0000	1.0499	1.0393	1.0334
45	50	55	0.9836	0.9892	0.9905	1.0000	1.0000	1.0000	0.9972	1.0030	1.0021
		60	0.9945	0.9963	0.9968	1.0000	1.0000	1.0000	0.9921	0.9940	0.9951
		65	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999

TABLE B2—Continued

ENTRY Age <i>y</i>	VESTING Age <i>z</i>	ATTAINED Age <i>x</i>	EMPLOYMENT TURNOVER TABLE VIII			EMPLOYMENT TURNOVER TABLE IX			EMPLOYMENT TURNOVER TABLE X		
			Vesting Conditions			Vesting Conditions			Vesting Conditions		
			1	2	3	1	2	3	1	2	3
45	55	50	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0158	1.0105	1.0092
		55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0158	1.0105	1.0092
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9983	0.9993	0.9992
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
45	60	50	1.0000	1.0000		1.0000	1.0000		1.0010	1.0005	
		55	1.0000	1.0000		1.0000	1.0000		1.0010	1.0005	
		60	1.0000	1.0000		1.0000	1.0000		1.0010	1.0005	
		65	1.0000	1.0000		1.0000	1.0000		0.9999	0.9999	
50	50	55	0.9843	0.9882	0.9902	1.0000	1.0000	1.0000	0.9972	1.0005	1.0006
		60	0.9960	0.9970	0.9975	1.0000	1.0000	1.0000	0.9936	0.9944	0.9956
		65	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	0.9999	1.0000	0.9999
50	55	55	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0119	1.0080	1.0070
		60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9973	0.9985	0.9986
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
50	60	55	1.0000	1.0000		1.0000	1.0000		1.0009	1.0005	
		60	1.0000	1.0000		1.0000	1.0000		1.0009	1.0005	
		65	1.0000	1.0000		1.0000	1.0000		0.9999	1.0000	
55	55	60	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9981	0.9988	0.9989
		65	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
55	60	60	1.0000	1.0000		1.0000	1.0000	1.0000	1.0007	1.0004	
		65	1.0000	1.0000		1.0000	1.0000	1.0000	0.9999	0.9999	
60	60	65	1.0000	1.0000		1.0000	1.0000	1.0000	0.9999	0.9999	

APPENDIX C

TABLE C1

NON-SELECT EMPLOYMENT TURNOVER TABLES

AGE	TABLE					
	I	II	IV	VI	IX	X
20.....	0.054384	0.054000	0.108000	0.216000	0.330000	0.079384
21.....	.054083	.050900	.101800	.203600	.318000	.078989
22.....	.053344	.048000	.096000	.192100	.297000	.078587
23.....	.052213	.045350	.090700	.181400	.273000	.078169
24.....	.050732	.042900	.085800	.171500	.252000	.077723
25.....	.048948	.040600	.081200	.162400	.231000	.077242
26.....	.046904	.038350	.076700	.153300	.211000	.076700
27.....	.044648	.036150	.072300	.144600	.191000	.075912
28.....	.042220	.034050	.068100	.136200	.172000	.074890
29.....	.039665	.032050	.064100	.128200	.156000	.073654
30.....	.037020	.030150	.060300	.120600	.140000	.072219
31.....	.034319	.028350	.056700	.113400	.123000	.070603
32.....	.031593	.026600	.053200	.106500	.109000	.068829
33.....	.028866	.025050	.050100	.100100	.097000	.066916
34.....	.026160	.023550	.047100	.094200	.086000	.064888
35.....	.023492	.022150	.044300	.088600	.075000	.062764
36.....	.020879	.020850	.041700	.083400	.066000	.060572
37.....	.018335	.019650	.039300	.078500	.058000	.058332
38.....	.015875	.018500	.037000	.074000	.051000	.056066
39.....	.013518	.017450	.034900	.066900	.046000	.053786
40.....	.011283	.016550	.033100	.066300	.040000	.051504
41.....	.009186	.015650	.031300	.062700	.035000	.049206
42.....	.007250	.014750	.029500	.059000	.031000	.046881
43.....	.005500	.013800	.027600	.055300	.028000	.044533
44.....	.003960	.012900	.025800	.051700	.025000	.042159
45.....	.002653	.012000	.024000	.048000	.022000	.039753
46.....	.001595	.011100	.022200	.044300	.020000	.037270
47.....	.000798	.010150	.020300	.040500	.017000	.034650
48.....	.000274	.009150	.018300	.036500	.013000	.031850
49.....	.000015	.008100	.016200	.032400	.013000	.028841
50.....	0.000000	.006950	.013900	.027900	0.000000	.025627
51.....		.005800	.011600	.023100		.022260
52.....		.004500	.009000	.018000		.018837
53.....		.003100	.006200	.012500		.015472
54.....		.001600	.003200	.006500		.012288
55.....		0.000000	0.000000	0.000000		.009394
56.....						.006847
57.....						.004713
58.....						.003024
59.....						.001767
60.....						.000901
61.....						.000367
62.....						.000094
63.....						.000000
64.....						.000000
65.....						0.000000

TABLE C2
SELECT EMPLOYMENT TURNOVER TABLES

DURATION	ENTRY AGE y						
	20	25	30	35	40	45	50
Employment Turnover Table III							
0.....	0.081250	0.068750	0.056250	0.037500	0.025000	0.016250	0.010000
1.....	.073750	.061250	.048750	.031250	.022500	.015000	.008750
2.....	.061250	.050000	.037500	.025000	.018750	.012500	.006250
3.....	.050000	.038750	.030000	.022500	.016250	.010000	.003750
4.....	.043750	.032500	.025000	.020000	.015000	.008750	.002500
5.....	0.040600	0.030150	0.022150	0.016550	0.012000	0.006950	0.000000
(Same as Employment Turnover Table II at attained ages $y+5$ and over)							
Employment Turnover Table V							
0.....	0.162500	0.137500	0.112500	0.075000	0.050000	0.032500	0.020000
1.....	.147500	.122500	.097500	.062500	.045000	.030000	.017500
2.....	.122500	.100000	.075000	.050000	.037500	.025000	.012500
3.....	.100000	.077500	.060000	.045000	.032500	.020000	.007500
4.....	.087500	.065000	.050000	.040000	.030000	.017500	.005000
5.....	0.081200	0.060300	0.044300	0.033100	0.024000	0.013900	0.000000
(Same as Employment Turnover Table IV at attained ages $y+5$ and over)							
Employment Turnover Table VII							
0.....	0.325000	0.275000	0.225000	0.150000	0.100000	0.065000	0.040000
1.....	.295000	.245000	.195000	.125000	.090000	.060000	.035000
2.....	.245000	.200000	.150000	.100000	.075000	.050000	.025000
3.....	.200000	.155000	.120000	.090000	.065000	.040000	.015000
4.....	.175000	.130000	.100000	.080000	.060000	.035000	.010000
5.....	0.162400	0.120600	0.088600	0.066300	0.048000	0.027900	0.000000
(Same as Employment Turnover Table VI at attained ages $y+5$ and over)							
Employment Turnover Table VIII							
0.....	0.276000	0.234000	0.191000	0.127000	0.085000	0.055000	0.034000
1.....	.255000	.214000	.171000	.109000	.079000	.050000	.031000
2.....	.220000	.180000	.135000	.090000	.067000	.045000	.024000
3.....	.190000	.150000	.111000	.080000	.061000	.040000	.016000
4.....	.175000	.134000	.100000	.075000	.057000	.036000	.010000
5.....	0.162400	0.120600	0.088600	0.066300	0.048000	0.027900	0.000000
(Same as Employment Turnover Table VI at attained ages $y+5$ and over)							

APPENDIX D

TABLE D1

ENTRY AGE LEVEL ANNUAL COSTS AND ACCRUED LIABILITIES WITHOUT PROVISION FOR VESTING MALE LIVES

ENTRY AGE <i>y</i>	ATTAINED AGE <i>z</i>	EMPLOYMENT TURNOVER TABLES									
		I	II	III	IV	V	VI	VII	VIII	IX	X
20]		32.6048	30.8724	30.5665	19.7587	19.1523	6.2112	5.4533	5.8081	5.4549	13.8770
		Entry Age Level Annual Cost (<i>NC</i>) _{<i>y</i>}									
20	25	216.1438	200.6975	205.2709	151.0471	157.7626	68.7789	73.8032	71.4509	96.3636	100.5622
	30	537.0254	484.4697	489.1909	404.6855	412.2200	238.4539	244.8051	241.8311	398.5051	279.3509
	35	962.6173	857.8476	862.5178	774.3128	782.2811	556.7439	564.2427	560.7307	911.6164	578.7530
	40	1,471.5178	1,330.9038	1,335.3249	1,271.3588	1,279.3109	1,061.5007	1,069.7279	1,065.8737	1,534.3648	1,039.8009
	45	2,042.7475	1,923.0906	1,927.0753	1,915.6070	1,923.0941	1,790.8758	1,799.3016	1,795.3526	2,225.2426	1,696.7959
	50	2,701.8534	2,660.0328	2,663.3869	2,719.5988	2,726.1185	2,738.5200	2,746.3715	2,742.6889	3,004.6207	2,567.1163
	55	3,543.6437	3,557.7545	3,560.2482	3,648.2818	3,653.2084	3,758.6083	3,764.7484	3,761.8646	3,764.7311	3,595.1696
	60	4,673.0522	4,680.8950	4,682.2824	4,731.2124	4,733.9417	4,792.5174	4,795.9061	4,794.3076	4,795.8936	4,751.2251
	65	6,262.1935	6,262.1924	6,262.1957	6,262.1923	6,262.1713	6,262.1512	6,262.0957	6,262.1060	6,262.0890	6,262.1600
		Accrued Liability (<i>AL</i>) _{<i>y</i>} , <i>z</i>									

NOTE.—Benefit equal to one unit per year of service from entry age to age 65.

TABLE D1—Continued

ENTRY AGE <i>y</i>	ATTAINED AGE <i>x</i>	EMPLOYMENT TURNOVER TABLES									
		I	II	III	IV	V	VI	VII	VIII	IX	X
25		Entry Age Level Annual Cost (NC) _y									
	42.1595	39.3759	39.0111	29.6850	28.9128	14.7418	13.3691	13.8285	19.8371	21.3611	
25	30	Accrued Liability (AL) _{y, x}									
	35	271.0614	246.4818	252.1120	209.0651	218.6611	134.6586	146.1662	142.3149	215.8301	153.1699
	40	644.3013	580.3687	585.9382	528.9254	539.0748	403.5884	417.1792	412.6306	627.9533	413.4794
	45	1,105.2939	1,010.5914	1,015.8639	971.0343	981.1643	843.3469	858.2644	853.2716	1,164.7115	820.1419
	50	1,635.7917	1,554.0138	1,558.7659	1,552.9511	1,562.4907	1,489.1889	1,504.4769	1,499.3600	1,786.1639	1,406.1810
	55	2,254.6696	2,233.6895	2,237.6896	2,286.8950	2,295.2048	2,338.4293	2,352.6908	2,347.9172	2,503.6439	2,189.4271
	60	3,042.5650	3,065.2430	3,068.2172	3,144.1782	3,150.4614	3,265.8972	3,277.0732	3,273.3317	3,224.3931	3,123.6220
30		Entry Age Level Annual Cost (NC) _y									
	52.0399	48.4298	48.0308	40.6931	39.8342	26.9537	25.1653	25.7758	37.8091	31.4054	
30	35	Accrued Liability (AL) _{y, x}									
	40	320.7631	294.4877	300.5777	269.3175	280.6082	213.9799	231.6876	225.6419	300.5931	219.5632
	45	734.0621	682.3255	688.0900	656.5156	667.7854	585.1776	604.6147	597.9780	747.3268	570.9423
	50	1,224.3911	1,177.7690	1,182.9637	1,176.9276	1,187.5414	1,146.4871	1,166.4081	1,159.6052	1,301.0988	1,086.6020
	55	1,803.8579	1,801.3133	1,805.6848	1,842.5455	1,851.7925	1,900.0681	1,918.6535	1,912.3053	1,962.5824	1,785.5005
	60	2,538.8393	2,568.2470	2,571.4955	2,631.2665	2,638.2606	2,743.1818	2,757.7489	2,752.7709	2,654.7574	2,631.6098
	65	3,513.7994	3,530.1459	3,531.9505	3,565.1737	3,569.0599	3,627.3795	3,635.4762	3,632.7056	3,578.2300	3,602.1855

TABLE D1—Continued

ENTRY Age <i>y</i>	ATTAINED Age <i>x</i>	EMPLOYMENT TURNOVER TABLES									
		I	II	III	IV	V	VI	VII	VIII	IX	X
		Entry Age Level Annual Cost (NC) _y									
35		61.7472	58.0475	57.7457	52.4397	51.7976	41.6269	40.2367	40.8857	53.5813	43.7423
Accrued Liability (AL) _{y, z}											
35	40	365.4913	345.9122	350.2736	332.3041	340.7299	300.2558	315.3651	308.3113	359.1745	295.2927
	45	815.3522	794.1812	798.1120	791.7739	799.7095	776.3651	791.8507	784.6211	844.1896	741.0910
	50	1,354.9735	1,362.7568	1,366.0655	1,390.2398	1,397.1539	1,436.1228	1,450.5704	1,443.8253	1,446.0521	1,358.0862
	55	2,036.5194	2,066.6569	2,069.1167	2,112.3341	2,117.5643	2,200.4102	2,211.7345	2,206.4475	2,103.0349	2,121.2825
	60	2,934.2177	2,950.9700	2,952.3382	2,976.3576	2,979.2649	3,025.3126	3,031.6073	3,028.6683	2,971.1887	3,011.4943
	65	4,174.7942	4,174.7965	4,174.7989	4,174.7942	4,174.7943	4,174.7939	4,174.7945	4,174.7936	4,174.7937	4,174.7994
Entry Age Level Annual Cost (NC) _y											
40		71.2544	68.3227	68.0724	64.8032	64.2766	57.7111	56.5682	57.0633	67.1732	57.7818
Accrued Liability (AL) _{y, z}											
40	45	409.0477	402.0334	405.2916	398.9992	405.5063	390.5283	403.2588	397.7431	415.1928	376.3213
	50	908.3229	916.9971	919.7387	931.2956	936.9645	957.5176	969.3946	964.2482	953.8442	913.2282
	55	1,535.8320	1,559.7143	1,561.7510	1,588.3823	1,592.6698	1,646.1507	1,655.4595	1,651.4253	1,569.0773	1,597.3521
	60	2,355.5456	2,368.8214	2,369.9520	2,384.7556	2,387.1375	2,416.8644	2,422.0380	2,419.7947	2,374.0254	2,413.0983
	65	3,478.9977	3,479.0009	3,478.9974	3,479.0003	3,478.9973	3,478.9991	3,478.9981	3,478.9959	3,479.0003	3,478.9980

TABLE D1—Continued

ENTRY AGE <i>y</i>	ATTAINED AGE <i>z</i>	EMPLOYMENT TURNOVER TABLES									
		I	II	III	IV	V	VI	VII	VIII	IX	X
Entry Age Level Annual Cost (NC) _y											
45		80.9624	79.3569	79.2039	77.6701	77.3531	74.2163	73.5314	73.8104	79.6865	72.8426
		Accrued Liability (AL) _{y, z}									
45	50	459.4316	462.9168	464.5926	466.9265	470.3401	474.5341	481.6514	478.7524	473.6622	457.9072
	55	1,033.5079	1,046.5859	1,047.8310	1,060.3244	1,062.9067	1,088.4577	1,094.0360	1,091.7639	1,043.9004	1,065.2633
	60	1,775.9628	1,783.2321	1,783.9233	1,790.8674	1,792.3028	1,806.5048	1,809.6049	1,808.3424	1,781.7389	1,810.0824
	65	2,783.1990	2,783.1997	2,783.1976	2,783.1974	2,783.1977	2,783.1976	2,783.1966	2,783.1976	2,783.1983	2,783.1961
Entry Age Level Annual Cost (NC) _y											
50		91.6154	91.1974	91.0260	90.7747	90.4269	89.9086	89.1991	89.3179	91.6154	88.1560
		Accrued Liability (AL) _{y, z}									
50	55	523.5998	526.8897	528.2865	530.3328	533.1659	537.3879	543.1668	542.1995	523.4856	531.1586
	60	1,192.3611	1,193.9925	1,194.7694	1,195.9062	1,197.4809	1,199.8277	1,203.0395	1,202.5022	1,192.1008	1,205.9266
	65	2,087.8554	2,087.3987	2,087.3999	2,087.3985	2,087.3986	2,087.3987	2,087.3981	2,087.3987	2,087.3996	2,087.3976

NOTE.—Benefit equal to one unit per year of service from entry age to age 65.

APPENDIX E

 DERIVATION OF ACCRUED LIABILITY INDICES
FOR VESTING CONDITION 1

Since

$$(NC)_y = \frac{(65-y) \ddot{a}_{65}^{(12)} \cdot v^{65-y} \cdot {}_{65-y}p_y^{(T)}}{\ddot{a}_{y:65-y}^{(T)}}$$

and

$$(AL)_{y,x} = (NC)_y \cdot \sum_{k=y}^{x-1} \frac{(1+i)^{x-k}}{x-k} \frac{1}{p_k^{(T)}} = (NC)_y \cdot \ddot{s}_{y:x-y}^{(T)},$$

then

$$(NC)_y^v = (1 + f_y^{z,65}) (NC)_y,$$

where

$$f_y^{z,65} = \sum_{k=z}^{64} \frac{k-y}{65-y} \cdot \frac{q_k^{(ws)} \cdot {}_{64-k}p_{k+1}}{{}_{65-k}p_k^{(T)}}.$$

Thus, for $x > z$

$$\begin{aligned} (AL)_{y,x}^v &= (1 + f_y^{z,65}) (NC)_y \cdot \ddot{s}_{y:x-y}^{(T)} \\ &\quad - \sum_{k=z}^{x-1} \left[\frac{k-y}{65-y} \cdot \frac{{}_{64-k}p_{k+1} \cdot q_k^{(ws)}}{{}_{65-k}p_k^{(T)}} \right] \left[(65-y) \ddot{a}_{65}^{(12)} \cdot {}_{65-x}p_x^{(T)} \cdot v^{65-x} \right]. \end{aligned}$$

If $(AL)_{y,x}^v$ is divided by $(AL)_{y,x}$, then

$$\begin{aligned} (I)_{y,x}^{AL} &= 1 + f_y^{z,65} - \sum_{k=z}^{x-1} \frac{k-y}{65-y} \cdot \frac{{}_{64-k}p_{k+1} \cdot q_k^{(ws)}}{{}_{65-k}p_k^{(T)}} \cdot \frac{\ddot{a}_{y:65-y}^{(T)}}{\ddot{a}_{y:x-y}^{(T)}} \\ &= 1 + f_y^{z,65} - f_y^{z,x} \cdot \frac{\ddot{a}_{y:65-y}^{(T)}}{\ddot{a}_{y:x-y}^{(T)}} \\ &= \frac{(1 + f_y^{z,x} + f_y^{x,65}) \ddot{a}_{y:x-y}^{(T)} - f_y^{z,x} \cdot \ddot{a}_{y:65-y}^{(T)}}{\ddot{a}_{y:x-y}^{(T)}} \\ &= \frac{\ddot{a}_{y:x-y}^{(T)} + f_y^{x,65} \cdot \ddot{a}_{y:x-y}^{(T)} - f_y^{z,x} [\ddot{a}_{y:65-y}^{(T)} - \ddot{a}_{y:x-y}^{(T)}]}{\ddot{a}_{y:x-y}^{(T)}} \end{aligned}$$

$$= 1 + f_y^{x, 65} - f_y^{z, x} \left(\frac{v^{x-y} \cdot {}_{x-y}p_y^{(T)} \cdot \ddot{a}_{x; \overline{65-x}|}^{(T)}}{\ddot{a}_{y; \overline{x-y}|}^{(T)}} \right)$$

$$= 1 + f_y^{x, 65} - f_y^{z, x} \cdot \frac{\ddot{a}_{x; \overline{65-x}|}^{(T)}}{\ddot{s}_{y; \overline{x-y}|}^{(T)}}.$$

DISCUSSION OF PRECEDING PAPER

WILLIAM K. WHITE:

In my discussion of the paper by Mr. Marples, entitled "Cost of Vesting in Pensions," it is stated that many of the comments would apply to a considerable extent equally to Mr. McGinn's paper.¹ This applies particularly to the comments that we are especially fortunate to have papers at this time presenting somewhat different mathematical techniques and that the concepts presented will be useful timesaving tools to many actuaries for their pension plan valuations. There are two additional comments that I would like to make.

First, while it is realized that it was not within the scope of what Mr. McGinn intended to cover, some discussion of why figures were developed for a number of different turnover assumptions would unquestionably be of interest to a layman who might read his paper. This, of course, would include a commentary on why a particular table would be used in a given situation, pointing out that, although the selection of the table will influence the level of deposits from time to time to provide for vesting, the actual experience under the plan will determine the real cost. The obvious objective is to select that table which seems, under the circumstances in a particular case, to be most likely to come closest to what may be expected in the way of actual experience.

I was particularly interested in Mr. McGinn's study of the impact of changes in mortality and interest assumptions on the estimated cost of vesting. His conclusion—that in both cases the degree of error introduced by applying the same set of factors to rather widely divergent assumptions is well within the degree of error one reasonably expects in the turnover assumption itself—confirms a suspicion that we had had but had not previously seen verified.

DOUGLAS R. BUTT:

When valuing the liabilities of a pension plan, the actuary cannot hope to produce precisely accurate forecasts, and thus any time that he spends trying to develop accurately, to the last decimal point, the cost of minor benefits is largely wasted. I am, therefore, always pleased to read a paper such as Mr. McGinn's, which provides the material for useful rules of thumb or approximate calculations.

¹ See pp. 277 ff.

My comments on the paper are minor and probably quite obvious to those actuaries actively involved with pension matters. They may, however, be of use to others, not as familiar with this field, who may decide to use Mr. McGinn's indices.

I believe that the indices can be used, regardless of the funding technique used in the valuation, since entry age normal costs, as Mr. McGinn states, should be a reasonable measure of long-range costs. If, say, the unit credit cost technique is used in the valuation and further indices are developed under this technique, these latter indices should be clearly identified as representing changes in the required deposit rates. The entry age normal indices should still give a reasonable indication of what the long term will bring.

Mr. McGinn's formulas cannot be used for contributory pension plans, as his no-vesting normal cost formulas do not allow for withdrawal values equal to the employees' contributions, usually with interest.

A feature of contributory plans, which would make a revision of these formulas of little value, is the provision that an employee may withdraw his contributions in lieu of taking a vested pension. It is very likely that all younger employees and most older ones will take their cash, even to their financial disadvantage. Various surveys bear this out. (This observation does not, of course, apply to legislated vesting, of the form emerging in the various provinces of Canada—compulsory vesting and locking-in of employee contributions after age 45 and ten years of service. However, the cost of such vesting is somewhat academic, as the employer has no choice in the matter.)

Thus the employer's cost of noncompulsory vesting under a contributory plan is the excess of the cost of the vested benefit over the employee's accumulated contributions, when the employee decides to leave his contributions in the fund. Under most plans the employee's accumulated contributions are more than enough to purchase his pension accrual up to age 40 and in many cases to higher ages. Considering the relatively low incidence of termination after age 40, and the high likelihood that terminating employees will take their cash, the conclusion one reaches is that, for most normal contributory plans, any reasonable vesting provision can be included at virtually no cost to the employer.

CHARLES E. FARR:

Dan McGinn is to be complimented for bringing us the results of his work. He has defined and derived expressions for the additional estimated cost associated with the inclusion of a vesting provision in a pension plan. These expressions are termed cost indices.

Mr. McGinn has limited his investigation to a single actuarial cost method and to a benefit formula under which the earned retirement benefit increases uniformly with service. Within these boundaries, he has produced a considerable volume of numerical cost indices, using ten different turnover assumptions, three basic types of vesting schedules, one mortality assumption, one interest assumption, one retirement-age assumption, and various combinations of entry age, vesting age, and attained age. Further, he demonstrates that changes in the mortality assumption or the interest assumption have a minor effect on the indices.

The projected benefit actuarial cost method dealt with by Mr. McGinn is individual level cost with supplemental liability (entry age normal cost method). He derives two cost indices, one applicable to the level cost and the other applicable to the supplemental liability. His example, demonstrating how the indices can be used, shows how the level cost indices are applied to the level costs computed without provision for vesting, grouped by entry age, and how the supplemental liability indices are applied to the supplemental liabilities computed without provision for vesting, grouped by entry age and attained age. The approach developed by Mr. McGinn uses a precise and detailed calculation of an estimated cost under a particular actuarial cost method. It may be helpful to mention briefly a modification of his approach which is less detailed in application but which can be used with several actuarial cost methods.

Projected benefit actuarial cost methods generally involve the calculation of the present value of the normal retirement benefit. It is proposed here to adjust for the estimated additional value of the superimposed vesting provision by applying a single factor to the present value of the normal retirement benefit. The use of a single factor rather than a whole family of factors is made possible by assuming a single entry age. The nature of this factor is clear, being the ratio, at the assumed single age of entry, of (*a*) the present value of vested benefits originating upon the assumed future occurrence of employee withdrawal to (*b*) the present value of the deferred normal retirement benefit.

These factors are already calculated by Mr. McGinn and are shown in his Appendix B tables of Entry Age Level Annual Cost Indices. His Entry Age 30, Vesting Condition 1, and Employment Turnover Tables I and IV will be taken as examples. These cost indices, reduced in each case by unity in order to represent the additional cost, are reproduced in the table on page 246.

When a plan begins, some employees will be younger than the age at which vesting commences and some will be older. It should be clear that the percentage opposite the vesting age in the table applies to those em-

ployees at or below the vesting age but that the percentage opposite the attained age in the table applies to those employees older than the vesting age. In other words, the table could be entered with the greater of the employee's attained age or his age when vested. For example, if vesting occurs after fifteen years (assumed to be at age 45 since assumed entry age is 30), the 0.0816 figure in the Table IV column is appropriate on the effective date of the plan not only for new entrants but also for all employees age 45 or younger. For employees already age 50 on the effective date, the 0.0276 figure applies, and so on.

Although it may at first seem appropriate to proceed as above in using the lower percentages for those in the initial group older than the age at which vesting occurs, the long-range estimated cost of vesting associated with specified assumptions of turnover and entry age is best represented

FOR VESTING AFTER:		ADDITIONAL COST OF VESTING, ASSUMING ENTRY AGE 30	
Years	Age	Table I	Table IV
0	30	0.0445	0.2292
5	35	.0344	.2030
10	40	.0150	.1447
15	45	.0024	.0816
20	50	0.0000	0.0276

by the single percentage opposite the vesting age. The estimated cost for new entrants becomes dominant as the effect of the initial employee group wears off.

To implement this method, then, the factors of the type shown above are applied to the present value of normal retirement benefits appropriately grouped. If the long-range view is taken, a single factor is applied to the total present value of normal retirement benefits. The result is the estimated additional present value due to the vesting provision.

As brought out in Mr. McGinn's paper, factors can be calculated for many different vesting provisions. Factors for full vesting after n years of service are illustrated above. Factors for partial vesting after n years, graded up to full vesting over an additional period of years, are easily available. Under the assumption of a single entry age, a vesting provision which involves both years of service and attainment of a given age can be reduced to a single parameter. These types account for most vesting provisions.

Although the assumption that the accrued pension (in which there is

partial or full vesting) is proportionate to service will permit the application of this method to many plans which are a function of service only, it nevertheless restricts its application to some extent. A large proportion of existing and new plans involves salary as well as service in the benefit formula. In those valuations in which a salary-increase assumption is not employed, accrued pension is assumed to be proportionate to service, and the method is applicable. Actuarial losses will appear to the extent that the assumption of level salary fails. However, when a salary increase assumption is felt necessary, appropriate modifications can be made in the calculation of the factors, if desired, in order to reflect the less-than-proportionate benefit accrual. The modification is greater under a final average salary benefit formula than under a career average salary benefit formula because of the greater disparity in the former case between the normal retirement benefit based on final average salary and the retirement benefit accrued for vesting purposes based on salary at or just prior to withdrawal.

In conclusion, I wish to express my appreciation to Dan McGinn for his timely examination of one aspect of pension costs.

EDWIN F. BOYNTON:

The tables presented in Mr. McGinn's paper should be useful to all pension actuaries in estimating the cost of vesting, particularly in indicating the relative cost of alternative vesting schedules. The paper presents an approach to the cost of vesting that we have been using for several years in The Wyatt Company. About six or seven years ago we were casting about for some fairly simple approach for estimating vesting costs, particularly for negotiation purposes. We arrived at the same conclusion as Mr. McGinn did—that the present value of prospective vested benefits, expressed as a percentage of the corresponding age retirement value, is independent of the interest rate, making it possible to derive a single set of factors for each service table in common use.

Our approach was somewhat simpler than proposed by Mr. McGinn. For one thing, we discovered, as did the author, that the application of the factors would produce a relatively large increase in normal cost but would often result in a net decrease in the accrued liability function. However correct it might be theoretically, it is sometimes awkward to explain to an unsophisticated employer or at the bargaining table why an improvement in vested benefits reduces the accrued liability under the plan. Therefore, we determined "vesting loading factors," as we call them, for the present value of benefits only. We commonly apply the factors only to the present value of age benefits and determine the total present value

of future vested benefits as a percentage of total present value of age benefits. The percentage so determined is applied uniformly to both normal cost and accrued liability items. The net effect on the contribution level, including amortization of the accrued liability, is probably about the same as using separate factors for normal cost and accrued liability.

We also applied the vesting loading factors to sample age distributions to get some measure of the effect on vesting costs of the age distribution, withdrawal assumption, and vesting eligibility conditions. Table 1 shows illustrative cost increases for vesting as a percentage of age retirement costs for males. The "young" group has an average age of 36; the "average" group, 41; and the "old" group, 45. Scale 1 turnover starts at 7.5 per cent turnover at age 20; Scale 2 at 15 per cent; Scale 3 at 22.5 per cent, and Scale 4 at 30 per cent, all grading down to no turnover around age 50 or so.

TABLE 1
PERCENTAGE INCREASE IN NORMAL RETIREMENT COSTS FOR VESTING

VESTING REQUIREMENTS		WITHDRAWAL SCALE			
Age	Service	1	2	3	4
Young Group (Age 36)					
Any	10	3.4%	8.0%	12.5%	19.8%
Any	15	1.4	3.7	5.8	9.3
40	10	2.3	6.2	10.2	14.8
40	15	1.3	3.6	5.7	8.5
45	15	0.7	2.4	4.3	6.4
Average Group (Age 41)					
Any	10	1.9%	4.7%	7.5%	10.0%
Any	15	0.7	2.1	3.4	4.6
40	10	1.5	4.1	6.7	9.2
40	15	0.7	2.0	3.4	4.6
45	15	0.5	1.6	2.9	4.1
Old Group (Age 45)					
Any	10	1.0%	2.5%	3.9%	5.3%
Any	15	0.4	1.1	1.9	2.6
40	10	0.8	2.2	3.7	5.0
40	15	0.3	1.1	1.9	2.6
45	15	0.2	0.9	1.7	2.4

Before deriving the vesting factors, we first arrived at the "loading factor" concept in connection with the calculation of disability costs. Several of our offices normally apply entry age normal cost funding for disability benefits. At one point, several years ago, we had developed several new service tables for a wide range of interest rates and did not feel inclined to work out all the commutation functions necessary to calculate disability costs for each separate table. After some testing, we found a scale of loading factors varying by attained age which, when multiplied by the age retirement cost functions, would satisfactorily approximate the corresponding disability cost functions, and we have since used such factors extensively in developing disability costs. We discovered that, as a practical matter, the variation by service table was not particularly significant, since disability costs are negligible at the younger ages, where withdrawal rates vary the widest. Unlike the vesting factors, there is some variation by interest rate, and we adopted a rule of thumb of a fixed percentage increase in the disability loading factor for each $\frac{1}{4}$ per cent increase in interest. The variation by interest rate is due to the lesser impact of interest on a temporary life annuity than on a deferred life annuity.

These so-called loading factors are particularly adaptable to computer valuation programs, and we have built these factors directly into our principal pension-valuation systems. Instead of calculating the entire cost function from several stored commutation columns, only a single multiplication is required to multiply the age retirement cost by the applicable factor. It saves a considerable amount of computer storage and simplifies coding of tables for development of disability and vesting costs.

Despite the usefulness of the vesting loading factors or indices, in the majority of our cases we empirically adjust for vesting costs by selection of conservative withdrawal rates, and the principal use of the vesting factors has been in the derivation of the costs of alternative vesting schedules. Except under the most stable of employment conditions, no great credence should be given to the dollar level of vesting costs, per se, determined by any actuarial formula.

ROBERT F. LINK:

Mr. McGinn's paper is an interesting collection of tables and mathematics. Actuaries who wish to avoid voluminous calculations may find some useful material for horseback guesses.

When we read papers on vesting costs and the associated mathematical techniques, our thoughts are drawn to the paradox of such costs. The actuary tends to be conservative by nature and therefore uses termination

rates for pension plans that are lower than rates he actually expects. This produces age pension costs that are higher than most probable costs. That is, they are conservative. When he is asked to quote the cost of vesting, he falls into a trap; use of the same turnover rates produces a cost of vesting that is lower than the expected cost.

When an actuary uses the unit credit cost method, he frequently omits entirely any allowance for withdrawal from employment. Thus, as Mr. Marples points out in his paper (pp. 277 ff.), he does not change his cost figures when a vesting provision is introduced. Yet the vesting has a cost, and he may be asked to quote it.

Mr. McGinn shows that the interest assumption does not affect the additional normal cost of a vesting provision when expressed as a percentage of the normal cost of a plan with no vesting. I tried to "general reason" this and decided that I agreed. Mr. McGinn's tables also reveal that the percentage relationship between basic plan normal costs and normal costs for vesting is not affected by the withdrawal rates applicable to the period before vesting commences. This is a little more surprising but plausible after thought.

A major group of funding methods requires a total liability for all future benefits. To use Mr. McGinn's principles, we need factors for future benefit liability, recognizing entry age, attained age, and vesting age. I think that the entry age level annual cost indices of Mr. McGinn's Appendix B may be used for this purpose. Omitting the complications of graded vesting, you would select the factor under Vesting Condition No. 1 on the basis of entry age as given and vesting age equal to the higher of the actual vesting age or attained age. In Mr. McGinn's notation, slightly extended,

$$(I)_{x,y}^{TL} = (I)_y^{NC},$$

taking z equal to the higher of x or vesting age.

Mr. McGinn has used as far as possible the multiple-decrement notation from chapter xii of Jordan's *Life Contingencies*. Jordan says, on page 265:

A uniform notation for multiple-decrement functions has never been generally adopted, and in view of the complexity of the situations analyzed by multiple-decrement techniques, it is probably most convenient to assign symbols according to the nature of each problem under consideration.

Mr. McGinn has followed this advice in developing additional notation. One problem in the development of notation is to decide how much use to make of the corners. That is, what provision should be made for displaying parameters? There are no precise guides, and it is easy to over-

load. However, Mr. McGinn might have considered the merits of a notation that indicates, at least, retirement age (r), the nature of the additional benefit under consideration (B), qualitative description of the additional benefit (i), and age at which vesting commences (z). In this case, the additional benefit is vesting, and the qualitative description is "vesting condition." With this extension, $(I)_y^{NC}$ might become ${}_z(INC)_y^{B_i}$.

It would now be slightly easier to express the total liability, as the following shows:

$${}_{z'}^{65}(ITL)_{x,y}^{V_1} = {}_{z'}^{65}(INC)_y^{V_1},$$

where z' equals the higher of z and x .

Also, this revised notation would encompass other ancillary benefits, such as widows' benefits or disability pensions. The literature on this subject should be surveyed before making any changes. As Jordan suggests, a uniform, complete, and consistent notation is probably unattainable.

(AUTHOR'S REVIEW OF DISCUSSION)

DANIEL F. MCGINN:

I will first comment on the various points raised by those who discussed my paper and then indicate the relatively simple modifications to my paper's formulas which may be made to make the indices adaptable to virtually any situation.

Mr. White has asked why figures were developed for so many different turnover tables and has suggested that I comment on when an actuary should use a specified turnover table. The following are the reasons why so many figures were developed:

1. Since the President's Committee Report on Private Pensions was published, numerous broad-brush statements have been made concerning the cost of vesting. I believe that the variety of tables included in my paper provides a useful reference to demonstrate both the wide range that exists in the relative cost of vesting schedules and the relationship between the relative cost of vesting schedules and the underlying employment turnover tables.

2. I believe that the relationships which exist among several of the turnover tables are such that the tables of indices provide a useful insight into the effect of variations in turnover levels on the relative cost of vesting. For example, among the Nonselect Turnover Tables, Table II has turnover rates equal to one-half the Table IV rates, while the Table IV rates are one-half the Table VI rates. Also, the Select Employment Turnover Tables grade into these Nonselect Turnover Tables. These relationships show both the change in the relative cost of vesting if turnover rates double or quadruple and also demonstrate the minor effect of select turnover rates on the relative cost of vesting.

3. As a practical matter, I think that these tables provide a reasonably wide range of vesting factors useful to pension actuaries without computer facilities and give pension actuaries handy reference tables for making quick approximations of the relative cost of various vesting schedules. Such tables may be very valuable at the bargaining table in evaluating alternative vesting schedules.

With regard to when a specific turnover table should be used, the choice must depend upon the judgment of the actuary. Since employment turnover is extremely volatile and greatly influenced by the movement of the economy, neither past nor present turnover experience of an employed group is necessarily a proper criterion in choosing the correct turnover table. However, since most actuaries must rely upon prior experience in establishing the actuarial basis of a pension plan, usually a pension actuary will attempt to review an employer's particular turnover experience and analyze current trends—both geographically and in the industry—to arrive at a reasonable index to the level of a company's turnover and to fix the effect of age and service on the incidence of turnover rates. Unfortunately, the actuary must usually base his choice of turnover rates on an employer's impressions rather than on statistical information.

As difficult as it is to establish a reasonable index to the level of turnover for an employer group, the difficulties are often compounded when union-negotiated pension plans are involved, because turnover experience must be measured by the proportion of accrued pension credits forfeited on account of breaks in service. Sometimes a break in service occurs when no hours are worked in one, two, or three plan years; in other instances a break in service occurs when 500 hours are not worked in a two-year period; and so forth. In addition, the existence of reciprocity agreements may preclude the forfeiture of accrued pension credits.

Because of these reasons and many other factors, no pension actuary can predict the actual cost of vesting; rather, all an actuary can do is to adopt that specific turnover table which reflects the nature of the plan, the covered group of employees, and the composition of plan provisions. After a given plan has been in existence for several years, the pension actuary must modify his assumptions to reflect actual emerging experience.

Perhaps in this discussion it will be useful to demonstrate concretely the range in which an actuary's cost estimates may fall, depending upon his choice of turnover table and vesting schedule. If we refer to the numerical illustration included in my paper, the "no vesting" level annual cost is \$3,583, if Turnover Table IV is appropriate. If a 100 per cent immediate vesting schedule is adopted (i.e., Vesting Condition 1, vesting

age = entry age), the resulting level annual cost is \$4,595. In other words, the addition of a 100 per cent immediate vesting schedule produces a "maximum" cost equal to 128 per cent of the "minimum" cost ($= \$4,595 \div \$3,583$). If this Table IV "minimum" cost is represented by an index of 100, it is now obvious that the level annual costs of all possible vesting schedules for Table IV will fall within the range 100-128. By making similar calculations for all turnover tables shown in Appendix C of the paper and relating all costs to the "minimum" level annual cost for Turnover Table IV (with an index of 100), we develop the range of cost indices shown in the accompanying tabulation for this particular hypothetical group of participants.

EMPLOYMENT TURNOVER TABLE	INDEX TO LEVEL ANNUAL COST		RATIO (=MAXI- MUM COST÷ MINIMUM COST)
	Minimum Cost (No Vesting)	Maximum Cost (100 Per Cent Immediate Vesting)	
I.....	132	141	107%
II.....	124	139	112
III.....	123	138	112
IV.....	100	128	128
V.....	98	126	129
VI.....	64	111	173
VII.....	61	104	170
VIII.....	62	106	171
IX.....	81	111	137
X.....	78	120	154

This tabulation shows that the selection of a particular turnover table from among the ten tables shown in Appendix C results in "no vesting" costs ranging from an index of 61 to an index of 132. Likewise, in this illustration the 100 per cent immediate vesting costs range from an index of 104 to an index of 141, and a choice of alternative vesting schedules can increase estimated maximum vesting costs from 7 to 73 per cent, depending upon which turnover table is chosen.

The foregoing calculations demonstrate the dramatic impact of variations in turnover assumptions on the total estimated pension plan costs and the estimated cost of vesting. However, as is the case with all the actuaries' other assumptions, the vesting factors employed in valuations are estimates; actual experience alone will determine the true cost of vesting.

The value of the mathematical technique presented in my paper is that it permits the actuary to provide for the precise relative costs of

vesting based on his judgment of appropriate and realistic assumptions of employment turnover.

Turning now to the points brought out in the other discussions, I heartily agree with Mr. Butt's comments regarding the cost of vested pension benefits under contributory pension plans. Since virtually every contributory plan involves conditional vesting, our experience has illustrated that, when employees with vested pensions terminate, most employer funds are released because terminating employees prefer to cash-out their accumulated contributions, even though it means forfeiture of the employer-paid portion of their vested pensions. Therefore, most vesting schedules in contributory plans add little to the employer's cost for the retirement benefits.

Mr. Farr suggests that "the long-range estimated cost of vesting associated with specified assumptions of turnover and entry age is best represented by the single percentage opposite the vesting age." If a quick, simple approximation to the cost of vesting is required, a single average entry age approach may be acceptable, but, for purposes of regular annual valuations, I believe that a more precise approach is mandatory since the age/service distribution of a covered group can change radically from one year to another as a result of economic and other factors. Certainly it seems that use of a single average entry age is not appropriate when vesting is a function of service only. Also, with the increasing use of computers in actuarial valuations, more precision can be attained at little additional cost.

Mr. Farr also states that "actuarial losses appear to the extent the assumption of level salary fails." I believe that this statement refers to the fact that the indices developed in my paper were based on accruals of level units of pension credits for each year of service and implies that the use of indices based on a salary scale would produce a more conservative estimate of the cost of vesting. Admittedly, actuarial losses will accrue to any salary plan in which salaries increase more than anticipated. However, as Mr. Marples points out in his paper, the most conservative estimate of the relative cost of vesting is determined by assuming an accrual of a level unit of pension credit each year. Consequently, use of the indices included in my paper with salary plans produces conservative results—reducing the degree of actuarial loss attributable to salary increases.

Mr. Boynton has pointed out the difficulty inherent in explaining why a more liberal vesting schedule may increase the normal cost of a plan substantially while decreasing the accrued liability. I agree that this apparent anomaly may be cumbersome to explain, but I think that a pension

actuary is often faced with other equally difficult explanations. It seems to me that the difficulty lies primarily with the characteristics of the actuarial cost method and that the problem could be eliminated (if necessary) by using another cost method. I do find Mr. Boynton's reference to "loading factors" quite intriguing; it certainly would be helpful if Mr. Boynton would demonstrate the precise approach that he used to develop these.

As useful as Mr. Boynton's "loading factors" may be, the approach described by him in the last paragraph of his discussion is not appropriate. First, it is quite obvious—by simply referring to the tables included in my paper—that choosing a conservative turnover table automatically produces age pension costs that are higher than most probable costs. (This fact was indicated in Mr. Link's discussion.) Consequently, all estimated vesting costs will be lower than the expected costs, and the total of the estimated age pension and vesting costs is too high. Again, Mr. Boynton's last sentence—if accepted—must also be true of the dollar cost of the basic retirement pension. If we cannot convert, with some relative validity, our actuarial assumptions into dollar estimates of costs, then what is the value of actuarial judgment in the field of pension plans?

In addition to his comments about the effects of using conservative turnover tables, Mr. Link has indicated some apparent deficiencies in the symbolism I have used. I agree that the symbolism can be improved and welcome his suggestions.

As I stated in the "Comments" section of my paper, "Numerous modifications can be made to these formulas so that the indices could be developed for virtually any combination of factors and actuarial cost methods."

By generalizing the definition of $f_y^{z,x}$, it can be shown that indices can be developed quite easily to reflect salary factors and different actuarial cost methods. Thus let

$$f_y^{z,x} = \sum_{k=s}^{x-1} (t)_y \cdot \frac{q_k^{(ws)} \cdot Q_{-k-1} p_{k+1}}{Q_{-k} p_k^{(T)}}.$$

With this generalized definition, we can examine (1) the nature of the indices required for the unit credit and aggregate cost methods (including "frozen liability" cost methods) when the benefit is a level unit of pension credit for each year of service and (2) the definition of $(t)_y$ and the method of applying the indices when salary scales and different actuarial cost methods are employed.

Unit Credit Cost Method

Under the unit credit cost method, the number of level units of pension credits earned from entry age to attained age x is $(x - y)$. Thus, when the normal retirement age is Q , the "no vesting" accrued liability at age x for an accrued pension credit is

$$(x - y) \cdot v^{Q-x} \cdot {}_{Q-x}p_x^{(T)} \cdot \ddot{a}_Q^{(12)},$$

and the current-year cost for a level unit of pension credit is

$$v^{Q-x} \cdot {}_{Q-x}p_x^{(T)} \cdot \ddot{a}_Q^{(12)}.$$

By letting $(t)_y = 1$, the vesting indices are determined by the following relationship.

1. For Vesting Condition 1:

$$x \leq z,$$

$$(I)_{y,x} = 1 + f_y^{z,Q}.$$

$$x > z,$$

$$(I)_{y,x} = 1 + f_y^{x,Q}.$$

2. For Vesting Conditions 2 and 3:

$$x \leq z,$$

$$(I)_{y,x} = 1 + 0.5 f_y^{z,Q} + r \sum_{s=1}^m f_y^{z+s,Q}.$$

$$z < x \leq z + m,$$

$$(I)_{y,x} = 1 + [0.5 + (x - z - 1)r] f_y^{x,Q} + r \sum_{s=x-z}^m f_y^{z+s,Q}.$$

$$x > z + m,$$

$$(I)_{y,x} = 1 + f_y^{x,Q}.$$

NOTE.—The same indices apply to both the accrued liability and current-year costs. Also, the indices must be applied to the costs of both accrued and current service pension credits after they are summarized by attained age for each entry age. You should also note that, when $(t)_y = 1$, $f_y^{z,x} = W_{z,x}^Q$, as defined in the paper.

Aggregate Cost Method

As with the entry age normal cost method, this method is of the "projected benefit" variety, and so $(t)_y = (k - y)/(Q - y)$ in the formula for $f_y^{z,x}$ when level units of pension credits are earned from entry age until normal retirement age Q . Under this method, it is necessary to separate for each entry age-attained age combination the "no vesting" single-sum liability, so that appropriate indices may be applied. For example, for

entry age y , the projected benefit is $(Q - y)$, and the single-sum liability at attained age x for projected benefits is

$$(Q - y) \cdot v^{Q-x} \cdot {}_{Q-x}p_x^{(T)} \cdot \ddot{a}_Q^{(12)}.$$

Thus, the indices required for Vesting Condition 1 would be

$$x \leq z,$$

$$(I)_{y,x} = 1 + f_y^{z,Q}.$$

$$x > z,$$

$$(I)_{y,x} = 1 + f_y^{z,Q}.$$

Since the vesting age z can differ for any specified entry age, the formulas for the level annual cost under this method become:

1. For Vesting Condition 1:

$$\begin{aligned} & \left[\sum_{u=y}^{Q-1} \sum_{k=u}^{z_u} (Q-u) \cdot v^{Q-k} \cdot {}_{Q-k}p_k^{(T)} \cdot \ddot{a}_Q^{(12)} \cdot (1 + f_u^{z_u, Q}) \cdot l_{u,k} \right. \\ & \quad \left. + \sum_{u=y}^{Q-1} \sum_{k=z_u+1}^{Q-1} (Q-u) \cdot v^{Q-k} \cdot {}_{Q-k}p_k^{(T)} \cdot \ddot{a}_Q^{(12)} \cdot (1 + f_u^{k, Q}) \cdot l_{u,k} - F \right] / \\ & \quad \sum_{k=y}^{Q-1} l_k \cdot \ddot{a}_{k:Q-k}^{(T)}. \end{aligned}$$

2. For Vesting Conditions 2 and 3:

$$\begin{aligned} & \left[\sum_{u=y}^{Q-1} \sum_{k=u}^{z_u} (Q-u) \cdot v^{Q-k} \cdot {}_{Q-k}p_k^{(T)} \cdot \ddot{a}_Q^{(12)} \cdot \left(1 + 0.5 f_u^{z_u, Q} \right. \right. \\ & \quad \left. \left. + r \sum_{s=1}^m f_u^{z_u+s, Q} \right) \cdot l_{u,k} + \sum_{u=y}^{Q-1} \sum_{k=z_u+1}^{z_u+m} (Q-u) \cdot v^{Q-k} \cdot {}_{Q-k}p_k^{(T)} \cdot \ddot{a}_Q^{(12)} \right. \\ & \quad \left. \cdot \left\{ 1 + [0.5 + (k - z - 1)r] f_u^{k, Q} + r \sum_{s=k-z_u}^m f_u^{z_u+s, Q} \right\} l_{u,k} \right. \\ & \quad \left. + \sum_{u=y}^{Q-1} \sum_{k=z_u+m+1}^{Q-1} (Q-u) \cdot v^{Q-k} \cdot {}_{Q-k}p_k^{(T)} \cdot \ddot{a}_Q^{(12)} \cdot (1 + f_u^{k, Q}) l_{u,k} - F \right] / \\ & \quad \sum_{k=y}^{Q-1} l_k \cdot \ddot{a}_{k:Q-k}^{(T)}, \end{aligned}$$

where

F = the fund asset;

$l_{u,k}$ = number of participants at attained age k with entry age u ;

l_k = total number of participants at attained age k ;

z_u = vesting age corresponding to specified entry age u .

Effect on Indices of Other Factors

All the formulas presented in this paper relate to a most simple type of benefit formula. Indices based on the assumption of a level unit of pension credit per year of service should be quite appropriate in cost calculations made for collectively bargained (or negotiated) pension plans. Also, when salary scales are not employed, such indices can be used to develop a reasonably accurate measure of vesting costs for corporate-type "salary" pension plans even though the assumption of a level unit accrual will normally result in a conservative cost estimate.

In order to develop more precise measures of vesting costs for pension plans involving projected benefit formulas, the factors, $f_y^{z,x}$, may be modified to reflect salary scales and care must be taken in applying the resulting indices in actuarial cost estimates. For example, if all assumptions used in this paper are retained *except* that a salary scale is utilized in the actuarial cost calculations, then the projected units of benefit at retirement age Q for entry age y will not be $Q - y$ but rather

$$\frac{1}{s_y} \sum_{g=y}^{Q-1} s_g$$

for a career average benefit formula and

$$\frac{1}{5s_y} \cdot \left(\sum_{g=Q-5}^{Q-1} s_g \right) \cdot (Q - y)$$

for a final five-year-average benefit formula. Thus, the formula for $(t)_y$ has to be changed as follows for attained age k .

1. Career average benefit formula:

$$(t)_y = \frac{\sum_{g=y}^{k-1} s_g}{\sum_{g=y}^{Q-1} s_g}.$$

2. Final five-year average benefit formula:

$$t)_y = \frac{\sum_{g=k-5}^{k-1} s_g}{\sum_{g=Q-5}^{Q-1} s_g} \cdot \frac{k - y}{Q - y}.$$

It can be readily seen that, if the salary factor s_k is not in a select form, introduction of a salary scale does not complicate unduly the calculation of indices in this theoretical situation.

Practical Complications

Complications can arise when vesting costs are required for actuarial valuations of existing pension plans—primarily when benefits are based on career average earnings and a salary scale is employed. In effect, the projected benefits for a particular participant comprise specific accrued pension credits *plus* projected future benefit accruals based on an assumed salary scale. This situation can be handled by applying indices based on level unit accruals to accrued pension benefit costs and applying indices based on salary scales to projected future benefit costs.

Comments

Because of the complexities involved in providing for the costs of vesting, many actuaries may question the need for such a degree of apparent accuracy when the basic actuarial assumptions depend on the actuary's judgment. On the other hand, the relative cost of vesting—for a specified employment turnover assumption—can have a dramatic effect on the total cost of a pension plan, and the additional work required to measure the cost of vesting in most instances is well worthwhile. Also, by the use of computers, a large portion of the additional work can be absorbed without an inordinate increase in the actuary's time and effort.

Included as a part of this discussion are the commutation functions ${}_kW_{z,65}^{65}$ and $W_{z,65}^{65}$ (see Tables 1–5), to which reference was made in Appendix A of the paper. In addition, the basic 1966 Group Annuity Table (see Table 6) and a table of $N_x^{(T)}$ for each employment turnover table (see Tables 7–11) are also included, so that additional vesting cost indices may be developed.

TABLE 1
 NONSELECT COMMUTATION FUNCTIONS— ${}_hW_{z,65}^{65}$ AND $W_{z,65}^{65}$

z	EMPLOYMENT TURNOVER TABLE I		EMPLOYMENT TURNOVER TABLE II		EMPLOYMENT TURNOVER TABLE IV	
	${}_hW_{z,65}^{65}$	$W_{z,65}^{65}$	${}_hW_{z,65}^{65}$	$W_{z,65}^{65}$	${}_hW_{z,65}^{65}$	$W_{z,65}^{65}$
20.....	32.669884	1.217145	36.257898	1.258298	114.744770	4.244635
21.....	30.257216	1.096512	33.817604	1.136283	103.409390	3.677866
22.....	27.874990	0.983072	31.532856	1.027486	93.402673	3.201355
23.....	25.546602	0.877237	29.390611	0.930111	84.523553	2.797759
24.....	23.291134	0.779173	27.376239	0.842529	76.595646	2.453067
25.....	21.123794	0.688867	25.478039	0.763438	69.480055	2.156585
26.....	19.056082	0.606158	23.687055	0.691798	63.067491	1.900082
27.....	17.096357	0.530784	21.999107	0.626877	57.279768	1.677477
28.....	15.250038	0.462402	20.410178	0.568028	52.048987	1.483745
29.....	13.520320	0.400627	18.914237	0.514602	47.309159	1.314465
30.....	11.908321	0.345040	17.505541	0.466026	43.003217	1.165985
31.....	10.413680	0.295219	16.178597	0.421794	39.081573	1.035263
32.....	9.034916	0.250743	14.928156	0.381458	35.501008	0.919761
33.....	7.769690	0.211204	13.751385	0.344683	32.229797	0.817536
34.....	6.615213	0.176220	12.638948	0.310973	29.222009	0.726391
35.....	5.568366	0.145431	11.588414	0.280075	26.454648	0.644998
36.....	4.625940	0.118504	10.595211	0.251698	23.901478	0.572050
37.....	3.784637	0.095135	9.654874	0.225577	21.539029	0.506426
38.....	3.041160	0.075041	8.763014	0.201473	19.346143	0.447159
39.....	2.392139	0.057961	7.917576	0.179225	17.309122	0.393553
40.....	1.833922	0.043648	7.114256	0.158627	15.410114	0.344861
41.....	1.362490	0.031862	6.346439	0.139431	13.627319	0.300291
42.....	0.973496	0.022374	5.614503	0.121579	11.956486	0.259539
43.....	0.661858	0.014954	4.918840	0.105016	10.393750	0.222331
44.....	0.421541	0.009366	4.262249	0.089746	8.940912	0.188544
45.....	0.245436	0.005363	3.642792	0.075668	7.589466	0.157829
46.....	0.125232	0.002692	3.060978	0.062739	6.336774	0.129992
47.....	0.051540	0.001090	2.517349	0.050921	5.180558	0.104856
48.....	0.013921	0.000290	2.014987	0.040232	4.124142	0.082380
49.....	0.000737	0.000015	1.557084	0.030692	3.171134	0.062525
50.....	0.000000	0.000000	1.146978	0.022323	2.325518	0.045268
51.....	0.000000	0.000000	0.790739	0.015198	1.597001	0.030697
52.....	0.000000	0.000000	0.489530	0.009292	0.985355	0.018704
53.....	0.000000	0.000000	0.252567	0.004735	0.506994	0.009505
54.....	0.000000	0.000000	0.086885	0.001609	0.174051	0.003223
55.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
56.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
57.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
58.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
59.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
60.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
61.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
62.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
63.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
64.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 1—*Continued*

Z	EMPLOYMENT TURNOVER TABLE VI		EMPLOYMENT TURNOVER TABLE IX		EMPLOYMENT TURNOVER TABLE X	
	$kW_{z,66}^{65}$	$W_{z,66}^{65}$	$kW_{z,66}^{65}$	$W_{z,66}^{65}$	$kW_{z,66}^{65}$	$W_{z,66}^{65}$
20.....	728.805280	30.060751	183.080180	6.387745	1,149.706200	51.476011
21.....	594.544130	23.347694	171.342530	5.800862	803.177840	34.149591
22.....	490.378840	18.387442	160.053210	5.263276	568.316630	22.965724
23.....	408.390390	14.660694	149.216320	4.770689	411.632380	15.843713
24.....	343.006180	11.817902	138.833100	4.319245	305.805640	11.242551
25.....	290.209520	9.618041	128.902690	3.905478	231.714790	8.155433
26.....	247.067330	7.892354	119.421870	3.526245	178.806340	6.039095
27.....	211.595650	6.528058	110.387610	3.178773	140.162570	4.552796
28.....	182.179730	5.438579	101.814790	2.861261	111.505780	3.491433
29.....	157.603640	4.560862	93.710246	2.571813	89.858499	2.718316
30.....	136.910070	3.847290	86.073337	2.308471	73.023518	2.137799
31.....	119.355240	3.262129	78.897815	2.069287	59.833944	1.698147
32.....	104.356630	2.778303	72.172785	1.852351	49.537208	1.365994
33.....	91.466170	2.375476	65.883281	1.655804	41.277421	1.107876
34.....	80.303287	2.037206	60.011749	1.477879	34.524070	0.903229
35.....	70.564166	1.750762	54.538365	1.316897	28.953909	0.739400
36.....	62.023465	1.506742	49.442237	1.171293	24.383659	0.608822
37.....	54.487506	1.297410	44.701244	1.039599	20.557386	0.502537
38.....	47.805721	1.116821	40.293135	0.920461	17.329723	0.415302
39.....	41.844885	0.959957	36.195688	0.812633	14.584030	0.343047
40.....	36.494079	0.822756	32.387723	0.714993	12.172028	0.281201
41.....	31.652814	0.701725	28.849055	0.626526	10.119828	0.229896
42.....	27.271265	0.594858	25.562265	0.546361	8.352864	0.186799
43.....	23.312598	0.500604	22.512212	0.473741	6.805719	0.149963
44.....	19.737960	0.417473	19.684912	0.407989	5.419286	0.117720
45.....	16.507331	0.344049	17.067914	0.348512	4.187962	0.089735
46.....	13.598238	0.279403	14.650435	0.294790	3.107358	0.065722
47.....	10.985314	0.222600	12.425504	0.246422	2.125116	0.044369
48.....	8.652567	0.172967	10.390599	0.203126	1.288994	0.026579
49.....	6.592275	0.130044	8.546333	0.164704	0.646991	0.013203
50.....	4.793278	0.093330	6.895610	0.131016	0.000000	0.000000
51.....	3.263568	0.062736	5.441879	0.101941	0.000000	0.000000
52.....	2.007566	0.038108	4.186704	0.077330	0.000000	0.000000
53.....	1.032557	0.019358	3.127635	0.056963	0.000000	0.000000
54.....	0.354725	0.006569	2.257548	0.040547	0.000000	0.000000
55.....	0.000000	0.000000	1.564211	0.027707	0.000000	0.000000
56.....	0.000000	0.000000	1.030840	0.018009	0.000000	0.000000
57.....	0.000000	0.000000	0.638615	0.011005	0.000000	0.000000
58.....	0.000000	0.000000	0.365604	0.006216	0.000000	0.000000
59.....	0.000000	0.000000	0.188132	0.003156	0.000000	0.000000
60.....	0.000000	0.000000	0.082918	0.001373	0.000000	0.000000
61.....	0.000000	0.000000	0.028430	0.000465	0.000000	0.000000
62.....	0.000000	0.000000	0.005874	0.000095	0.000000	0.000000
63.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
64.....	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

TABLE 2
SELECT COMMUTATION FUNCTIONS— $kW_{x,65}^{65}$ AND $W_{x,65}^{65}$
EMPLOYMENT TURNOVER TABLE III
(Entry Age y)

DURATION	ENTRY AGE 20		ENTRY AGE 25	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	39.837877	1.429842	28.961915	0.898099
1.....	35.889664	1.232431	25.699929	.767620
2.....	32.432603	1.067810	22.885528	.659373
3.....	29.646918	0.941188	20.643939	.576352
4.....	27.413255	0.844072	18.934681	.515307
5.....	25.478039	0.763438	17.505541	0.466026
(Same as those calculated under Table II at attained ages $y+5$ and over)				
	ENTRY AGE 30		ENTRY AGE 35	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	20.611725	0.566488	13.408437	0.330504
1.....	17.968760	.478390	11.660674	.280568
2.....	15.735218	.406340	10.221083	.240579
3.....	14.046349	.353563	9.072506	.209537
4.....	12.705290	.312925	8.037366	.182296
5.....	11.588414	0.280075	7.114256	0.158627
(Same as those calculated under Table II at attained ages $y+5$ and over)				
	ENTRY AGE 40		ENTRY AGE 45	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	8.285682	0.187038	4.245378	0.088736
1.....	7.097375	.157331	3.450371	.071069
2.....	6.028527	.131261	2.709970	.054973
3.....	5.138947	.110081	2.088906	.041759
4.....	4.364630	.092073	1.587730	.031318
5.....	3.642792	0.075668	1.146978	0.022323
(Same as those calculated under Table II at attained ages $y+5$ and over)				
	ENTRY AGE 50			
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$		
0.....	1.633430	0.031813		
1.....	1.116084	.021466		
2.....	0.661471	.012552		
3.....	0.333931	.006253		
4.....	0.135881	.002516		
5.....	0.000000	0.000000		
(Same as those calculated under Table II at attained ages $y+5$ and over)				

TABLE 3
SELECT COMMUTATION FUNCTIONS— $kW_{z,65}^{65}$ AND $W_{z,65}^{65}$
EMPLOYMENT TURNOVER TABLE V
(Entry Age y)

DURATION	ENTRY AGE 20		ENTRY AGE 25	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	133.303630	5.137842	83.194681	2.688213
1.....	113.343660	4.139843	70.507320	2.180718
2.....	97.412987	3.381239	60.369140	1.790788
3.....	85.597803	2.844186	52.828158	1.511493
4.....	76.750159	2.459506	47.373824	1.316695
5.....	69.480055	2.156585	43.003217	1.165985
(Same as those calculated under Table IV at attained ages $y+5$ and over)				
	ENTRY AGE 30		ENTRY AGE 35	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	52.916554	1.487293	31.361001	0.781161
1.....	44.514724	1.207232	26.680626	.647436
2.....	37.837430	0.991835	22.969952	.544362
3.....	33.052619	0.842310	20.109745	.467059
4.....	29.401376	0.731666	17.598209	.400966
5.....	26.454648	0.644998	15.410114	0.344861
(Same as those calculated under Table IV at attained ages $y+5$ and over)				
	ENTRY AGE 40		ENTRY AGE 45	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	18.226872	0.413246	8.922607	0.186752
1.....	15.396815	.342494	7.183817	.148113
2.....	12.916642	.282002	5.596267	.113601
3.....	10.894657	.233860	4.284948	.085700
4.....	9.167737	.193699	3.240206	.063935
5.....	7.589466	0.157829	2.325518	0.045268
(Same as those calculated under Table IV at attained ages $y+5$ and over)				
	ENTRY AGE 50			
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$		
0.....	3.348183	0.065222		
1.....	2.279889	.043856		
2.....	1.345330	.025531		
3.....	0.676471	.012668		
4.....	0.272451	.005045		
5.....	0.000000	0.000000		
(Same as those calculated under Table IV at attained ages $y+5$ and over)				

TABLE 4
SELECT COMMUTATION FUNCTIONS— $kW_{z, 65}^{65}$ AND $W_{z, 65}^{65}$
EMPLOYMENT TURNOVER TABLE VII
(Entry Age y)

DURATION	ENTRY AGE 20		ENTRY AGE 25	
	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$
0.....	1012.407600	43.824863	404.503140	14.073014
1.....	720.881470	29.248552	300.800760	9.924919
2.....	533.378990	20.319863	231.154770	7.246227
3.....	418.391250	15.093148	186.588040	5.595607
4.....	344.312390	11.872328	157.937650	4.572379
5.....	290.209520	9.618041	136.910070	3.847290
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 30		ENTRY AGE 35	
	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$
0.....	189.419380	5.556084	88.686079	2.255249
1.....	145.123780	4.079564	71.574962	1.766360
2.....	114.386730	3.088046	59.110303	1.420120
3.....	94.742976	2.474179	50.143834	1.177783
4.....	80.969643	2.056805	42.685377	0.981508
5.....	70.564166	1.750762	36.494079	0.822756
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 40		ENTRY AGE 45	
	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$	$kW_{z, 65}^{65}$	$kW_{z, 65}^{65}$
0.....	44.601991	1.020040	19.744873	0.414362
1.....	36.509399	0.817725	15.599154	.322235
2.....	29.791228	0.653867	11.941569	.242722
3.....	24.572748	0.529618	9.014160	.180437
4.....	20.289807	0.430015	6.741901	.133098
5.....	16.507331	0.344049	4.793278	0.093330
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 50			
	$kW_{z, 65}^{65}$	$W_{z, 65}^{65}$		
0.....	6.969480	0.135830		
1.....	4.690856	.090258		
2.....	2.738388	.051974		
3.....	1.366060	.025583		
4.....	0.547676	.010142		
5.....	0.000000	0.000000		
(Same as those calculated under Table VI at attained ages $y+5$ and over)				

TABLE 5
SELECT COMMUTATION FUNCTIONS— $kW_{x,65}^{65}$ AND $W_{x,65}^{65}$
EMPLOYMENT TURNOVER TABLE VIII
(Entry Age y)

DURA- TION	ENTRY AGE 20		ENTRY AGE 25	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	867.930200	36.802162	360.546410	12.350552
1.....	659.142480	26.362776	282.387940	9.224214
2.....	512.525660	19.381023	225.455620	7.034508
3.....	413.817870	14.894305	186.375220	5.587086
4.....	344.312390	11.872328	158.684850	4.598145
5.....	290.209520	9.618041	136.910070	3.847290
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 30		ENTRY AGE 35	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	170.255830	4.931596	80.501918	2.028127
1.....	136.235110	3.797572	67.025261	1.643080
2.....	110.777250	2.976350	56.640443	1.354613
3.....	93.580853	2.438963	48.789054	1.142413
4.....	80.969643	2.056805	42.266994	0.970780
5.....	70.564166	1.750762	36.494079	0.822756
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 40		ENTRY AGE 45	
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$	$kW_{x,65}^{65}$	$W_{x,65}^{65}$
0.....	40.977405	0.931745	18.120433	0.378759
1.....	34.399435	.767296	14.700908	.302769
2.....	28.666029	.627457	11.697844	.237485
3.....	24.078664	.518234	9.074294	.181665
4.....	20.089220	.425456	6.799665	.134277
5.....	16.507331	0.344049	4.793278	0.093330
(Same as those calculated under Table VI at attained ages $y+5$ and over)				
	ENTRY AGE 50			
	$kW_{x,65}^{65}$	$W_{x,65}^{65}$		
0.....	6.377789	0.124066		
1.....	4.461067	.085732		
2.....	2.738936	.051965		
3.....	1.421513	.026630		
4.....	0.547676	.010142		
5.....	0.000000	0.000000		
(Same as those calculated under Table VI at attained ages $y+5$ and over)				

TABLE 6
1966 GROUP ANNUITY TABLE
(Male 1951 Group Annuity Table with Mortality Improvement
Projected According to Scale C to 1966)

Age x	$q_x^{(d)}$	Age x	$q_x^{(d)}$	Age x	$q_x^{(d)}$
10.....	0.000400	45.....	0.002960	80.....	0.090120
11.....	.000400	46.....	.003370	81.....	0.099320
12.....	.000410	47.....	.003810	82.....	0.108780
13.....	.000420	48.....	.004290	83.....	0.118740
14.....	.000430	49.....	.004810	84.....	0.129080
15.....	.000440	50.....	.005360	85.....	0.139540
16.....	.000450	51.....	.005950	86.....	0.150600
17.....	.000460	52.....	.006570	87.....	0.162180
18.....	.000480	53.....	.007230	88.....	0.174090
19.....	.000490	54.....	.007920	89.....	0.186950
20.....	.000510	55.....	.008640	90.....	0.200590
21.....	.000530	56.....	.009400	91.....	0.212560
22.....	.000550	57.....	.010180	92.....	0.225160
23.....	.000570	58.....	.011020	93.....	0.238520
24.....	.000600	59.....	.011910	94.....	0.252770
25.....	.000630	60.....	.012880	95.....	0.268030
26.....	.000660	61.....	.013970	96.....	0.284460
27.....	.000690	62.....	.015200	97.....	0.302220
28.....	.000730	63.....	.016620	98.....	0.321520
29.....	.000770	64.....	.018270	99.....	0.342530
30.....	.000820	65.....	.020220	100.....	0.365460
31.....	.000870	66.....	.022520	101.....	0.390540
32.....	.000930	67.....	.024930	102.....	0.417980
33.....	.000990	68.....	.027310	103.....	0.450100
34.....	.001060	69.....	.029760	104.....	0.489200
35.....	.001140	70.....	.032550	105.....	0.537610
36.....	.001220	71.....	.036030	106.....	0.597620
37.....	.001310	72.....	.039910	107.....	0.671550
38.....	.001420	73.....	.044120	108.....	0.761720
39.....	.001530	74.....	.048720	109.....	0.870430
40.....	.001660	75.....	.053690	110.....	1.000000
41.....	.001820	76.....	.059320		
42.....	.002030	77.....	.065900		
43.....	.002290	78.....	.073300		
44.....	0.002610	79.....	0.081360		

TABLE 7
 NONSELECT EMPLOYEE TURNOVER TABLES— $N_x^{(T)}$

Age	Table I	Table II	Table IV
20.....	67,719,646.0	70,307,809.6	46,387,316.0
21.....	62,951,656.9	65,518,886.2	41,598,392.6
22.....	58,608,278.3	61,154,665.3	37,483,426.5
23.....	54,650,520.3	57,164,526.7	33,923,058.8
24.....	51,041,403.6	53,505,329.1	30,822,707.9
25.....	47,746,351.4	50,140,348.7	28,107,159.3
26.....	44,733,425.7	47,038,079.9	25,715,906.2
27.....	41,973,376.7	44,171,224.8	23,599,687.5
28.....	39,439,622.6	41,515,784.6	21,717,751.9
29.....	37,108,173.4	39,050,614.7	20,036,235.6
30.....	34,957,509.9	36,757,187.1	18,527,052.6
31.....	32,968,400.1	34,619,204.3	17,166,780.5
32.....	31,123,733.3	32,622,318.1	15,935,809.7
33.....	29,408,306.9	30,753,848.5	14,817,637.3
34.....	27,808,658.1	29,002,493.7	13,798,219.7
35.....	26,312,860.9	27,358,397.8	12,865,847.9
36.....	24,910,372.7	25,812,725.7	12,010,456.4
37.....	23,591,874.2	24,357,618.8	11,223,446.9
38.....	22,349,116.7	22,986,059.5	10,497,441.0
39.....	21,174,809.6	21,691,783.5	9,826,093.6
40.....	20,062,523.0	20,469,138.6	9,203,872.8
41.....	19,006,572.3	19,313,052.8	8,625,990.0
42.....	18,001,961.7	18,219,044.6	8,088,355.7
43.....	17,044,319.5	17,183,000.4	7,587,316.7
44.....	16,129,856.6	16,201,159.9	7,119,614.2
45.....	15,255,312.7	15,270,034.4	6,682,290.9
46.....	14,417,916.8	14,386,483.8	6,272,749.8
47.....	13,615,318.6	13,547,608.9	5,888,653.6
48.....	12,845,570.9	12,750,754.5	5,527,906.7
49.....	12,107,064.3	11,993,424.0	5,188,582.1
50.....	11,398,499.4	11,273,277.6	4,868,910.4
51.....	10,718,840.5	10,588,121.6	4,567,266.6
52.....	10,067,258.9	9,935,859.7	4,282,125.2
53.....	9,442,965.2	9,314,560.7	4,012,113.5
54.....	8,845,189.7	8,722,347.5	3,755,913.5
55.....	8,273,186.3	8,157,436.1	3,512,290.2
56.....	7,726,224.3	7,618,126.7	3,280,083.7
57.....	7,203,587.0	7,102,801.7	3,058,203.9
58.....	6,704,575.4	6,610,771.9	2,846,354.2
59.....	6,228,496.7	6,141,354.2	2,644,240.3
60.....	5,774,682.4	5,693,889.4	2,451,578.5
61.....	5,342,480.6	5,267,734.6	2,268,092.0
62.....	4,931,266.1	4,862,273.5	2,093,515.4
63.....	4,540,451.8	4,476,927.2	1,927,599.5
64.....	4,169,489.0	4,111,154.6	1,770,111.4
65.....	3,817,877.0	3,764,462.1	1,620,838.4
66.....	3,485,165.7	3,436,405.8	1,479,589.5

TABLE 7—*Continued*

Age	Table VI	Table IX	Table X
20.....	24,320,010.9	46,193,440.8	16,297,532.5
21.....	19,531,087.5	41,425,451.7	11,529,543.4
22.....	15,914,631.0	37,196,964.8	8,452,800.2
23.....	13,140,434.1	33,445,406.5	6,431,876.5
24.....	10,981,641.0	30,115,602.9	5,063,589.4
25.....	9,279,513.2	27,158,862.3	4,105,550.9
26.....	7,921,256.2	24,532,202.1	3,415,393.7
27.....	6,825,525.8	22,197,631.7	2,904,264.9
28.....	5,932,001.3	20,121,517.7	2,515,885.7
29.....	5,195,900.8	18,273,730.3	2,213,301.9
30.....	4,583,557.5	16,627,409.7	1,972,030.9
31.....	4,069,466.5	15,158,691.9	1,775,937.6
32.....	3,634,121.7	13,846,456.5	1,613,547.7
33.....	3,262,461.1	12,672,050.8	1,476,415.4
34.....	2,942,718.5	11,619,057.9	1,358,769.9
35.....	2,665,687.3	10,673,045.0	1,256,488.1
36.....	2,424,105.5	9,821,357.9	1,166,485.9
37.....	2,212,151.5	9,052,913.6	1,086,341.9
38.....	2,025,145.7	8,358,011.9	1,014,287.2
39.....	1,859,284.7	7,728,174.4	948,956.0
40.....	1,711,475.7	7,156,000.3	889,287.1
41.....	1,579,185.6	6,635,013.4	834,508.4
42.....	1,460,342.6	6,159,554.1	783,909.3
43.....	1,353,185.7	5,724,663.9	736,934.7
44.....	1,256,205.4	5,325,994.8	693,153.4
45.....	1,168,113.7	4,959,727.5	652,232.8
46.....	1,087,817.4	4,622,503.5	613,880.3
47.....	1,014,367.3	4,311,351.5	577,836.7
48.....	946,946.9	4,023,634.1	543,907.9
49.....	884,842.7	3,756,981.7	511,886.0
50.....	827,425.0	3,509,255.9	481,555.2
51.....	774,141.8	3,278,518.9	452,841.4
52.....	724,492.7	3,063,013.2	425,313.8
53.....	678,028.4	2,861,156.6	398,939.1
54.....	634,343.9	2,671,539.3	373,684.7
55.....	593,069.1	2,492,924.7	349,519.1
56.....	553,859.8	2,324,245.0	326,411.4
57.....	516,394.3	2,164,594.2	304,331.3
58.....	480,622.4	2,013,214.0	283,249.3
59.....	446,494.5	1,869,478.3	263,136.2
60.....	413,962.6	1,732,883.5	243,963.8
61.....	382,980.0	1,603,026.5	225,704.4
62.....	353,501.8	1,479,588.0	208,331.7
63.....	325,486.0	1,362,316.9	191,820.8
64.....	298,893.3	1,251,013.2	176,148.6
65.....	273,687.8	1,145,515.6	161,294.0
66.....	249,837.2	1,045,688.9	147,237.9

TABLE 8
SELECT EMPLOYEE TURNOVER TABLE— $N_x^{(T)}$
TABLE III

Age	Entry Age 20	Entry Age 25	Entry Age 30	Entry Age 35
20.....	65,960,802.4			
21.....	61,171,879.0			
22.....	56,933,208.7			
23.....	53,151,007.5			
24.....	49,730,621.6			
25.....	46,600,581.3	43,667,912.8		
26.....	43,717,323.9	40,784,655.4		
27.....	41,052,860.4	38,198,283.2		
28.....	38,584,886.8	35,859,604.2		
29.....	36,293,751.1	33,719,718.6		
30.....	34,162,233.1	31,738,504.3	29,923,942.9	
31.....	32,175,185.9	29,892,432.8	28,077,871.4	
32.....	30,319,274.4	28,168,193.0	26,399,983.6	
33.....	28,582,713.6	26,554,836.3	24,862,908.9	
34.....	26,954,999.6	25,042,603.9	23,438,325.9	
35.....	25,426,972.2	23,622,985.8	22,107,786.1	21,365,314.3
36.....	23,990,420.4	22,288,353.3	20,858,758.8	20,116,287.0
37.....	22,638,040.2	21,031,920.7	19,682,915.5	18,958,923.1
38.....	21,363,309.4	19,847,628.5	18,574,585.6	17,879,557.3
39.....	20,160,405.7	18,730,067.5	17,528,706.8	16,866,576.4
40.....	19,024,076.2	17,674,357.6	16,540,711.7	15,913,564.1
41.....	17,949,606.7	16,676,119.1	15,606,501.6	15,014,774.7
42.....	16,932,832.4	15,731,482.5	14,722,455.3	14,164,247.1
43.....	15,969,930.0	14,836,895.7	13,885,248.4	13,358,783.0
44.....	15,057,404.8	13,989,111.9	13,091,842.5	12,595,459.2
45.....	14,192,014.2	13,185,118.7	12,339,418.4	11,871,563.3
46.....	13,370,839.9	12,422,204.7	11,625,438.6	11,184,654.2
47.....	12,591,187.3	11,697,866.7	10,947,560.3	10,532,477.9
48.....	11,850,588.7	11,009,811.8	10,303,637.9	9,912,970.2
49.....	11,146,723.7	10,355,884.5	9,691,654.1	9,324,190.1
50.....	10,477,417.7	9,734,064.3	9,109,718.0	8,764,318.3
51.....	9,840,632.0	9,142,457.1	8,556,057.0	8,231,649.6
52.....	9,234,418.1	8,579,252.8	8,028,977.0	7,724,554.1
53.....	8,656,981.2	8,042,783.7	7,526,917.4	7,241,530.3
54.....	8,106,576.6	7,531,429.2	7,048,361.4	6,781,119.0
55.....	7,581,546.2	7,043,648.7	6,591,867.5	6,341,933.3
56.....	7,080,310.4	6,577,974.7	6,156,062.1	5,922,651.7
57.....	6,601,365.7	6,133,010.4	5,739,637.9	5,522,016.5
58.....	6,144,071.6	5,708,160.6	5,342,038.1	5,139,491.9
59.....	5,707,793.2	5,302,835.5	4,962,710.7	4,774,546.8
60.....	5,291,917.9	4,916,465.9	4,601,122.9	4,426,668.9
61.....	4,895,848.3	4,548,496.8	4,256,755.4	4,095,358.3
62.....	4,519,011.4	4,198,395.9	3,929,110.0	3,780,135.8
63.....	4,160,869.3	3,865,663.3	3,617,719.0	3,480,551.4
64.....	3,820,919.1	3,549,831.9	3,322,145.2	3,196,184.4
65.....	3,498,702.0	3,250,475.5	3,041,989.6	2,926,651.1
66.....	3,193,805.4	2,967,210.8	2,776,893.6	2,671,606.4

TABLE 8—*Continued*

Age	Entry Age 40	Entry Age 45	Entry Age 50
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40.	15,578,702.2		
41.	14,679,912.8		
42.	13,836,705.4		
43.	13,043,741.1		
44.	12,295,282.1		
45.	11,587,214.7	11,464,387.9	
46.	10,916,758.5	10,793,931.7	
47.	10,280,203.1	10,160,090.5	
48.	9,675,533.8	9,560,381.9	
49.	9,100,856.1	8,991,777.3	
50.	8,554,394.4	8,451,556.3	8,383,881.6
51.	8,034,484.1	7,937,896.0	7,870,221.3
52.	7,539,534.6	7,448,896.4	7,382,731.7
53.	7,068,080.2	6,983,109.5	6,919,745.8
54.	6,618,696.7	6,539,128.2	6,479,193.0
55.	6,190,030.3	6,115,615.0	6,059,204.9
56.	5,780,791.2	5,711,295.7	5,658,615.1
57.	5,389,752.0	5,324,957.4	5,275,840.5
58.	5,016,389.7	4,956,083.4	4,910,369.2
59.	4,660,185.8	4,604,161.6	4,561,693.8
60.	4,320,640.2	4,268,697.9	4,229,324.6
61.	3,997,265.2	3,949,210.4	3,912,784.2
62.	3,689,593.0	3,645,237.0	3,611,614.7
63.	3,397,184.3	3,356,343.6	3,325,386.1
64.	3,119,628.6	3,082,124.6	3,053,696.5
65.	2,856,551.2	2,822,209.9	2,796,179.3
66.	2,607,615.3	2,576,266.7	2,552,504.7

TABLE 9
SELECT EMPLOYEE TURNOVER TABLE— $N_x^{(T)}$
TABLE V

Age	Entry Age 20	Entry Age 25	Entry Age 30	Entry Age 35
20.....	40,848,388.1			
21.....	36,059,464.7			
22.....	32,196,061.3			
23.....	29,023,527.4			
24.....	26,341,933.7			
25.....	24,017,205.2	21,072,877.9		
26.....	21,973,909.0	19,029,581.7		
27.....	20,165,626.2	17,332,178.7		
28.....	18,557,535.5	15,897,623.6		
29.....	17,120,700.6	14,654,144.5		
30.....	15,831,122.1	13,549,371.7	12,031,247.6	
31.....	14,668,786.3	12,554,564.9	11,036,440.8	
32.....	13,616,936.8	11,654,319.9	10,186,247.7	
33.....	12,661,472.2	10,836,567.6	9,447,395.0	
34.....	11,790,392.5	10,091,037.7	8,789,321.3	
35.....	10,993,692.5	9,409,167.2	8,193,718.6	7,649,227.1
36.....	10,262,771.2	8,783,594.5	7,648,955.8	7,104,464.3
37.....	9,590,281.4	8,208,031.5	7,147,742.5	6,619,370.7
38.....	8,969,918.5	7,677,082.2	6,685,379.8	6,181,603.6
39.....	8,396,260.8	7,186,106.3	6,257,826.9	5,781,309.3
40.....	7,864,581.1	6,731,058.1	5,861,560.6	5,413,393.6
41.....	7,370,787.8	6,308,435.6	5,493,531.3	5,073,503.2
42.....	6,911,386.3	5,915,248.1	5,151,134.6	4,757,285.5
43.....	6,483,255.2	5,548,824.0	4,832,044.0	4,462,591.9
44.....	6,083,609.8	5,206,779.8	4,534,184.0	4,187,505.7
45.....	5,709,923.1	4,886,952.9	4,255,671.3	3,930,287.5
46.....	5,359,975.9	4,587,443.8	3,994,851.8	3,689,409.8
47.....	5,031,770.9	4,306,543.3	3,750,237.0	3,463,497.8
48.....	4,723,517.5	4,042,718.8	3,520,492.5	3,251,319.1
49.....	4,433,569.2	3,794,560.9	3,304,390.9	3,051,740.2
50.....	4,160,414.1	3,560,775.8	3,100,805.4	2,863,720.5
51.....	3,902,663.6	3,340,175.0	2,908,701.2	2,686,304.2
52.....	3,659,014.3	3,131,642.9	2,727,106.7	2,518,594.2
53.....	3,428,293.0	2,934,175.6	2,555,147.6	2,359,782.9
54.....	3,209,373.6	2,746,809.2	2,391,984.5	2,209,095.1
55.....	3,001,200.8	2,568,640.5	2,236,830.9	2,065,804.4
56.....	2,802,783.6	2,398,821.2	2,088,948.3	1,929,228.7
57.....	2,613,190.5	2,236,554.1	1,947,642.4	1,798,726.9
58.....	2,432,167.9	2,081,622.2	1,812,724.2	1,674,124.4
59.....	2,259,464.5	1,933,810.4	1,684,006.3	1,555,248.2
60.....	2,094,837.8	1,792,911.2	1,561,308.1	1,441,931.4
61.....	1,938,051.3	1,658,722.2	1,444,453.4	1,334,011.2
62.....	1,788,878.2	1,531,049.3	1,333,273.0	1,231,331.5
63.....	1,647,105.5	1,409,710.2	1,227,608.2	1,133,745.7
64.....	1,512,534.2	1,294,534.5	1,127,310.6	1,041,116.8
65.....	1,384,982.6	1,185,366.8	1,032,244.9	953,319.8
66.....	1,264,287.5	1,082,067.3	942,289.4	870,242.2

TABLE 9—*Continued*

Age	Entry Age 40	Entry Age 45	Entry Age 50
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40.	5,186,076.0		
41.	4,846,185.6		
42.	4,535,504.5		
43.	4,250,073.2		
44.	3,985,834.0		
45.	3,740,006.2	3,660,054.1	
46.	3,510,790.4	3,430,838.3	
47.	3,295,815.7	3,217,741.6	
48.	3,093,909.4	3,019,201.2	
49.	2,903,993.0	2,833,350.3	
50.	2,725,076.3	2,658,568.0	2,615,317.9
51.	2,556,249.6	2,493,861.6	2,450,611.5
52.	2,396,659.1	2,338,166.0	2,295,884.4
53.	2,245,536.6	2,190,731.7	2,150,247.0
54.	2,102,144.3	2,050,838.9	2,012,550.5
55.	1,965,791.0	1,917,813.3	1,881,785.9
56.	1,835,827.6	1,791,021.6	1,757,376.1
57.	1,711,644.0	1,669,868.6	1,638,499.2
58.	1,593,074.0	1,554,192.3	1,524,996.0
59.	1,479,953.1	1,443,832.1	1,416,709.1
60.	1,372,122.5	1,338,633.0	1,313,486.4
61.	1,269,427.2	1,238,443.9	1,215,179.6
62.	1,171,718.7	1,143,119.9	1,121,646.5
63.	1,078,857.4	1,052,525.0	1,032,753.5
64.	990,713.0	966,531.8	948,375.8
65.	907,166.6	885,024.4	868,399.6
66.	828,111.1	807,898.4	792,722.5

TABLE 10
SELECT EMPLOYEE TURNOVER TABLE— $N_x^{(T)}$
TABLE VII

Age	Entry Age 20	Entry Age 25	Entry Age 30	Entry Age 35
20.....	19,168,216.7			
21.....	14,379,293.3			
22.....	11,265,962.3			
23.....	9,151,987.9			
24.....	7,614,746.3			
25.....	6,430,247.8	4,981,275.7		
26.....	5,489,042.3	4,040,070.2		
27.....	4,729,754.9	3,382,932.0		
28.....	4,110,586.1	2,905,143.6		
29.....	3,600,504.5	2,537,046.3		
30.....	3,176,180.7	2,237,505.6	1,764,942.1	
31.....	2,819,941.0	1,986,546.8	1,513,983.3	
32.....	2,518,268.8	1,774,029.0	1,326,718.6	
33.....	2,260,726.5	1,592,599.1	1,181,576.2	
34.....	2,039,160.8	1,436,513.3	1,062,794.4	
35.....	1,847,192.1	1,301,277.7	962,157.8	838,250.7
36.....	1,679,788.1	1,183,347.1	874,961.3	751,054.2
37.....	1,532,914.5	1,079,879.6	798,458.8	679,712.0
38.....	1,403,328.8	988,590.9	730,961.0	619,627.7
39.....	1,288,395.4	907,624.3	671,095.1	567,582.3
40.....	1,185,971.2	835,470.1	617,744.9	522,004.1
41.....	1,094,300.7	770,891.5	569,996.1	481,654.9
42.....	1,011,948.4	712,877.3	527,100.7	445,407.1
43.....	937,694.0	660,567.7	488,423.3	412,723.7
44.....	870,491.4	613,225.9	453,419.0	383,144.1
45.....	809,448.1	570,223.1	421,623.1	356,275.6
46.....	753,806.6	531,025.6	392,640.8	331,784.7
47.....	702,909.3	495,170.2	366,129.7	309,381.9
48.....	656,190.2	462,258.2	341,795.0	288,818.3
49.....	613,155.1	431,941.4	319,379.2	269,876.2
50.....	573,367.5	403,912.3	298,654.9	252,363.5
51.....	536,444.9	377,901.6	279,422.8	236,112.0
52.....	502,040.5	353,664.9	261,502.3	220,968.9
53.....	469,843.1	330,982.9	244,731.3	206,797.2
54.....	439,571.8	309,657.9	228,963.7	193,473.4
55.....	410,970.4	289,509.2	214,065.9	180,884.5
56.....	383,800.3	270,368.8	199,913.5	168,925.6
57.....	357,838.5	252,079.7	186,390.5	157,498.6
58.....	333,050.3	234,617.4	173,478.8	146,588.1
59.....	309,401.3	217,957.5	161,160.5	136,179.1
60.....	286,858.2	202,076.7	149,418.3	126,256.8
61.....	265,388.7	186,952.3	138,235.3	116,807.1
62.....	244,961.7	172,562.3	127,595.3	107,816.2
63.....	225,548.1	158,886.2	117,483.1	99,271.4
64.....	207,120.6	145,904.7	107,884.5	91,160.6
65.....	189,654.4	133,600.4	98,786.6	83,473.0
66.....	173,127.0	121,957.6	90,177.8	76,198.6

TABLE 10—*Continued*

Age	Entry Age 40	Entry Age 45	Entry Age 50
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40.	479,005.9		
41.	438,656.7		
42.	403,719.5		
43.	373,137.0		
44.	345,930.6		
45.	321,472.1	307,660.2	
46.	299,373.7	285,561.8	
47.	279,159.6	265,709.7	
48.	260,604.8	247,787.7	
49.	243,513.2	231,442.9	
50.	227,711.3	216,386.7	209,368.9
51.	213,047.3	202,452.4	195,434.6
52.	199,383.4	189,468.4	182,613.3
53.	186,596.0	177,317.3	170,761.4
54.	174,573.6	165,893.2	159,698.6
55.	163,214.4	155,099.3	149,272.7
56.	152,423.6	144,845.5	139,403.7
57.	142,112.7	135,047.7	129,973.7
58.	132,268.0	125,692.8	120,970.0
59.	122,875.7	116,767.8	112,380.0
60.	113,922.7	108,260.2	104,191.8
61.	105,396.1	100,157.7	96,393.6
62.	97,283.5	92,448.7	88,974.1
63.	89,573.4	85,122.1	81,922.7
64.	82,255.0	78,167.6	75,229.4
65.	75,318.4	71,575.8	68,885.2
66.	68,754.7	65,338.4	62,882.1

TABLE 11
SELECT EMPLOYEE TURNOVER TABLE— $N_x^{(T)}$
TABLE VIII

Age	Entry Age 20	Entry Age 25	Entry Age 30	Entry Age 35
20.....	21,354,317.1			
21.....	16,565,393.7			
22.....	13,225,887.0			
23.....	10,829,585.7			
24.....	9,029,299.5			
25.....	7,624,763.9	6,453,010.8		
26.....	6,508,716.7	5,336,963.6		
27.....	5,608,381.3	4,513,649.0		
28.....	4,874,193.4	3,890,437.6		
29.....	4,269,356.9	3,398,289.7		
30.....	3,766,208.8	2,995,430.6	2,553,101.2	
31.....	3,343,792.2	2,659,463.5	2,217,134.1	
32.....	2,986,079.8	2,374,958.5	1,955,426.4	
33.....	2,680,695.1	2,132,072.1	1,746,532.1	
34.....	2,417,970.0	1,923,115.0	1,572,556.9	
35.....	2,190,339.9	1,742,070.4	1,423,649.3	1,314,295.6
36.....	1,991,837.8	1,584,192.4	1,294,628.6	1,185,274.9
37.....	1,817,680.1	1,445,676.6	1,181,431.1	1,076,852.7
38.....	1,664,021.9	1,323,465.0	1,081,557.6	983,867.8
39.....	1,527,737.7	1,215,071.9	992,976.8	902,427.5
40.....	1,406,286.5	1,118,476.2	914,037.0	830,322.0
41.....	1,297,586.8	1,032,022.2	843,385.3	766,141.6
42.....	1,199,936.2	954,356.1	779,915.2	708,485.1
43.....	1,111,887.7	884,327.1	722,686.3	656,498.1
44.....	1,032,201.0	820,948.7	670,892.4	609,448.3
45.....	959,817.8	763,379.2	623,845.5	566,710.9
46.....	893,840.0	710,904.1	580,961.9	527,755.3
47.....	833,487.5	662,903.2	541,734.7	492,121.3
48.....	778,089.5	618,842.9	505,727.7	459,412.6
49.....	727,059.9	578,256.8	472,560.0	429,283.0
50.....	679,880.9	540,733.4	441,895.2	401,427.0
51.....	636,099.3	505,912.1	413,438.6	375,576.9
52.....	595,303.7	473,465.6	386,922.9	351,489.8
53.....	557,124.9	443,100.4	362,108.0	328,947.7
54.....	521,230.1	414,551.8	338,777.6	307,754.1
55.....	487,315.3	387,578.1	316,734.2	287,729.5
56.....	455,097.6	361,954.3	295,793.9	268,707.0
57.....	424,312.7	337,470.0	275,784.9	250,530.5
58.....	394,919.4	314,092.6	256,680.4	233,175.7
59.....	366,877.0	291,789.5	238,453.9	216,618.4
60.....	340,146.0	270,529.4	221,079.8	200,835.4
61.....	314,688.1	250,281.8	204,533.1	185,804.1
62.....	290,466.4	231,017.5	188,789.9	171,502.7
63.....	267,446.4	212,708.8	173,827.8	157,910.8
64.....	245,595.7	195,330.1	159,625.7	145,009.3
65.....	224,884.8	178,857.9	146,164.4	132,780.8
66.....	205,287.2	163,271.2	133,426.8	121,209.6

TABLE 11—*Continued*

Age	Entry Age 40	Entry Age 45	Entry Age 50
20.....			
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40.....	790,935.5		
41.....	726,755.1		
42.....	670,255.3		
43.....	620,199.0		
44.....	575,282.5		
45.....	534,729.5	523,446.6	
46.....	497,972.3	486,689.4	
47.....	464,349.0	453,314.2	
48.....	433,485.9	422,862.3	
49.....	405,056.4	394,943.6	
50.....	378,772.2	369,225.8	360,686.3
51.....	354,380.8	345,449.2	336,909.7
52.....	331,653.0	323,294.2	314,894.5
53.....	310,383.0	302,560.3	294,459.2
54.....	290,385.4	283,066.8	275,364.6
55.....	271,490.9	264,648.6	257,387.7
56.....	253,541.9	247,152.1	240,371.1
57.....	236,391.2	230,433.7	224,111.3
58.....	220,015.8	214,471.0	208,586.5
59.....	204,392.9	199,241.9	193,775.2
60.....	189,500.6	184,725.1	179,656.6
61.....	175,317.6	170,899.6	166,210.4
62.....	161,823.3	157,745.4	153,417.2
63.....	148,998.5	145,243.8	141,258.6
64.....	136,825.1	133,377.2	129,717.6
65.....	125,286.7	122,129.5	118,778.6
66.....	114,368.6	111,486.5	108,427.6

COST OF VESTING IN PENSIONS

WILLIAM F. MARPLES

INTRODUCTION

ONE of the major trends in retirement plans is toward the extension and development of vesting provisions. The trend has been given force by automation with its concomitant involuntary mobility of the labor force and has been given direction by collective bargaining, where the argument is heard that benefits are the equivalent of contributions which are the equivalent of wages and that therefore accrued benefits should be permanently available to the employee. This contention receives encouragement from government, partly by force of example in the OASDI system of the United States and the Old Age Security System in Canada; partly by direct pressure, such as legislation on portability in Canada; and partly by indirect pressure through the machinery for granting tax exemption for contributions and investment yield.

Public reports and published books have recently focused so much attention on vesting that a discussion of the mathematics of the determination of cost is timely. This paper presents a theoretical treatment of the determination of the cost of vesting by a method not hitherto demonstrated to the Society. It is followed by illustrative calculations of cost factors developed on the basis of certain actuarial cost determinants.

DEFINITION OF VESTING

Termination of employment arises from inability to continue to work, discharge for cause, or merely voluntary termination of employment. In employments where pensions are provided, in some or all of these circumstances, vesting may occur. A vested benefit, in pension plan parlance, is a benefit to which an employee has a right, contingent upon survival, but unaffected by his work history subsequent to his acquiring the right.

It may be noted that this definition differs from the legal definition of a vested right which indicates a nondefeatable right to current enjoyment of the benefits or the object to which the rights attach. For example, in a single-employer pension plan a vested right may be acquired upon attainment of 50 years of age and the accumulation of 15 years of credited service. To such an employee his vested right is not defeated by subse-

quent termination of employment, but it would be defeated by death prior to the normal retirement age since his vested pension becomes payable only when he survives to the normal retirement age.

In a multi-employer pension plan, accumulation of pension credits may be related to hours of work credited under the plan and may result from hours of employment by more than one employer. The accumulation of pension credits is not terminated by change of employer since the plan specifically provides for this type of vesting. However, accumulated pension credits could be canceled by failure to work minimum hours in employment covered by the plan, thus making this type of vesting contingent upon preserving a current working status. At some age and accumulated total of service credits, the vesting provision may assume a form similar to that seen in single-employer plans, where the accrued pension rights are then not defeatable by subsequent changes in employment patterns. In both types of plan, the vested right is usually a right to a deferred pension contingent upon survival to the earliest age at which a normal pension can be claimed and is thus defeatable by death.

When an employee retires upon pension, he acquires a vested right to his pension. This means that he can collect his pension each month if he is alive at the payment date and provided the plan has sufficient assets to make the payment. The vesting rights here also are seen to be contingent. In fact, the only way of satisfying the legal definition is to pay a lump sum in lieu of pension, and this procedure runs contrary to the purpose and the philosophy of the pension plan.

FACTORS AFFECTING THE COST OF VESTING

In a retirement plan covering a stated list of participants, the basic items in determining the theoretical cost to the employer of the plan benefits are the estimates of the number of survivors of the participants to retirement age, of the amount of the benefits provided for each retiring participant, and of the time interval until those benefits become due. The method described below uses the costs of nonvested benefits as the basis for determining the cost of vesting. Vested benefits mature at the same time as nonvested benefits. Vested benefits become "frozen" at an age and duration prior to maturity and mature on survival of the deferment period. Regular valuation functions can thus be adjusted by this additional survivorship to provide the value of accrued and prospective vesting benefits.

In establishing the survivorship, the use of mortality rates only is appropriate since termination of employment can no longer cancel entitlement to benefit. The deferment in time must involve the use of time

and the same rate of interest as that employed in the basic calculation of pension values, in order to make use of these relationships.

The value of prospectively vesting pension benefits will be calculated as a percentage of the normal contribution rate required or as a percentage of the value of accrued or prospective nonvested pensions for participants covered by the plan. These percentages will be referred to as "vesting cost ratios." The method will be found to be very flexible. The value of vesting in any segment of the benefits is directly related to the value of that segment of the nonvested benefits and, correspondingly, to the contributions required to provide it. In consequence, the cost of vesting may be incorporated into any of the recognized systems of funding prospective pensions.

The computed vesting cost ratios will vary (in size) according to (a) the annual rates of termination of employment used in the calculations, (b) the benefits of the plan, (c) the vesting formula, and (d) a vesting benefit claim rate where there are alternative benefits upon withdrawal.

It may be noted that there cannot be any theoretical additional cost of vesting unless the original calculation incorporated a reduction of cost on account of anticipated terminations of employment. If there are no anticipated terminations, the cost of vested rights is an implicit part of the calculated cost of the whole plan. An unanticipated termination in such circumstances produces an experience gain. A lesser gain develops if some part of the gain is earmarked for a vested benefit.

Since the vesting of benefits is usually effective at some stated age or duration of participation in the plan or when a combination of these criteria is satisfied, it will generally be found necessary to develop the calculations of actuarial liabilities by entry age and duration in order to be able to apply vesting cost ratios to the appropriate actuarial cost values. Whether the computation of cost is made by an accrued-benefit method or by a projected-benefit method, the dollar amount of the cost of the vesting benefit according to the rules of the plan will require determination under two heads: (1) the amount to be added to the annual cost and (2) the amount to be added to the estimate of pension liabilities in respect to (a) benefits already vested in inactive participants and (b) benefits expected to vest in the future in respect to participants now active but expected to terminate participation in the future.

For liabilities under 2(a), that is, terminated vested former participants, direct calculation is effected on the basis of the amount of vested pension and the age of the former participant. To determine liabilities under 1 and 2(b), the vesting cost ratios method may be applied in each case.

DEVELOPMENT OF PRIMARY VESTING COST RATIOS

No mathematical problems arise in the treatment of a vested immediate annuity, but the treatment of vesting rights in a deferred pension requires some analysis. Consider now how a vested benefit changes the status of the participant. If he quits before vesting, his accrued pension lapses and may not be restored on re-employment. A vested participant who quits work, however, remains entitled to a pension at the normal retirement age, and the present value of such liabilities, with respect to all such pensions, remains a claim on the assets of the plan. The gross liabilities of the plan are thus increased by the value of the deferred pensions to former participants with vested rights.

There remains the question of determining the value of prospective vested rights which may be expected to arise from the current group of participants, some of whom may have to complete many years of employment before becoming entitled to vested rights. In brief, the vested benefit turns a termination of employment with complete severance of connection with the pension plan into a cessation of the employer-employee relationship(s) with a contingent liability in the pension plan covering such former participants. This concept of continuing relationship is the basis of the mathematics which follow.

Suppose the pension plan liabilities are determined on the basis of a service table exhibiting the decrements of death, withdrawal, and disablement. The last decrement is not essential and would be merged in whole or in part into the first two if a benefit on disability were not payable from the plan. In order that a cost of vesting may be separately determinable, it is necessary to show deaths separately from terminations of employment. If there are no termination decrements, then there is no separately projectable cost of vesting. This is so because, where death is the sole decrement, termination of employment occurs but the survival of any deferment period does not and so the benefit and its present value do not exist. The equation determining the service table figures is

$$l_x - l_{x+1} = d_x + w_x + ir_x. \quad (1)$$

Summing from x to r (the normal retirement age), we have

$$l_x - l_r = \sum_{t=0}^{r-1} d_{x+t} + \sum_{t=0}^{r-1} w_{x+t} + \sum_{t=0}^{r-1} ir_{x+t}. \quad (2)$$

In words, the difference between the number of persons who start at age x and survive to age r is the sum of the deaths at the intervening ages, the terminations at the same ages, and the disablements before the

final age, and equation (2) holds even if values of any decrements are 0 at any age or series of ages.

In order to isolate the cost of the vesting benefit, it is necessary to assume that the rates of mortality and disablement remain unchanged, and, with that assumption, the use of rates of termination has the effect of reducing the value of l_r to a figure lower than it would otherwise be. High rates of termination reduce l_r more than low rates. The effect of vesting may be considered, from one point of view, to keep some of these terminations in the l_x column and so increase the value of l_r to a figure greater than it would otherwise have been. The expression for the increase in l_r is developed below.

From the w_{x+t} terminations in the service at age $x+t$, the survivors at age r are

$$w_{x+t} \cdot {}_{r-x-t-1/2} p'_{x+t+1/2} \quad (3a)$$

(prime indicates mortality decrement only) or

$$w_{x+t} \cdot l'_r / l'_{x+t+1/2} \cdot \quad (3b)$$

The sum of such survivors is

$$\sum_{t=0}^{r-x-1} w_{x+t} \cdot {}_{r-x-t-1/2} p'_{x+t+1/2} \quad (4a)$$

or

$$\sum_{t=0}^{r-x-1} w_{x+t} \cdot l'_r / l'_{x+t+1/2} \cdot \quad (4b)$$

The number of survivors at age r adjusted for the vesting provision is now

$$l_r + \sum_{t=0}^{r-x-1} w_{x+t} \cdot {}_{r-x-t-1/2} p'_{x+t+1/2}, \quad (5)$$

which becomes

$$l_r \left(1 + \frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot {}_{r-x-t-1/2} p'_{x+t+1/2} \right). \quad (6)$$

The primary vesting cost ratio is thus

$$\frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot {}_{r-x-t-1/2} p'_{x+t+1/2} \quad (7)$$

or

$$\frac{1}{l_r} \sum_{t=0}^{r-x-1} w_{x+t} \cdot l'_r / l'_{x+t+1/2} \cdot \quad (8)$$

Formulas (7) and (8) show the proportionate increase in l_r from projected survivors from those who withdraw, which is used in combinations with vesting benefit provisions to develop the fully adjusted vesting cost ratios. A table of $r_{-x-t-1/2}p'_{x+t+1/2}$ (or $l'_r/l'_{x+t+1/2}$) can readily be calculated on a single decrement (mortality) table so that the operation of developing the number of survivors is a series of multiplications followed by a summation. The value of taking the ratio of the survivors of terminations to the value of l_r lies in the fact that l_r is a principal component in the formula for valuing the pension payable at the normal retirement age.

To obtain the addition to the normal cost arising from, for instance, the vesting of a portion of the normal retirement benefit, the vesting cost ratios, formulas (7) and (8) are applicable to the normal cost for normal retirement pension. They should not be applied to any other part of the normal cost.

To obtain the addition to the pension liabilities, it might have been sufficient to apply the appropriate primary vesting cost ratio to the present value of normal pension at attained ages had not the vagaries of the qualifying requirements for vesting intervened. If the qualification for 100 per cent vesting is solely the attained age, then application of the vesting cost ratios to present value of normal pension by attained age could suffice; but if, as appears to be most usual, the vesting qualifications include both age and service components, then a development of present values of normal pension by entry age and duration or some equivalent is necessary for the proper application of the ratios.

ADJUSTMENTS TO PRIMARY VESTING COST RATIOS

Before they can be applied to provide cost estimates applicable to a specific plan, the primary ratios as developed above will require adjustment under either or both of two heads: (1) the vesting qualifications and (2) the vested pension.

1. Let us assume that the entry age is x , the credited service is t , so that the attained age is $x + t$. Further, let the vesting qualification be attainment of age y and completion of n years of credited service. The formulas for the various cases which occur are as follows:

a) If (i) $y \geq x + n \geq x + t$ or if (ii) $y \geq x + t \geq x + n$, vesting occurs at age y and the primary vesting cost ratio is

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} w_{y+s} \cdot r_{-y-s-1/2} p'_{y+s+1/2} = \frac{1}{l_r} \sum_{s=0}^{r-y-1} w_{y+s} \cdot l'_r / l'_{y+s+1/2}. \quad (9)$$

The ratio at age y is applicable at all ages preceding y , and the ratios thereafter change with attained age $y + s$.

b) If $x + t \geq y \geq x + n$, let $x + t = y + k$. Then the above summation in (i) is made for values of s from k to $r - y - 1$, and the ratio is reduced accordingly.

c) If (i) $x + n \geq y \geq x + t$ or (ii) $x + n \geq x + t \geq y$, then vesting occurs at $x + n$ and the summation is adjusted accordingly by substituting $x + n$ for y .

d) If $x + t \geq x + n \geq y$, then the summation in (i) is made after substituting $x + t$ for y .

Numerical examples of the cases cited are given below:

Case	Entry Age x	Entry Age plus Duration $x+t$	Vesting Age y	Entry Age plus Vesting Minimum Duration $x+n$	Summation from:
a) (i).....	25	30	50	35\}	50 (=y)
a) (ii).....	25	37	50	35\}	
b).....	25	52	50	35	52 (=x+t=y+s)
c) (i).....	45	47	50	55\}	55 (=x+n)
c) (ii).....	45	52	50	55\}	
d).....	35	52	50	52	52 (=x+t)

2. The adjustment for the reduced vested pension has now to be introduced. It arises from the fact that the effect of applying the ratio to the value of normal pension is to provide on retirement at age r the full amount of pension accruing with respect to all service between x and r . This may not be consistent with the vested pension benefit to be provided under the plan, and, where it is not, adjustment is necessary. Consider the simple case in which a participant is credited with a pension of \$1 a year for each year of credited service. The entrant at age x has accrued \$ t a year at attained age $(x+t)$ in which he vests on termination at that age and could accrue a total of \$($r-x$) if he remained in employment until age r . By applying the primary vesting cost ratios to the value of the normal retirement pension, he would be credited, on vesting, with a pension of \$($r-x$) to commence at age r . To be consistent with the facts, this has to be cut down to \$ t by application of the ratio $t/(r-x)$. The effective vesting cost ratio, formula (9), then becomes

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot r^{-y-s-1/2} p'_{y+s+1/2} \quad (10a)$$

or

$$\frac{1}{l_r} \sum_{s=0}^{r-s-1} \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot l'_r / l'_{y+s+1/2} \cdot \quad (10b)$$

In Appendix I will be found the calculation of the vesting cost ratios on the basis of the formula and on certain assumptions regarding termination rates. The cost of a number of vesting benefit formulas can be derived from this table, as is shown in the accompanying table.

BENEFIT VESTS 100 PER CENT AT:	VESTING COST RATIOS	
	Entry Age 20	Entry Age 25
10 years of credited service.....	+45.6%	+30.2%
20 years of credited service.....	+24.4	+14.7
Age 45 and 20 years of credited service.....	+15.7	+14.7
Age 50 and 15 years of credited service.....	+ 8.4	+ 8.1
Age 55 and 15 years of credited service.....	+ 3.1	+ 3.0

It is to be emphasized that these percentage increases only apply to normal cost rates calculated upon the table described, if a pro rata vesting of pension benefits occurs and on the assumption of 100 per cent vesting in the accrued benefit.

GRADED VESTING

Not infrequently vesting may be on a graded basis, such as, for instance, 50 per cent on reaching a qualifying age or period of service (or a combination of the two) and increasing by 10 per cent for each subsequent year of credited service up to 100 per cent. This can be incorporated into the calculations by including a function f_{y+s} , where f_y is the starting percentage and f_{y+s} increases according to the scale required up to unity. The ratio then is:

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} f_{y+s} \cdot \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot r^{-y-s-1/2} p'_{y+s+1/2} \quad (11a)$$

or

$$\frac{1}{l_r} \sum_{s=0}^{r-y-1} f_{y+s} \cdot \frac{y+s-x}{r-x} \cdot w_{y+s} \cdot l'_r / l'_{y+s+1/2} \cdot \quad (11b)$$

In Appendix II is shown the calculations adjusting the values in Appendix I to a graded vesting formula. The results are compared with the previously quoted vesting cost ratios in the tabulation on page 285.

The cost ratios applicable under other grading formulas can be calculated in a similar manner. The grading adjustment procedure has to be followed for each entry age to develop vesting cost ratios applicable to entry age normal cost rates. For additions to pension liabilities, vesting cost ratios at attained ages are required. If the attained age is within the grading range, the vesting cost ratio can be taken from that table;

if the attained age exceeds the last age of the grading range, the value of the vesting cost ratio is not affected and can be taken from the ungraded table.

ADJUSTMENTS FOR BENEFITS RELATED TO SALARY

The adjustment factor allowing for benefit accruals to the date of termination assumes different forms according to the type of benefit provided by the plan. It has been shown so far on a uniform benefit accrual basis. Other forms have now to be considered.

COMPARISON OF VESTING COST RATIOS

Entry Age	Vesting 100 Per Cent at 10 Years of Credited Service	Vesting on Graded Scale 50 Per Cent at 10 Years of Credited Service Increasing by 10 Per Cent per Annum to 100 Per Cent
20.....	+45.6%	+42.3%
25.....	+30.2	+27.8

1. If the plan offers a benefit based on career average salary, and the valuation cost determinants included a salary scale S_x , then the form is

$$\sum_{s=0}^{t-1} S_{x+s} / \sum_{s=0}^{r-x-1} S_{x+s}. \quad (12)$$

When $S_x = S_{x+1} = S_{x+2} = \dots = S_{x+t} = \dots = S_{r-1}$, the expression reduces to $t/(r-x)$.

Further, because S_x can be assumed to be a function which does not decline with an increase in x , then

$$\sum_{s=0}^{t-1} S_{x+s} / \sum_{s=0}^{r-x-1} S_{x+s} \leq t/(r-x), \quad (13)$$

and consequently the value $t/(r-x)$ is the upper limit of the adjustment factor.

2. For a "final average" benefit based on the average salary of, say, the five years preceding retirement, the adjustment factor at attained age $(x+t)$ becomes

$$\frac{t}{r-x} \cdot \frac{\sum_{s=0}^4 (S_{x+t+s} + S_{x+t+s-1})}{\sum_{s=0}^4 (S_{r-s} + S_{r-s-1})}. \quad (14)$$

Again, unless S_x declines in value with an increase in x , the upper limit of formula (14) is $t/(r - x)$.

Values of the adjustment factors on two salary scales are shown in Appendix IV. Again it has to be stated that these adjustment factors only apply numerically to cost estimates calculated on the actuarial cost determinants indicated. It may seem from the vesting cost ratios in Appendix IV that the cost of vesting would be less in salary plans than it is in others and less still the steeper the salary scale. However, the vesting cost ratios may be less but they are applied to higher normal costs and thus produce higher absolute costs of vesting.

3. The method of dealing with a split-level benefit formula such as 1 per cent of salary under \$4,800 plus 2 per cent of the excess of salary over \$4,800 requires a combination of two sets of vesting cost ratios. The formula can be expressed as 2 per cent of salary less 1 per cent of \$4,800, so that the adjustment in formula (12) is required for the costs of the pension related to salary and the adjustment in formula (10) is applied to the deductive term. As a practical measure, a set of compromise values can be obtained and employed having regard to the relative weight of the two terms in the benefit formula.

CALCULATION OF VESTING COST RATIOS BY PRECEDING FORMULAS

The formulas set out above are more formidable in appearance than they are in calculation. No adjustments are required in the work sheet when select withdrawals are used. In use with a select table, at the appropriate age w_{x+t} will become $w_{[x]+t}$ in the formulas given earlier and calculation will proceed as before. In fact, whether select withdrawal rates are used or not, the calculations for many plans have to be made in select form, that is, by entry age and duration.

In Appendix I is set out the type of work sheet required for the calculation of a vesting cost ratio at every fifth entry age assuming the use of our aggregate or ultimate service table. The full calculations are shown for entry age 20 and entry age 25. The adjustments to give effect to a graded vesting formula are shown in Appendix II. The vesting cost ratios are different for service tables constructed with different rates of termination. For the same vesting formulas the vesting cost ratios are higher for heavier withdrawal rates. This is illustrated in Appendix III, where at every fifth age the vesting cost ratios are shown on the table of withdrawal rates used for Appendixes I and II together with vesting cost ratios calculated with respect to withdrawal rates of two-thirds and one-third the original withdrawal rates. The vesting cost

ratios show almost pro rata reductions because of the early 100 per cent vesting.

In Appendix IV salary scale adjustment factors are shown at fifth ages on two salary scales and are compared with the ratios without salary scale.

VESTING IN CONTRIBUTORY PLANS

Vested benefits in contributory plans are not infrequently made conditional upon the participant's leaving his contributions in the fund. If this is the case, vesting will not occur automatically upon termination of employment under conditions satisfying the vesting formula. It appears probable that a high proportion of such terminated participants will take the refund of contributions and allow their vested benefits to be voided. In such cases the cost of vesting will not emerge at the levels calculated earlier.

An estimate of the lower level of costs can be obtained by incorporating in the calculations still another adjustment factor—this time a graduated claim rate with respect to the vested benefit. Examples of graduated claim rates are to be found in Appendix XII to the *Report on the Canadian Civil Service Superannuation Fund* as of December 31, 1957. It would be a service if members of the Society would draw attention to other sources of statistics on this point.

Immediate total and permanent vesting in participant contributions occurs in most contributory pension plans, and, consequently, an increase in cost with respect to a vesting formula is only incurred on that portion of the benefit financed by employer contributions. A practical approach to the determination of this vesting cost may be made on the basis of the vesting cost ratios indicated earlier in this paper coupled with an estimate (for each entry age) of the proportion of the benefit financed by the employer. If a more sophisticated approach is warranted or desired, the full cost of the vesting benefit obtained by means of the vesting cost ratios or some other equivalent method has to be offset by the projected value at the normal retirement age of the participant contributions at termination. A valuation procedure by entry age and duration enables an estimate of the accumulated contributions of participants at termination to be derived in much the same way as the value of future pension accruals in a career average plan. If a claim rate were appropriate, this would again have to be incorporated in the calculations. The projected increase in expected survivors to normal retirement age as calculated earlier would complete all requirements for the calculation.

In a plan which is partly contributory, for example, where a basic noncontributory benefit is provided together with a supplementary benefit conditional on contributions, a combination situation occurs. The cost of vesting will depend upon the vesting benefit provided, and the actuary may find himself using one set of vesting cost ratios on the noncontributory benefit and another (incorporating, *inter alia*, a claim rate) for the contributory benefit.

EXPERIENCE GAINS AND LOSSES ON VESTING

Benefits to vested terminated participants are valued on an accrued benefit basis. They may be a source of gain if claims for benefit at the normal retirement age fail to be made. In order to avoid a mounting list of unclaimed benefits and potential escheat to a state exchequer, it may be desirable to adopt a rule of the plan voiding the benefit if it is not claimed within a definite period after the normal retirement age.

There is also the question of gains and losses on the experience of the plan. If terminations do not occur or occur in lesser numbers than expected, operating losses in respect to the major benefits will occur and the unused contributions in respect to vesting benefits will help offset the loss. If terminations occur in greater numbers than expected, gains will emerge. Some part of those gains may be absorbed in providing for the vesting benefit. In fact, the contributions with respect to the vesting benefit and more may be required to provide for the vested benefits. This should not involve the plan in over-all loss since, in order to develop vested benefits in excess of expected amounts, terminations in excess of expected numbers must first have occurred. Thus, the release of reserves on excess terminations must precede the re-absorption of those reserves on excess vesting.

COST ESTIMATES WITH MODEL PENSION FUNDS

No attempt has been made to present as part of this paper any over-all estimates of the cost of vesting as related to age and service distributions of participants in a model pension fund. For such an estimate to have relevance, it would be necessary to develop an age and service distribution which could have been the result of termination of employment experience for which the service table withdrawal rates would have been appropriate. For example, the results of applying vesting cost ratios derived from a high withdrawal rate to an age and service distribution developed from a low withdrawal experience would produce incorrect estimates of the over-all cost of vesting. A step in the direction of an over-all cost estimate would consist of assuming an age distribution

proportionate to the numbers of l_x in the service table and applying the appropriate vesting cost ratios like to like, that is, high withdrawal rate cost ratios to the l_x derived from a high withdrawal rate, medium to medium, and low to low. However, even these values of l_x stem from a number of entrants at a single entry age, and that not only is a rare feature in a pension plan but also returns us to the theoretical vesting cost ratios appropriate to that age.

A more elaborate statistical device would establish relative proportions of new entrants at selected entry ages. This expands the necessary arithmetic to the stage where access to a computer becomes desirable. Furthermore, the result is a somewhat stilted model of the mature population type, which is rarely met in practice. The estimated cost of the introduction of a vesting benefit or the change from one vesting schedule to another would in such circumstance not be applicable to any existing pension plan and would probably not be sufficiently accurate to warrant publication. The application of the appropriate vesting cost ratio to existing plans and the tabulation of results for examination would also require the statement of so many ancillary details if a sound judgment is to be made by any person other than the actuary responsible for the calculations that the project appears to be larger than can be accommodated in this paper.

CONCLUDING REMARKS

While the mathematics of determining the cost of vesting in benefits forms the major portion of this paper, the philosophical or managerial aspects must not be overlooked. The original pensions were granted on retirement from active employment, and persons who failed to stay the course failed to obtain the prize. No second thoughts were wasted on them. This is a far cry from the current concept that staying half the course might be worth a half-prize! This would have been a logical approach on the pension-equivalent-to-wage argument, but the probability is that it may have arisen from a general fair-treatment idea applied, first of all, retroactively in a case of hardship and then gradually as a basic concept applicable to all active employees.

The fact is that, as with many other things, concepts either stagnate and wither or else develop and flourish. The pension concept started on what would now be termed a "subsistence basis," developed, possibly thousands of years ago, into a cash transaction, and became consolidated in a flourishing condition under the pension fund practices of the past two or three centuries. It is now extending the philosophy of *retirement* allowances to cover early retirement, whether on account of

age or disability, and vested benefits for terminated employees not preparing to retire immediately from employment. It almost appears as if an attempt is made to establish a social right to accrued pension benefits. This "right" is entrenched in governmental systems of social security and appears, under the influence of writers on the social aspects of pensions, to be creeping over toward private pension plans.

The granting of vested rights to pensions is not favorable to management aspirations. In general, management desires to retain its skilled labor, white-collar or blue-collar, and provides a pension plan as part of the incentive to stay. The freezing of accrued benefits, in a form which can be collected later, favors the employee who wishes to break away from his employer. Occasionally, the employer concurs in the break and, in that case, the plan provisions on vesting help to forward the joint desires of the parties. In the main, however, vesting benefits aid mobility of labor and may be against the interests of management. The assistance afforded by a vested benefit to mobility of labor explains why governments are in favor of vesting, since economists have long preached that full employment rests at least in part upon mobility of labor.

Let us consider for a moment what could happen if vesting in benefits were carried to a final logical conclusion. This would seem to be a condition of affairs in which every employee vested in a pension benefit for every hour worked. A little thought suggests that, before this could be carried out or enforced, a system of uniform pension plans might have to be set up, one for *every* employer regardless of his financial or managerial capacity. In fact, rather than accomplish this, by such individual pension plans, it is probable that a central pension plan would be established and a staff set up to manage it. It needs only a short step from this to arrive at a situation in which all pension plans have been absorbed into a central government retirement system, with a gigantic financial trust to hold and invest the funds accumulating and a vast bureaucracy to manage it.

This picture cannot be contemplated with equanimity, and one wonders where the point of resistance should be established. There are many points at which resistance can be offered. The vesting benefit is clearly one of the most important points of resistance, and serious consideration should be given both by employer and employee to the consequences of unfettered development of the benefit.

This paper provides the means of determining the cost of vesting in benefits, or some part of them, accrued at termination of employment. Acceptance of these costs as a necessary burden by the employer and

enjoyment of the benefit by employees (or rather by *former* employees) should not be permitted to form a platform for continuous extension of the benefit under the guise of "liberalization." A reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer but overdevelopment of the concept might lead to unpalatable consequences. It is hoped that in the discussion which this paper will open, some consideration may be given to this philosophical aspect of the subject and that suggestions will be made with regard to the reasonable limitations to be placed on the benefits.

APPENDIX I

COST OF VESTING IN PENSIONS

(Vesting: 100 Per Cent at 10 Years of Credited Service; Service Table: q_x^d , $a-49$; q_x^w as Shown; $q_x^r = 0$ for All Ages; $S_x = 1$)

x	q_x^w (1)	w_x (2)	$65-x-1/2$ (3)	Survivors to 65 (2) × (3) (4)	Benefit Proportion ($x-20$)/45 (5)	(4) × (5) (6)	Σ (6) (7)	Vesting Cost Ratio (7) ÷ (8) (8)	Benefit Proportion ($x-25$)/40 (9)	(4) × (8) (10)	Σ (10) (11)	Vesting Cost Ratio (11) ÷ (12) (12)
Entry Age 20												
30.....	0.0555	2,143	0.7822	1,676	0.2222	372.4	7,118.6	0.4594	0.250	245.2	4,671.8	0.3015
31.....	0.0525	1,912	0.7830	1,497	.2444	365.9	6,746.2	.4354	.275	248.9	4,426.6	.2857
32.....	0.0495	1,706	0.7839	1,337	.2667	356.6	6,380.3	.4118	.300	241.5	4,177.7	.2696
33.....	0.0480	1,570	0.7848	1,232	.2889	355.9	6,023.7	.3888	.325	241.8	3,936.2	.2540
34.....	0.0450	1,401	0.7858	1,101	.3111	342.5	5,667.8	.3658	.350	240.4	3,694.4	.2384
35.....	0.0420	1,247	0.7869	981	.3333	327.0	5,325.3	.3437	.375	237.7	3,454.0	.2229
36.....	0.0405	1,149	0.7880	905	.3556	321.8	4,998.3	.3226	.400	234.4	3,216.3	.2076
37.....	0.0375	1,020	0.7892	805	.3778	304.1	4,676.5	.3018	.425	241.0	2,981.9	.1924
38.....	0.0360	941	0.7905	744	.4000	297.6	4,372.4	.2822	.450	235.4	2,740.9	.1769
39.....	0.0345	868	0.7920	687	.4222	290.0	4,074.8	.2630	.475	228.9	2,505.5	.1617
40.....	0.0330	799	0.7935	634	.4444	281.7	3,784.8	.2442	.500	222.0	2,276.6	.1469
41.....	0.0315	737	0.7951	586	.4667	273.5	3,503.1	.2261	.525	214.2	2,054.6	.1326
42.....	0.0315	711	0.7970	567	.4889	277.2	3,229.6	.2084	.550	206.3	1,840.4	.1188
43.....	0.0300	655	0.7991	523	.5111	267.3	2,952.4	.1905	.575	196.6	1,634.1	.1054
44.....	0.0285	601	0.8015	482	.5333	257.1	2,685.1	.1733	.600	187.2	1,437.5	.0928
45.....	0.0270	552	0.8044	444	.5556	246.7	2,428.0	.1567	.625	177.5	1,250.3	.0807
46.....	0.0255	505	0.8074	408	.5778	235.7	2,181.3	.1408	.650	167.1	1,072.8	.0692
47.....	0.0240	462	0.8109	375	.6000	225.0	1,945.6	.1256	.675	156.6	905.7	.0585
48.....	0.0225	420	0.8150	342	.6222	212.8	1,720.6	.1110	.700	145.6	749.1	.0483
49.....	0.0210	381	0.8196	312	.6444	201.1	1,507.8	.0973	.725	132.7	603.5	.0389
50.....	0.0195	345	0.8247	284	.6667	189.3	1,306.7	.0843	.750	121.5	470.8	.0304
51.....	0.0180	309	0.8305	257	.6889	177.0	1,117.4	.0721	.775	107.7	349.3	.0225
52.....	0.0165	277	0.8369	232	.7111	165.0	940.4	.0607	.800	94.4	241.6	.0156
53.....	0.0150	246	0.8440	208	.7333	152.5	775.4	.0500	.825	80.0	147.2	.0095
54.....	0.0135	215	0.8519	183	.7556	138.3	622.9	.0402	.850	67.2	67.2	0.0043
55.....	0.0120	188	0.8606	162	.7778	126.0	484.6	.0313				
56.....	0.0105	160	0.8702	139	.8000	111.2	358.6	.0231				
57.....	0.0090	134	0.8807	118	.8222	97.0	247.4	.0160				
58.....	0.0075	109	0.8923	97	.8444	81.9	150.4	.0097				
59.....	0.0060	87	0.9050	79	.8667	68.5	68.5	0.0044				
60 and over.....												

APPENDIX II

COST OF VESTING IN PENSIONS

(Vesting at 10 Years 50 Per Cent + 10 Per Cent per Year to 100 Per Cent at 15 Years;
Service Table: q_x^d , $a-49$; q_x^w , as Appendix I; $q_x^{*w} = 0$ at All Ages)

x	Entry Age 20					Entry Age 25				
	SURVIVORS × BENEFIT PROPORTION (1)	VESTING PROPORTION (2)	(1) × (2) (3)	Σ (3) (4)	VESTING COST RATIO (5)	SURVIVORS × BENEFIT PROPORTION (6)	VESTING PROPORTION (7)	(6) × (7) (8)	Σ (8) (9)	VESTING COST RATIO (10)
Appendix I, Col. 4 30..... 31..... 32..... 33..... 34..... 35..... 36..... 37..... 38..... 39..... 40.....	372.4 365.9 356.6 355.9 342.5 327.0 Other Values as Appendix I Col. 6 Col. 7	0.5 0.6 0.7 0.8 0.9 1.0	186.2 219.5 249.6 284.7 308.2 327.0	6,573.5 6,387.3 6,167.8 5,918.2 5,633.5 5,325.3	0.4242 .4122 .3980 .3819 .3636 0.3437	245.2 248.9 241.5 241.8 240.4 237.7	0.5 0.6 0.7 0.8 0.9 1.0	122.6 149.3 169.0 193.4 216.4 237.7 etc.	4,305.1 4,182.5 4,033.2 3,864.2 3,670.8 3,454.4	0.2778 .2699 .2603 .2494 .2369 0.2229

APPENDIX III

COST OF VESTING IN PENSIONS

(Vesting Cost Ratios: 100 Per Cent Vesting at 10 Years of Service;
Service Table: q_x^d , $a-49$; q_x^w , as Shown; $q_x^r = 0$; $S_x = 1$)

x	TERMINATION TABLE NO. 1			TERMINATION TABLE NO. 2			TERMINATION TABLE NO. 3		
	q_x^w	Vesting Cost Ratios		q_x^w	Vesting Cost Ratios		q_x^w	Vesting Cost Ratios	
		Entry Age x	Entry Age 20, Attained Age x		Entry Age x	Entry Age 20, Attained Age x		Entry Age x	Entry Age 20, Attained Age x
		(1)	(2)		(4)	(5)		(7)	(8)
20.....		0.4565	0.4565		0.3248	0.3248		0.1447	0.1447
25.....		.3015	.4565		.2218	.3248		.1036	.1447
30.....	0.0555	.1954	.4565	0.0370	.1484	.3248	0.0185	.0697	.1447
35.....	.0420	.1176	.3437	.0280	.0914	.2521	.0140	.0439	.1147
40.....	.0330	.0610	.2442	.0220	.0483	.1846	.0110	.0236	.0863
45.....	.0270	0.0224	.1567	.0180	0.0180	.1214	.0090	0.0088	.0582
50.....	.0195		.0973	.0130		.0666	.0065		.0325
55.....	.0120		.0313	.0080		.0251	.0040		.0124
59.....	0.0060		0.0044	0.0040		0.0036	0.0020		0.0017
60.....									

Vested benefit: $t/(65 - x)$ of benefit at normal retirement age.

APPENDIX IV COST OF VESTING IN PENSIONS SALARY SCALE ADJUSTMENTS

ENTRY AGE 20										ENTRY AGE 25			
No. 1: $S_x = 1$ ($x - 20$)/ (65 - 20) (1)		No. 2: $S_x = p^x - x$, 3 Per Cent			No. 3: $S_x = S3^*$			No. 1: $S_x = 1$ ($x - 25$)/ (65 - 25) (8)		No. 2: $S_x = p^x - x$, 3 Per Cent		No. 3: $S_x = S3^*$	
		S_x (2)	Career Av. Adjustment Formula (8) (3)	Final Av. Adjustment Formula (10) (4)	S_x (5)	Career Av. Adjustment Formula (8) (6)	Final Av. Adjustment Formula (10) (7)			Career Av. Adjustment Formula (8) (9)	Final Av. Adjustment Formula (10) (10)		
30.....	0.2222	3.554	0.1236	0.2890	0.0847	0.0571	0.2500	0.1520	0.1030	0.1192	0.0853		
35.....	.3333	4.120	.2006	.3770	.1504	.1180	.3750	.2467	.1791	.2068	.1640		
40.....	.4444	4.776	.2898	.4758	.2349	.1943	.5000	.3564	.2768	.3160	.2717		
45.....	.5556	5.537	.3932	.5838	.3403	.3019	.6250	.4835	.4012	.4490	.4128		
50.....	.6667	6.419	.5131	.7051	.4685	.4403	.7500	.6310	.5581	.6089	.5950		
55.....	.7778	7.441	.6521	.8403	.6228	.6170	0.8750	0.8019	0.7550	0.7960	0.8072		
60.....	0.8889	8.626	0.8132	0.9450	0.8032	0.8200							

* *Actuary's Pension Handbook*, by T. F. Crocker, H. M. Sarason, and B. W. Straight (published by the authors).

DISCUSSION OF PRECEDING PAPER

ROBERT F. LINK:

This discussion takes off from Mr. Marples' "Concluding Remarks," particularly the invitation in his last sentence to consider philosophical aspects.

People are quite excited these days about vesting (among other things). Some of them are dissatisfied with the current state of vesting provisions, or lack thereof, in pension plans. They hope to see significant or even drastic improvement.

Thinking of unilateral plans, the primary agency for improvement is employers. Why should employers introduce or enlarge vesting? Perhaps they have some "social" aims. For example, they may just think vesting is "good." Or they may think that enlarged vesting will inhibit increases in social security, and they consider this good.

Aside from social aims, an employer has two major objectives in setting up or continuing a pension plan. These are (1) to help attract and hold desirable employees and (2) to facilitate the removal of employees whose usefulness (either in fact or by adopted criteria) has substantially ended. We would expect employers to include vesting provisions in their plans that they regard as appropriate to further both these objectives. Such vesting provisions are likely to be measured against the norms in the labor markets in which an employer competes. If he gives too little vesting, an employer must balance this by other attractions. If he gives too much, he must justify the resulting cost by some alleged or demonstrated advantage.

If employers do not increase vesting of their own accord fast enough to appease critics of the present situation, we should not be surprised. Are there other forces to accelerate progress toward greater vesting? Employees covered under unilateral plans are probably an ineffective force. They choose an employer for a variety of reasons, and the pension plan is usually a minor factor. Most employees have no voice in the terms or management of unilateral plans.

Union employees, of course, have a very effective voice. Vesting under collectively bargained pension plans will increase about as fast as it is demanded. Any such demand, of course, reflects a priority for vesting as against alternatives such as higher benefits for age retirement.

The other major potential influence is government. The states have in general not been heard from. It seems that the federal government

wants earlier vesting. There are several levers that may be pushed to encourage the desired result. These include (1) specific vesting provisions as a requirement for plan qualification; (2) the "second layer" idea; (3) the threat of increased social security; and (4) possible tax incentives.

Mr. Marples' paper reflects a widely held view that private pensions are a private matter and that governmental requirements on such subjects as vesting represent unwarranted interference. This is a difficult and confusing subject. In the interest of balanced consideration, I want to list several points supporting the opposite view, with brief comments on each:

1. *Mobility of labor.*—It is alleged that absence of vesting ties employees to unwanted jobs and prevents the most effective employment of the labor force. There is little statistical evidence. Of course, many of us are acquainted with situations in which an employer and an employee would both be quite happy to part company, except for the destruction of a valuable pension expectation. It is a question whether such cases justify a major change.

2. *Frustrated pension expectations.*—There is concern, expressed by Bernstein and others, that too many persons covered by private pensions will receive no pension or a much smaller pension than they had hoped for. Serious questions have been raised about the over-all effectiveness of the private pension industry's leg of the three-legged stool. It would be pleasant to reply with facts.

3. *Pensions as compensation.*—A critic could say that the compensation afforded by many pension plans is quite strange. Suppose a plan provides vesting after fifteen years. This plan could be described as providing nothing for the first fifteen years of service, a whopping deferred-benefit bonus at the end of that period, and reasonable deferred benefits for service thereafter. These benefits bear little relationship to the value of the services rendered by the employee in expectation of a pension.

4. *Individual financial planning for retirement.*—An employee covered by the plan described above is considering what he should personally save for retirement. If he were sure that he would be employed for fifteen years, he might save at one level. The uncertainty on this point may cause him to save at a higher rate (or, more likely, to lie awake nights worrying because he has not). In short, it is difficult to make rational allowance for some typical pension plans in establishing one's personal savings program.

5. *Federal income tax treatment of private pensions.*—This one is perhaps more of an excuse than a reason. It is alleged that federal government intervention in private pensions is justified by the existence of a substantial tax subsidy. We in the private pension industry should have a better articulated position on this complicated subject. In a rather convincing article, entitled "The Myth of Special Tax Concessions for Qualified Pension Plans" (*Iowa Law Review* [Spring 1966]), Raymond Goetz argues that special tax concessions for qualified pension plans are a myth.

Most of the arguments for greater vesting derive more or less from the concept of pensions as compensation. The arguments against are associated with the concept of pensions as a reward for faithful service. An interesting socioeconomic study could be made of (a) how the present characteristics of pensions derive from the conditions of earlier times, (b) how these two concepts of pensions relate to the historical background, and (c) which concept better serves the needs of society today.

The actuary has a problem. He is inclined by nature to be conservative and to support freedom of contract and so forth. He is concerned about the interests of his pension clients and wary of threatened governmental action that is potentially adverse to those interests. On the other hand, he sees that governmental action might take a more palatable form if he were to contribute constructively in the planning stages. We should not sacrifice any principles. However, I wonder whether we would do better in some cases to help show how something can be done rather than concentrating on why it cannot or should not be done.

By way of illustration, some have suggested that any mandatory vesting should apply only to future-service benefits. For this purpose, future-service benefits would be those accruing for service after a plan is introduced or after a plan is amended to introduce the mandated vesting. Such an approach would greatly reduce vesting costs. It seems to go as far as necessary to meet the current criticism. However, many plans do not directly associate benefit accruals with periods of service. Pension experts have pointed out the difficulties of defining a benefit for vesting purposes under such plans. An effort to split an accrued pension into past and future service would compound these difficulties.

If anyone can solve these problems, it is the actuaries. Maybe we should give it a serious try. There is scope here to substitute new facts for old impressions, whether or not we advocate mandatory vesting.

WILLIAM K. WHITE:

I am certainly in full accord with the introductory comment in Mr. Marples' interesting paper to the effect that a discussion of vesting is most timely. This seems to be an area in which there is a dearth of literature in the *Transactions*, and thus we are especially fortunate to have two papers at this time presenting somewhat different mathematical techniques for estimating the possible cost of vesting. I am sure that the mathematical concepts presented in both these papers will be useful, timesaving tools to many actuaries for many of their pension plan valuations.

The comments in this discussion will be confined primarily to what

I shall term the practical, rather than the theoretical, aspects of the subject and perhaps to a considerable extent to what these papers did not say rather than to what they did say.

While many of the following comments will apply, to a considerable extent, equally to Mr. Marples' paper and the one by Mr. McGinn, they are being appended to Mr. Marples' primarily because it seems to delve into the practical considerations of vesting cost somewhat more than Mr. McGinn's paper does.

It is well known that many unions have become increasingly interested in having vesting provisions in the pension plans that they are negotiating, and many employers with unilateral plans have been adding a vesting provision when none existed or liberalizing an existing one. More recently, great interest in the subject of vesting has been generated by the "Report to the President on Private Employee Retirement Plans" by the President's Committee on Corporate Pension Funds (hereinafter referred to as the President's Committee's Report) issued in January, 1965. The Committee concluded that some form of mandatory vesting should be legislated, but it reached this conclusion on the basis of certain pardonable misunderstandings. I use the term "pardonable" because they were based on quite popular misconceptions which actuaries should be able to clear up but seem to have done very little about heretofore. For example, the President's Committee's Report opens the section on vesting with the following statement:

A vesting provision, as usually defined, *guarantees* to pension plan participants whose employment is terminated before becoming eligible for a retirement benefit the right to all or part of their accrued pension benefits at retirement age, regardless of their employment status at that time [emphasis added].

Mr. Marples is to be commended for having exploded this myth when he points out near the end of the second section that the guarantee exists only "provided the plan has sufficient assets to make the payments." When people of the stature of those comprising the President's Committee have such a basic misunderstanding, it is not surprising that it is very widespread. Because of the importance of this point, it might well have been stressed considerably more in the paper.

Perhaps the area of greatest public misunderstanding on the subject of vesting is the very title of this paper "Cost of Vesting in Pensions." There is far too prevalent an impression among laymen that it is only necessary to develop a reasonable set of turnover or termination rate assumptions, apply an actuarial formula, and out comes the cost of vesting. While Mr. Marples does refer to "theoretical" or "estimated" costs in several instances, it is my impression that the paper does so in such a

way as to lend further credence to this popular misconception rather than to scotch it. Perhaps this can best be illustrated by the section headed "Factors Affecting the *Cost* of Vesting" (emphasis added). The first sentence states that, among the items needed to determine the "theoretical cost," are *estimates* "of the number of survivors" and "of the amount of benefits." With such tools, we find in the next sentence a statement that we are now ready to develop a method "for determining the cost of vesting." While an actuary would subconsciously read into all this that only estimates are being considered throughout, the layman could hardly be blamed for taking these statements as authoritative confirmation that the impression described above is correct—that is, that, given estimates of turnover or survivors, one can readily determine costs, *real* costs.

It is unfortunate that this very important difference between estimated and true costs was not brought out clearly, and it could have been emphasized by a more careful choice of words. In my opinion it is vitally important that this be made eminently clear lest employers and unions adopt vesting provisions with an erroneous understanding of what their cost really is. Of perhaps even greater importance is the avoidance of governmentally mandated vesting requirements based on such a misunderstanding of costs. This is by no means remote, when one considers the following statement from the President's Committee's Report:

On the basis of available information and actuarial estimates, the Committee is convinced that a modest vesting requirement, with rare exceptions, would not have a substantial impact on pension costs. Precise cost estimates will, of course, depend on the final detailed specifications of the vesting requirement.

Although a precise estimate is a somewhat unrealistic concept, at least reference is made to "estimates." However, when one considers the following summary quotation, it seems evident that the Committee suffered from this common misunderstanding:

The most complete available information indicates that the added cost of adopting the basic features of the Committee's recommendation would be quite modest; that is, under 5 percent for a large majority of pension plans. Rarely would the added cost exceed 10 percent of present expenditure, and special provision should be made available for any plan where the cost of the vesting recommendation . . . would have this effect.

It is hard to believe that in making this statement the Committee was thinking of anything but the estimates as representing precise costs. It is equally hard to believe that they could possibly have been contemplating

a situation in which, say, ten years hence, it is found that the proposed vesting provision has actually cost perhaps 35 per cent instead of the anticipated, let us say, 8 per cent.

The misunderstanding is equally apparent in the following question taken from the agenda of the Federal Interagency Task Force on Private Pension Plans for its meetings with several private groups: "How significant are the differences in cost between 5-, 10-, and 20-year vesting standards?"

Since this question was drafted, the Task Force has had several meetings, and I am confident that it would not ask the same question in the same form today. This group is to be commended for the earnestness with which it is seeking information on pension matters prior to considering possible legislation and for its interest in implementing the Society's motto of substituting facts for appearances. It seems evident, however, that initially it had the common misconception that actuarial estimates are real cost figures.

In reading Mr. Marples' paper, I was looking forward to the possibility of the section entitled "Experience Gains and Losses on Vesting" clearing up the misunderstanding between actuarial estimates and true costs. Instead, however, about a third of this brief section that might have been the most important in the paper relates to the "Enoch Arden" clause used almost universally—in this country at least—to provide for canceling benefits if they are not claimed within a period of time stated in the plan.

The remainder of this section gives a very broad-brush statement of the impact of gains and losses, which is correct only in limited situations. Consider, for example, the following statement: "In order to develop vested *benefits* in excess of the *expected amounts*, terminations in excess of the expected *numbers* must first have occurred" (emphasis added). On the contrary, it is well within the realm of possibility that the *number* of terminations may be equal to or less than expected over a period of time, but the total vested *benefits* could be greater than expected because the average amount per individual is greater than expected, perhaps because the terminations occur at a later age than anticipated. The fact that the individuals who were expected to terminate stayed to retirement and thereby receive the full benefit makes the resulting loss all the greater.

To further emphasize that it is not just the relation of actual to expected number of terminations that determines whether there will be a gain or loss, consider a case with the experience of vested terminations being heavier than expected and at the same time the average vested

benefits also being greater than expected. Is there an experience gain or loss? Actually, the answer can go either way, depending upon the degree of deviation in each item in the particular case at hand.

All this points up the fact that the average amount of vested benefits compared with the expected is just as important as the number of terminations compared with the expected. In fact, to make a complete analysis, the number of actual to expected terminations before and after attaining the vesting requirements should be considered separately.

A more careful and detailed explanation of the impact of gains and losses on vesting costs, together with an analysis in layman's language of the variety of consequences, would be a valuable addition to actuarial literature.

If, then, actuarial cost estimates do not represent the real cost of vesting, the question might be asked whether they represent what might reasonably be expected on the average. The answer to this is a qualified "Yes." The reasons for qualification are several. First, experience can deviate radically from expectations, no matter how carefully and scientifically they are developed. Second, the actuary has little to go on in this area except past experience, and there is a considerable degree of uncertainty as to whether historical experience is going to be appropriate to the future in any given situation. For example, the mere addition of a vesting provision to a plan that did not have one previously may well change the pattern of terminations, and the more liberal the plan is the more dramatic the change is likely to be. This is based on the premise that those who might be thinking of changing jobs just prior to completing the vesting requirement will often tend to stay on their present job, just long enough to complete the vesting requirement.

As another consideration, there is nearly an endless list of factors which will result in variations in the cost of vesting. Without going into detail, it is probably fair to say that nearly all the factors that would affect the true cost of vesting would fall into one or more of the following broad categories: (a) characteristics and composition of the employee group; (b) economic environment; and (c) characteristics of the pension plan.

It is important to note that factors in all three of these categories can and do change from time to time and correspondingly influence the cost of any particular vesting provision over the years. Furthermore, changes in turnover patterns may be so dynamic as to make the cost of vesting one of the most difficult phases of pension costs to estimate with any sense of security.

With all these potential elements of uncertainty, then, one may well ask, "Why bother with actuarial estimates of vesting costs?" The answer to this is simply that, if a vesting provision is included in a pension plan, realistically its cost must be recognized as appropriately as possible. The actuarial estimate may very well recognize differences in turnover pattern by industry and perhaps by company within the industry; perhaps by locale within the company or within the industry, or both; and perhaps by sex. But once again we must not lose sight of the fact that this is based on historical information, and changing conditions in one or more of the three categories mentioned above can have an important effect on the appropriateness of the assumptions used.

In the final analysis, however, the actuary has little foundation for any basis of anticipating the future except (perhaps the most unlikely) that past trends will continue. Under such circumstances and particularly for smaller groups in which chance fluctuations are likely to be the greatest, it is understandable that the actuary will quite appropriately recognize the unreality of extreme refinement in vesting cost estimates and pay closer attention to how experience is developing under the plan and making appropriate changes in the assumptions as circumstances dictate. Once again, this all points up the importance of making it clear to the layman that actuarial cost estimates are just that, although they may and undoubtedly do turn out to be reasonably accurate the majority of the time. Conditions can and do develop in particular situations, however, where they turn out to be meaningless.

Mention was made earlier in this discussion of the propriety of the use of averages or of something mentioned in Mr. Marples' paper, which is closely related, model groups. To illustrate how misleading averages can be, an analysis was made recently of actual turnover experience of some ten pension plans by both life insurance company and consulting actuaries. Unfortunately, it was not feasible to relate the turnover experience to the expected cost; instead it was related to the normal cost. It was found that for a five-year-service vesting requirement the cost ranged from 3 to 93 per cent of normal cost, depending on case and year involved. For these same cases, had the vesting provision been twenty years of service, the added cost would have ranged from 0 to 24 per cent. It is interesting to note that the 0 per cent related to a group of nearly 500 employees with more than twenty years' service in a company whose over-all turnover experience would be classified as heavy. In any event, with such wide ranges developing, it is clear that it is of little solace to the company whose actual turnover experience deviates sharply in the

wrong direction from what is expected so that it averages out with another company that has gained a windfall because its termination experience is more favorable than expected.

Every actuary knows that his pension vesting cost estimates are no more than estimates and that only experience over the years will determine the true cost of any given vesting provision. He cannot, for example, prognosticate that ten years, or even one year, hence a major plant or operation will be closed down. This sort of thing happens all the time and, depending on the relative size of the employee group involved, can have a major impact on true vesting cost, either favorably or unfavorably, depending on the circumstances. With these considerations in mind, it is probably more important than ever now, with so much interest in vesting in private pension plans on the part of various branches of the federal government, employers, unions, and others, for actuaries to lift the shroud of mystery on the full cost implications of vesting provisions and to do so in such a way that it will be clearly understandable to the layman.

At the conclusion of his paper Mr. Marples invited discussion of the philosophical aspects of the subject. I am not sure whether some of the foregoing falls in that category, but, if it does, I suspect that it is not the type of philosophical commentary Mr. Marples contemplated.

WILMER A. JENKINS:

This paper includes either one too many or one too few sections. The final section, headed "Concluding Remarks" and concerned with the philosophy of vesting, should either have been omitted or made complete.

The paper develops quite fully a general statement that it makes that management desires to retain its skilled labor and, therefore, designs its pension plan to provide an incentive for employees to stay put in their jobs. The paper does not, however, mention a number of other important considerations in plan design that today are recognized by informed employers. These considerations raise the question of the validity of the author's statement that "the granting of vested rights to pensions is not favorable to management aspirations." Management aspirations are much broader than the lowest possible turnover. They also indicate that vesting is not solely for the benefit of ex-employees, as stated by the author.

The considerations to which I refer are mainly these:

1. An employer cannot keep an employee that he cannot attract to his establishment in the first place. One major purpose of a pension plan is to attract prospective employees, especially superior people, and for this purpose a good

vesting provision can be important. The smart young man may be unwilling to select the employer whose pension plan pays nothing if the man's selection proves to be the wrong one and should be changed later. The older, much-needed, experienced man can be taken on by the employer without incurring a substantial pension liability if the man already has a vested benefit from a previous employer.

2. Independence and self-respect, though intangibles, are very important to employees and, consequently, to employers. Throughout his working career—in the early, middle, and late years alike—an employee does a better job if he knows that, if his present job does not work out well for him or for his employer, he will reach retirement age with retirement income based on his years at his present job. Nonvested benefits encourage a middle-aged or older man to "rest on his oars" and merely to keep out of trouble until he reaches retirement age. They also encourage the employer to put up with such actions.

3. Paradoxically, nonvested benefits have the least holding power on the very people that the employer wants to keep the most—the highly trained, skillful, key people. A new employer can afford to compensate such a person for his loss, especially if he will be very valuable. The holding power of nonvested benefits is strongest for the mediocre and inefficient, and creates difficult problems when the unsatisfactory have to be dismissed. Therefore, the objective of management stated by the author tends to backfire, downgrading the quality of the employer's staff.

4. The author's fear of a central, governmental pension plan resulting from a requirement of universal vesting is in opposition to the fact that many proponents of federal social security cite, as their principal argument, that, because private pensions are nonvested and therefore "do not do the job," the federal vested plan should be expanded more and more, even to the point of replacing private plans. We are already seeing some of this process. Unfortunately for the private plans, the record of retirement benefits actually paid lends weight to this argument.

While the author does not mention these and other considerations, he seems to acknowledge their importance on his final conclusion that "a reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer. . . ." One cannot disagree with this conclusion, although the word "reasonable" can have many meanings. No doubt, its meaning is in practice determined in the light of prevailing practices and the degree to which they are fulfilling the fundamental purpose of providing retirement income where needed, as modified by current informed opinion among the public, labor unions, and government officials. It is these broad forces, and not only the calculated normal cost of the employer, that have determined the recent and current trend, which has been gradually to more vesting. The prevailing vesting age is no longer 65; ages of 45 or 40 or less have become

more common. The prevailing nonvested period, likewise, has shortened to fifteen years or ten years or less. Most authorities agree with the author that vesting could reach a point at which it is overdeveloped, but there are many opinions as to where this point lies. How long and how far this trend will continue, and whether the point of maximum development has been approached or reached, are questions dependent on so many broad intangibles that they are largely unanswerable.

FRANK L. GRIFFIN, JR.:

Mr. Marples has made another interesting contribution to the actuarial literature, not the least important part of which is to be found in the "Concluding Remarks" section of his paper, which deals with the philosophical aspects of vesting. My comments will be confined to this subject.

Mr. Marples has pointed out the difference between the underlying philosophies that originally led to the adoption of private pension plans and the concept of pensions as deferred compensation, which has evolved in recent years. The idea that pension contributions "belong to the employee because he would otherwise have received larger cash compensation" may be open to question, but it is nonetheless one of the mainstays of the new philosophy that is developing. Strictly speaking, this latter concept logically would result only in money-purchase plans providing fully vested benefits for future service, but proponents of the idea are not deterred by logic from applying the concept to all types of plans. I agree with Mr. Marples' observations relative to the ultimate state of affairs if the principle of vesting were carried to its final conclusion.

With the present increasing tempo of governmental intrusion into private pension plans, one may well predict the following sequence of events:

1. If we begin by requiring private plans, as a condition of favorable tax treatment, to confer vesting after a certain period of time, would this not lead to a gradual shortening of the period mandated for vesting and the institution of a relatively short period over which past-service liabilities must be amortized?
2. Would not the developments mentioned in item 1 lead some employers to drop their funded plans in favor of unqualified pay-as-you-go plans, which provide even less security to employees?
3. Would not the planners then feel it necessary to legislate compulsory funding of pension plans and to outlaw pay-as-you-go plans?
4. Would not this, in turn, lead some employers to substitute profit-sharing plans for pensions?
5. Eventually, would not the planners therefore feel compelled to legislate *compulsory private* pension plans? (A conflict in terms.)

Thus, by yielding what may appear to be a relatively small point initially, we may eventually run the full gamut of regulation.

We have come to a point at which we should examine more closely the basic objectives and philosophy of private pensions. Have we indeed reached the point where many of the provisions of such plans must be uniform, without regard to the legitimate personnel and financial objectives of private employers? Is there any greater justification for the dictation of pension provisions than there is for the dictation of wages to be paid by all employers regardless of differences between industries or within the same industry?

Who, for example, can say with assurance that immediate or near-immediate vesting is socially desirable or that employers should be denied the right to incorporate provisions designed to hold down turnover and the costs of recruiting and training? This is a far more important cost factor in some occupations than in others, as we all know.

In 1757, Benjamin Franklin wrote in *Poor Richard's Almanac*:

For the want of a nail the shoe was lost,
For the want of a shoe the horse was lost,
For the want of a horse the rider was lost,
For the want of a rider the battle was lost,
For the want of a battle the kingdom was lost—
And all for the want of a horseshoe nail.

What may seem to be innocuous pension proposals constitute a new twist to the loss of a kingdom for want of a horseshoe nail. Here the kingdom is freedom of choice under private pension plans, all of which may be lost because we were willing to give up a relatively small initial point—the horseshoe nail.

HARRY M. SARASON:

Mr. Marples has shown how to shorten some actuarial calculations for vested benefits in retirement plans by using intermediate mathematical factors for more than one purpose. The discovery and use of commutation columns are the best-known actuarial example of such a mathematical shortcut.

More actuarial formulas of this "multiple use of factors family" probably have already been made effective by those who program our computations for electronic calculating machines—at least one to my personal knowledge. Still more formulas of this general family are likely to be discovered in developing future electronic calculation actuarial programs.

We should be alert to recognize such formulas, to generalize such discoveries, to publish the discoveries, and, eventually, to incorporate them in our textbooks. But our students should always be prepared to

derive such formulas from basic functions such as q_x and i . Many formulas are derived from basic functions in the first place; this is the way programmers do much of their thinking, and, incidentally, this is the way many difficult questions are developed for our actuarial examinations.

In proving an actuarial formula, as elsewhere, "The proof of the pudding is in the eating." Actuarial formulas should be tested numerically—not tested in one instance only (it takes two to test a formula based on averages or aggregates) or even in two instances. (I speak from a sad experience. My formula worked on both ends of a series but not on the middle terms and I had to redo two weeks' work—and an assistant had warned me!)

To strike a brighter note, a method that works numerically in one or two instances may prove to be the keynote to a general formula. (I speak from several happy experiences—well, fairly happy experiences. Half an hour of numerical testing was always followed by up to five days of spare-time algebraic drudgery.)

CRAWFORD E. LAING:*

These comments arise from the "Concluding Remarks" in Mr. Marples' paper. I am addressing myself mainly to the philosophy of the subject rather than to the detailed calculations.

I agree with Mr. Marples that the philosophy of retirement provision has developed over the years from what he refers to as "a subsistence basis" to what is commonly referred to in Britain as the concept of "deferred remuneration." As soon as this latter concept makes its appearance, the case for vesting is greatly strengthened; this is particularly so when the employer is seeking to take credit, in his presentation to the employees, for the hidden fringe benefits that accrue as a result of employment by the company. Further, as stated by Mr. Marples, it is generally recognized by economists that full employment to some extent rests on mobility of labor, and vesting of pension benefits is definitely an aid to such mobility. It seems to me that management would have to take a particularly narrow and selfish view to regard mobility of labor as being against its interests. Efficient, forward-looking, and aggressive managements will, in fact, benefit from mobility of labor, in that they will be able to recruit the best brains available, when they are required for their companies' further development, without inhibitions as to pension costs if some degree of vesting has been achieved generally.

It seems to me that the important aspect to consider is the degree of

* Mr. Laing, not a member of the Society, is a Fellow of the Faculty of Actuaries.

vesting or, to put it in other words, the level of benefits that ought to vest on termination of employment. The present trend in Canada which insists, under the Pension Benefits Acts of Ontario, Quebec and Alberta, that the whole of the benefit provided under a private plan should vest at age 45 with ten years' service, no matter how generous the private plan, is a philosophy that is open to question.

I am, of course, used to the concept in Britain whereby vesting is mandatory in respect to a level of benefits equal to the maximum benefit that can be obtained by the employee under the Government Graduated Scheme, but it seems to me quite logical that in any legislation it is only open to the government to insist on a minimum level of vesting in benefits and that this should be independent of the total benefits provided under the private plan. The course that has been adopted in Canada so far seems to me to be inimical to the sound development of pension provision. This seems to be contrary to the intention of the legislation which, for example, in the case of Alberta, declares the function of the superintendent to be "to promote the establishment, extension and improvement of pension plans throughout Alberta." The legislation may have, eventually, quite the opposite effect when employers begin to recognize that a large part of the money spent on providing benefits under a formal plan is in fact going to benefit employees who will have left them before reaching retirement age.

As Mr. G. N. Calvert has stated elsewhere recently, this may undo all the good work of recent years in the building-up of private plans, funded—as they should be—over the working lifetime of employees. The trend may well revert to the informal, unregulated, and haphazard method of granting retirement gratuities or, alternatively, of leaving all the pension provision to the state.

If the alternative of a minimum level of benefit vesting at certain ages and lengths of service is adopted, the possibility of 100 per cent state pension provision envisaged above, and by Mr. Marples, should be avoided. I quite agree with Mr. Marples' penultimate paragraph, particularly with the point that the vesting provision is one of the most important points of resistance to further state encroachment. This is a "pass" which must be defended, and, if it is "sold" or allowed to go by default, it is not clear whether there is a defensible position to be taken up at a later stage.

The pension industry in Britain has been through similar vicissitudes, and it was careless enough to "sell the pass" that was so carefully prepared by Lord Beveridge in his historic report. This occurred when they allowed the state to provide graduated benefits related to earnings up

to the modest level of originally £15 per week and recently £18 per week. Although, there, the option of "contracting out" of the graduated scheme was granted originally, the conditions became progressively more onerous as the ceiling was raised, and, if the precedents set by the Conservative governments in the past are followed by the present Labour government, they could make "contracting out" a dead letter overnight. Canada has already reached a higher level of state provision, but without granting any option to contract out, and is in the course of dealing a mortal blow to the remainder of the private pension industry by the current Pension Benefits Acts. It may be too late to stem the flow, but if these views have any merit, maybe the provincial governments should have this situation brought to their notice.

It is, of course, recognized that our personal interests are probably served better by allowing the trend toward vesting. I personally have suffered considerably from the lack of any vesting requirements, having lost twenty-five years' past-service benefits (except for a minimum vested amount of £16 per annum!) when I left the insurance industry in Britain. Nevertheless, I think that it is our duty as actuaries to look at the long-term problems of the industry generally and advise accordingly, even when this conflicts with our own personal best interests.

DANIEL F. MCGINN:

Mr. Marples' paper demonstrates that he has a very intuitive grasp of the effect of vesting schedules upon the cost of pension plans. Mr. Marples has followed the classic "service table" approach in developing a method of measuring the costs of vesting pension benefits. This approach is probably typical of the method that many consulting actuaries use in valuing large pension plans. However, this approach is not very adaptable to measuring the cost of vested pension benefits for smaller pension plans that are often underwritten by an insurance company.

Some of the more interesting points that Mr. Marples' paper covers are:

1. The vesting cost ratios essentially measure, in the form of a percentage increase, the increase in the number of retirees under a pension plan which results from adoption of a specific vesting schedule.
2. The approach basically is limited to measuring the increase in normal cost on account of adopting a vesting schedule.
3. The introduction of salary scales effectively reduces the relative cost of vesting (i.e., the vesting cost ratio) even though the absolute cost of the plan, including the cost of vesting, increases as a result of introducing a salary scale. Conversely, vesting cost ratios developed on the assumption

of a level unit of pension benefit accrual would always produce higher vesting cost ratios than those developed by incorporation of a salary scale in the calculation of the ratios.

4. Vesting costs depend upon the level of withdrawal rates (rates of employment termination); that is, as turnover rates increase, the cost of vesting increases both relatively and absolutely. However, the total cost of the pension plan for a specified vesting schedule will decrease with an increase in turnover rates.
5. There is no measurable cost of vesting unless withdrawal rates (or turnover rates) are assumed in the actuary's cost calculations.

Although Mr. Marples' paper does not include a large number of vesting cost ratios, I note that the table of withdrawal rates included in his paper is quite similar to that shown as Employment Turnover Table IV of my paper. A comparison of my "Entry Age Level Annual Cost Indices" for Employment Turnover Table IV with Mr. Marples' vesting cost ratios shows a reasonable correlation. For example, for entry age 20 and 100 per cent vesting at age 30, his ratio is 45.6 per cent, while the vesting factor included in my index is 43.74 per cent. An examination of factors for other entry age-vesting age combinations with the factors in my paper for Table IV show a reasonable correlation, but the relative cost of vesting in my paper is lower because the employment termination rates stop at a lower attained age.

Basically, the formula that I have used in my paper to derive level annual cost indices is the same as shown in Mr. Marples' paper except that his formula is based on a service table and mine is based on probabilities. Although he has presented a very complete picture of the concepts underlying vesting costs, Mr. Marples could have made a stronger and more readily understood presentation if he had demonstrated in some manner how these factors would have been used in deriving numerical results for an actuarial valuation. I believe that this paper is a significant contribution to actuarial knowledge of vesting costs and should give pension actuaries an insight which will be very practical in their everyday work.

MAX BLOCH* AND CLARK T. FOSTER:

How timely the subject of vesting, its cost, and its philosophy really is may be seen from the fact that not one but two papers on it have been put before this meeting of the Society. Our discussion of Mr. Marples' paper will often make reference to Mr. McGinn's paper as well. Both

* Mr. Bloch is not a member of the Society. He holds a degree in actuarial science from the Technical University in Prague in prewar Czechoslovakia and has been in the pension field in this country for over fifteen years.

gentlemen have given us a wealth of material, much new insight, and some valuable practical help with respect to a subject which, to our knowledge, has never before received systematic treatment in the *Transactions*.

The two gentlemen, however, are not the only ones who have been wrestling with these problems over the years. We have been doing some work and have obtained results which may be of general interest.

We started where our fathers left off—with the well-known commutation function:

$$C_{x+k}^{aw} = v^{1/2} \cdot D_{x+k}^t \cdot q_{x+k}^{(w)} \cdot r^{-1/2-x-k} / \ddot{a}_{x+k+1/2}^{(12)} \quad (1)$$

In it, D_{x+k}^t stands for

$$\frac{D_{x+k}}{t_{x+k}} = D_{x+k} / \prod_{t=0}^{r-x-k-1} [1 - q_{x+k+t}^{(w)}] \quad (2)$$

It may be debatable that the $-q^{(w)}$ in equation (1) should be a dependent probability. We have used an independent probability for reasons that will soon become clear. At worst, this practice results in an overstatement which is truly negligible at the young ages, where the withdrawals really count, and not significant at the higher ages, where assumed withdrawal rates tend to be low. (At age 50, where many turnover tables end, the overstatement is $\frac{3}{10}$ of 1 per cent on the unprojected 1951 Group Annuity Table and less on projected tables.) After all, Mr. McGinn has used independent q 's which, even for moderate turnover rates, differ from the dependent q 's by much larger percentages; moreover, he implicitly assumes that terminations will occur at the end of the year, while Mr. Marples, like us, assumes that they will occur at the middle of the year.

For $r-1/2-x-k/\ddot{a}_{x+k+1/2}^{(12)}$ we used $N_r^{(12)}/\frac{1}{2}(D_{x+k} + D_{x+k+1})$. We then rewrote formula (1) as follows:

$$C_{x+k}^{aw} = \frac{q_{x+k}^{(w)}}{t_{x+k}} \cdot N_r^{(12)} \cdot \frac{2 \cdot v^{1/2} \cdot D_{x+k}}{D_{x+k} + D_{x+k+1}} \quad (3)$$

If the fraction at the end is developed in terms of i and $-q_{x+k}$ and the series for $(1+i)^{1/2}$ is broken off after the fourth term, the fraction becomes

$$\frac{2+i-\frac{1}{4}i^2+\frac{1}{8}i^3}{2+i-q_{x+k}^{(d)}} \quad (4)$$

It is immediately apparent from this form that its value is very close to 1. As Table 1 indicates, for all practical purposes its value can be

assumed to be 1. We thus recognize that our original commutation function is very close to

$$C_{x+k}^{av} \doteq \frac{q_{x+k}^{(w)}}{t_{x+k}} \cdot N_r^{(12)}. \quad (5)$$

The fraction C_{x+k}^{av}/D_x^t represents the value at age x of an annual benefit of 1 that becomes vested at the midpoint between $x+k$ and $x+k+1$.

TABLE 1
VALUES OF THE FACTOR $2 \cdot v^{1/2} \cdot D_{x+k}/(D_{x+k} + D_{x+k+1})$

AGE	INTEREST RATE			
	3%	3½%	4%	5%
1937 Standard Annuity Table				
20.....	1.000547	1.000507	1.000461	1.000352
30.....	1.000909	1.000868	1.000821	1.000711
40.....	1.002041	1.001997	1.001947	1.001831
50.....	1.004487	1.004437	1.004381	1.004253
1951 Group Annuity Table				
20.....	1.000194	1.000155	1.000110	1.000003
30.....	1.000379	1.000339	1.000294	1.000186
40.....	1.000877	1.000836	1.000789	1.000679
50.....	1.003090	1.003044	1.002991	1.002870

Dividing it by $r-x/\ddot{a}_x^{t(12)}$ leaves us with the "basic vesting ratio" $(BVR)_{x+k}$.

$$(BVR)_{x+k} = \frac{q_{x+k}^{(w)}}{t_{x+k}}. \quad (6)$$

The meaning of this function is very similar to that of Mr. Marples' "primary vesting cost ratio," but it has two important advantages. It is easier to compute, and it is entirely independent of the interest and mortality assumptions. It also readily demonstrates a few general propositions:

- a) It furnishes the theoretical background for Mr. McGinn's empirical determination of the negligible influence of variations in the interest and mortality assumptions on his level cost indices. (Mr. Marples quite correctly

never suggests any connection between vesting cost ratios and the assumed rate of interest.)

- b) The basic vesting ratio is higher at any age than the withdrawal rate at that age.
- c) The basic vesting ratio is greatest at the lowest age and decreases steadily.
- d) Basic vesting ratios increase (decrease) with the use of higher (lower) withdrawal rates.
- e) The basic vesting ratio becomes zero when $q_x^{(w)}$ becomes zero.

Our $(BVR)_{x+k}$ function can be tabulated once and forever as an adjunct to each employment turnover table and can then be readily combined with any other set of actuarial assumptions.

Mr. Marples notes in passing that his cost ratios apply equally to normal cost and to the total cost of the vested benefit; he does not mention the "accrued liability" at all. Mr. McGinn furnishes the formal proof of the influence on this accrued liability (i.e., the accumulation of past normal costs) of changes in the interest rate and gives us ample tables that should be of great help in preparing cost estimates. However, two questions come to mind: Are we, apart from cost estimates, very much in need of accrued liability indices? Do we want to compute the normal cost always in the manner taken for granted by both authors?

Where the funding method is the frozen initial liability method, the accrued liability indices can be useful in the initial valuation and again at the time the liability is "unfrozen" in view of major plan amendments. Otherwise, it is the total present value of future vested benefits that is of principal interest, and only the normal cost indices are then needed. The same is true for applications of the attained age normal and aggregate cost methods.

More serious problems are raised if, for one reason or another, a plan is to be valued on the "orthodox" entry age normal cost method. Mr. McGinn's accrued liability cost indices fall below 1 at the higher ages, and necessarily so, because the total liability for vested benefits decreases toward zero prior to age r while normal cost payments continue to age r ; as a consequence, the accrued liability for vested benefits becomes negative at those ages. Such negative liabilities are a nuisance to the actuary who understands them, and they are almost incomprehensible to the layman who has the opportunity to look at detailed valuation results.

We have found that a very convenient way out is to compute a different kind of normal cost. Instead of dividing the lump-sum cost at entry age of an employee's potential vested benefits by a temporary annuity from entry age to retirement, we divide by one that runs only

to the last age for which the rate of turnover is not yet zero. If the first age at which the turnover rate disappears is z , then no normal cost and no accrued liability exist for ages from z up. This does not render our (*BVR*) function useless. It can be applied as before, provided that the normal cost so computed is then multiplied by the fraction $(N_x - N_r)/(N_x - N_z)$.

The practical application of the (*BVR*) function, while laborious in desk calculations, presents no particular difficulties when working with a computer. Suppose that a projected pension benefit payable at retirement has been computed (with or without a salary scale) and that the present lump-sum value has been determined. It is then necessary to project the benefit to the midpoint between every two ages from the "vesting age" on (and to reduce the result on account of any vesting percentage stipulated by the plan). Each such projected benefit is multiplied by the appropriate (*BVR*) factor, and the products for all ages included in the process are summed up. Their total is divided by the pension benefit at retirement, and the ratio so obtained is applied to the present lump-sum value of the pension benefit. The result is the present value of potential future vested benefits. The year-by-year projection is not carried beyond age z . Where the present age x is equal to or higher than the "vesting age," the first age to which the pension benefit is projected is $x + \frac{1}{2}$. This description applies to career average plans of all types. Modifications for a final pay plan are not difficult, provided that the concept of the "vested benefit" under such a plan has been clearly defined.

Some plans are valued on the accrued benefit cost method, and the addition of one-year term costs of vested benefits to the future service cost is then usually considered sufficient. The customary approach is to multiply the accrued benefit of each employee who already meets the vesting conditions by a "term cost factor."

$$v^{1/2} \cdot q_{x,r-1/2-x}^{(w)} / \bar{a}_{x+1/2}^{(12)} \quad (7)$$

Where exact work is desired, this factor should be applied to the benefit that is expected to have accrued at age $x + \frac{1}{2}$. The result is the "term cost," that is, the reserve required at age x to fund in full the potential vested benefit resulting from a possible termination at $x + \frac{1}{2}$. If we multiply this term cost by the fraction

$$\frac{N_r^{(12)} / D_x^t}{N_r^{(12)} / (D_x / t_x)}, \quad (8)$$

the value of which is, of course, 1, we obtain a new form:

$$\frac{q_x^{(w)}}{t_x} \cdot \frac{2 \cdot v^{1/2} \cdot D_x}{D_x + D_{x+1}} \left[B_{x+1/2} \cdot \frac{N_r^{(12)}}{D_x^t} \right]. \quad (9)$$

We see immediately that the first fraction is $(BVR)_x$ and that the second fraction can be "omitted," that is, assumed to be 1. The product in brackets is the sum of the accrued liability at x and one-half of the future service cost for the year beginning at x . Thus the labor usually devoted to computing "term cost factors" in accordance with formula (7) is unnecessary. The calculation of basic vesting factors instead provides an easy way of obtaining the desired results, since these factors are applicable to liabilities and costs which must be calculated in any event.

One-year term costs which, at least in theory, do not serve to build up reserves for future vested benefits may be considered to be inadequate and, therefore, undesirable. To some people, their simplicity is appealing; these people might contend that vesting represents only a "fringe benefit" when measured against the principal benefits of a pension plan, so that the inclusion of a term cost for vesting might be sufficient even where the principal benefits are valued on a level cost method. It would also be possible to take the opposite view: where principal benefits are valued on the accrued benefit method, vested benefits should still be valued on level cost in order to provide reserves.

While the practical significance of term costs may be debatable, their theoretical meaning is substantial and deserves a brief discussion. For an employee who has reached "vesting age," the accrued liability for vested benefits under a level cost method decreases from year to year until it reaches zero at age z . Without developing all the symbols that would be needed for a formal proof, we may say that this is so even though another annual normal cost is added to the accrued liability at age x , as well as interest and an expected release of reserves. What causes the reserve at age $x + 1$ to be smaller is the reduction of the reserve at x by an annual term cost with interest. It should be no surprise that our (BVR) function appears in formula (9). The true state of affairs is that the (BVR) function is basically a term cost function and that it appears in the level premium formulas because the annual reduction of level premium reserves is equal to the term cost for that year (with interest).

In order to demonstrate that the (BVR) function is related to term cost, we make the artificial assumption that all terminations occur at the beginning of the year. At that point, a reserve of $N_r^{(12)}/D_x^t$ is held—

on an accrued benefit valuation basis—for each dollar of accrued pension benefits for employees whose age is x . Where there is no vesting, a reserve of $-q_x^{(w)} \cdot N_r^{(12)} / D_x^t$ is expected to be released. When vesting is introduced (meaning 100 per cent immediate vesting), two things happen: the expected release must be restored and the reserve for those whose benefits become vested must be raised to $N_r^{(12)} / D_x$, because their benefit rights have become immune to “turnover.” The cost of restoring the reserves and raising them to the higher level is the term cost of vesting. This is

$$q_x^{(w)} \cdot \frac{N_r^{(12)}}{D_x} = \frac{q_x^{(w)}}{t_x} \cdot \frac{N_r^{(12)}}{D_x^t} \quad (10)$$

Thus, the (*BVR*) function is indeed a term cost factor applicable to the lump-sum liability for accrued pension benefits.

Up to this point we have not questioned whether the results of calculations made in accordance with the helpful methods proposed by Messrs. Marples and McGinn—or, for that matter, with our own (*BVR*) method—really represent “the cost of vested benefits.” Mr. Marples may have had slight doubts when he wrote, “If there are no termination decrements, then there is no *separately projectable* cost of vesting” (emphasis added). This is unquestionably true, but it is not something a pension consultant can tell his client—and get away with it. Where assumed turnover rates are conservative (and most of them are), the vesting cost figures shown in valuations represent no more than a formal accounting for a portion of the total cost, an accounting that is fully consistent with the over-all valuation method. To tell a client or the public that they represent “the cost of vesting” could be quite misleading.

How can a better approximation of the true cost of vesting be obtained? The true cost of vesting, like the true cost of a pension plan, can be ascertained only in hindsight. It should be possible, however, to prepare, as a practical matter not as an integral part of actuarial valuations, current estimates which come closer than the theoretical, formally consistent figures produced by the methods that we have been discussing. While we have made “educated guesses” in various concrete situations, we do not have a universally applicable answer. It may be that some of our fellow actuaries will wish to contribute to the solution of this problem.

At the end of his article, Mr. Marples discusses the philosophy of vesting. After reading his remarks, one senses a serious problem for the conscientious pension consultant who shares his basic feelings. Should he discourage his client from including vesting provisions in his plan? Should he make any recommendation in favor of vesting only to the accom-

paniment of half-spirited comments on overwhelming pressures—"keeping up with the Joneses"?

We do not believe that vesting has harmful consequences. Our most advanced companies these days are in dire need of highly qualified personnel, both at the executive and the skilled-labor level. Experience and mature knowledge are largely matters of age and long service. Do we want to restrict the availability of the various talents by forcing employees to consider the loss of accrued pension rights first and a new challenge to their abilities only second? The company which puts its good people to the test every day has little to fear from the adoption of a vesting provision; the company which allows its staff to become stale will not be protected by the absence of such a clause. We live in a competitive society and we want to keep it so. Vesting will help.

THOMAS P. BLEAKNEY:

Mr. Marples asks for statistics regarding claim rates for the vesting benefit under contributory plans. The statistics below are very limited but may be of interest.

In a public employees' plan with which I am familiar, a great increase in the number of employees becoming eligible for vesting recently occurred because of liberalization of the plan. Specifically, the plan now provides for full vesting for persons terminating with more than fifteen years of service or, if over age 50, with ten years of service. During the period of eight months immediately following the effective date of the change in the plan, there were twenty-eight participants who terminated service while eligible for vesting. Although some had rather substantial benefits accrued that were forfeited upon withdrawal of their contributions, every person forfeited his vested benefits by taking cash.

As a matter of fact, under this particular plan, many participants could withdraw their contributions instead of taking current retirement benefits. Over a ten-year period, over 600 retiring participants, or about 7 per cent of all retirees, actually forfeited substantial amounts of immediately payable retirement benefits in order to withdraw contributions.

The importance of these statistics is emphasized by the fact that the plan is now about twenty years old and has, therefore, not reached maturity. Since a substantial past-service benefit is provided—a benefit, of course, forfeited by withdrawal of contributions—there is substantial reason now to take the benefit instead of cash. In the future most participants will have significantly higher accumulated contributions relative to accrued benefit, with a corresponding increased likelihood of forfeiting the benefit for cash.

This natural tendency to take the bird in hand leads me to feel that the cost of vesting will be of far less importance if contributory plans should become more prevalent. If the United States tax laws should ever be modified to allow for the deduction of employee contributions, as those in Canada do, this might come to pass.

LEE H. KEMPER:

I wish to congratulate Mr. Marples on his very fine paper, "Cost of Vesting in Pensions." Not only has Mr. Marples introduced a very interesting and practical approach to determining vesting costs, but, in addition, he has presented some philosophical views with respect to the vesting benefit. In his paper the author has asked that a discussion include consideration of the philosophical aspects of the subject with regard to reasonable limitations to be placed on vesting benefits.

In arriving at the reason for including a vesting provision in a retirement plan, it is important for the employer to determine at the outset whether his plan is a retirement plan or a deferred-compensation plan. If it is a deferred-compensation plan, then, as mentioned by Mr. Marples in his paper, the employee would be entitled to the full amount of the benefit accrued to his credit at the time of termination. If the plan is considered a retirement plan without a deferred-compensation feature, the primary reason, from a theoretical point of view, for including a vesting provision in the plan is to secure the employee's benefit against nonvoluntary termination. Mr. Marples touches on the question of nonvoluntary termination when he states, "The provision may have arisen from a general fair-treatment idea applied, first of all, retroactively in a case of hardship and then gradually as a basic concept applied to all active employees."

This statement implies that the provision is for the purpose of alleviating the situations in which the employee is forced to terminate because of health or other circumstances beyond his control. In addition, it should probably include the case in which the employee is discharged by the employer. There have been a number of situations in which an employer has terminated the services of an employee, who is approaching retirement, for the sole purpose of reducing the employer's retirement plan cost. In this connection, we cannot overlook the fact that a vesting provision is also included to compete with other plans with similar benefits. These are probably the most important reasons for including a vesting provision in a plan which has been designed primarily for the purpose of providing for superannuated employees.

The next point to consider is at what point the vesting benefit should

commence. During the early years of participation in a plan, the cost of deferred pension benefits for younger employees is small as a result of the future interest earnings to be earned on the funds and the fact that the employee must survive to normal retirement age to receive the benefit. Under a contributory plan, the contributions plus interest might be larger than the present value of the deferred benefit; this is particularly true in the case where the vesting allowed is less than 100 per cent of the accrued benefits. The vesting benefit at this point would have little or no value to the participant. The logical point for the vesting benefit to begin is sometime after the present value of the benefit accrued to the employee exceeds his contributions with interest.

Another point to consider in connection with the commencement of the vesting provision is that the company must maintain records for all terminating employees with a vested interest. To avoid carrying a large number of records involving insignificant amounts of benefits, the vesting benefit should begin at the point when the benefit is sufficiently large to warrant the cost of the additional record keeping.

The fact that vesting is much more costly to the company at the older than at the younger ages for participants with the same length of service suggests the amount of vesting should vary by age as well as by length of service if complete equity is to be achieved. For example, a \$100 per month normal retirement benefit at age 65 for an employee with fifteen years' service at age 35 would be worth \$4,100, while the same benefit for an employee with fifteen years' service at age 55 would be worth \$8,400. I realize that vesting varying by age and length of service would introduce a number of practical problems and might in some respects defeat the purpose of the benefit provided; however, the inequities of the situation have always bothered me.

The final point on which I wish to comment is the level of the vested benefit. Based on the reasoning that the vesting provisions should be included to secure the pensions for such nonvoluntary terminations and assuming that the plan is not a pension-equivalent-to-wage plan, it is reasonable that a vesting provision should gradually increase until it becomes equal to the retirement benefit at normal retirement age or at the age at which an early retirement benefit is available. The grading of the benefit in this way, combined with the point at which vesting begins, will, to some extent, determine the level of the benefit.

In my opinion the gradual vesting provision overcomes many of the disadvantages to the employer mentioned in Mr. Marples' paper. Under this type of vesting provision, the employee can always look forward to additional credits if he remains with the company.

Again I would like to congratulate Mr. Marples on his paper which, in my opinion, represents a fine addition to our actuarial literature.

WALTER RIESE:

It is a pleasure to follow through the development of the formulas and tables contained in Mr. Marples' paper, which should prove a useful addition to actuarial literature. The main body of the paper deals with facts and demonstrations relating to the cost of vesting, and it remains to be seen whether some readers discover grounds for argument in this area. However, under the heading "Concluding Remarks," the author expresses his opinions regarding the philosophical and managerial aspects of vesting, and it is difficult to resist his invitation to follow him into the more fertile, if somewhat treacherous, land of appearances and impressions. Although the following discussion will not be one to fulfill the author's desire for suggestions of "reasonable limitations to be placed on benefits," it may help to show how difficult it is for the Society of Actuaries to speak with one voice.

It is possible that in Canada, where most pension plans are contributory, employees are more likely to be aware of accrued pension benefits and to think of these benefits as deferred compensation "equities" than are employees in the United States. However, it is probably true in both countries that considerable publicity is given to the contributions employers make to pension plans, expressed as percentages of salaries or wages. Employees, not unnaturally, tend to think of these contributions as being allocated on an individual basis. With such mental conditioning, a simple statement of fact like Mr. Marples' "If he quits before vesting, his accrued pension lapses . . ." is therefore likely to lead to as unfavorable an emotional reaction as might the discovery that traditional nonforfeiture benefits had been abandoned and that insurance companies had adopted the policy of using gains from lapsed policyholders for greater dividends to continuing policyholders who were considered to be of more value to the companies. If such a view had been taken—and it could have been taken, perhaps, more easily in Canada and the United Kingdom, where no statutory nonforfeiture laws exist—the insurance industry might not have withstood repeated attacks as successfully as it has. By analogy, one would think that improvement in vesting and portability—far from bringing on the downfall of the private pension movement—is likely to further its growth and general acceptance.

In retrospect, the gradual development of plans emphasizing accrual of pension benefits related to length of service as well as earnings, such

as money-purchase and unit benefit plans, seems natural not only because such plans provide desirable incentives to continued employment but because it seems reasonable to have pension payments reflect the differences in the quality and length of service rendered by different employees. It might be rationalized that such plans also reflect the philosophy that responsibility for retirement income should reasonably be prorated among employers, giving some weight to the value of services rendered by the employee to each enterprise (in contrast to the obsolescent notion that the only employer who might have any pension obligation to a given individual is the one who has had the employee's service over long periods of years and over the later years of his working life). While such a philosophy seems to pose a problem in the case of service with an employer who has no pension plan, this problem appears to be resolved by the deferred-wage theory, since it may be rationalized that there is simply no deferred compensation arising out of such service (certainly such service was rendered without any expectation of deferred compensation).

While stating that "the granting of vested rights to pensions is not favorable to management aspirations," Mr. Marples concedes that "reasonable vesting in benefits should be regarded as the mark of a well-designed plan and an enlightened employer." However, the author clearly believes that no restrictions should be placed on the employer's freedom to determine the degree of his own enlightenment. The question seems to be whether the perpetuation of shortcomings of the private pension movement through such a *laissez faire* attitude will not be more inimical to its survival than limited restrictions on the employer's freedom of choice.

The vesting requirements enacted by the governments of the provinces of Ontario, Quebec, and Alberta are based on the premise that it is the decision of the employer (subject to the importunities of collective bargaining) whether or not retirement income for employees is provided in addition to benefits under government-operated programs. This seems only natural since, apart from minimum wage laws, compensation for service is determined by the employer (again subject only to the importunities of collective bargaining). However, the statutes of the three provinces provide that, if a private employer-employee pension plan is operated, the benefits must vest after ten years of service and attainment of age 45. Furthermore, such vesting is not conditional on the employee's leaving his contributions in the fund, since any employee's contributions become "locked in" at this point.

Of course, there is no assurance that at future points in time the various legislatures concerned may not decide to make vesting requirements more stringent. However, Mr. Marples' vision of a system of uniform pension benefits or the absorption of all pension plans into a central government retirement system seems rather gloomy. Such a development seems hardly conceivable, unless a system of uniform wages had become acceptable to society—a rather remote contingency.

If we could detach ourselves momentarily from the absorbing activities and controversies of the social organisms in which we operate and view our free-enterprise society from the perspective of outer space, would our myriads of decision-making bodies—be they management, labor, or government—not be clearly discernible as agents responsible ultimately to society as a whole? Also, regardless of the media, intermediaries, and intricacies of accounting, would not pensions—whether public, private, funded, or unfunded—be recognizable as no more and no less than a charge on future production which, though charged initially to a particular enterprise, must ultimately be borne by society through higher prices, lower wages, or higher taxes than would obtain if the deferred claims on production did not have to be honored? If so, it would seem inevitable that, even though society may be happy to leave the determination of levels of compensation largely to the market place and even though elected representatives of employees may have agreed to the pension conditions, sooner or later the employer's exclusive right to determine conditions of pension payment will be questioned. Indeed, how reasonable is it and how necessary from a managerial point of view that retirement income in respect to service rendered to a particular enterprise in the first instance, but presumably ultimately to society as a whole, should be conditional on the employee's attachment to the enterprise for long periods of years and until attainment of a relatively advanced age?

There is no doubt that management desires to retain its skilled labor, but are there not sufficient incentive devices in longevity pay, longer vacations, seniority rights, and so forth, not to mention increases in pension expectation produced by longer service and increases in earnings, without having to make the receipt of retirement income liable to full or partial forfeiture over long periods of years?

Mr. Marples expresses the view that "acceptance of these costs [of vesting] as a necessary burden by the employer . . . should not be permitted to form a platform for continuous extension of the benefit." While in noncontributory pension plans it would be difficult to obtain visible

contributions toward plan liberalization from employees, and while it may be equally difficult to offset the cost of vesting by a reduction in the unit benefit, it could well be argued that any cost of vesting borne by the employer is by necessity offset by the denial of some other form of employee benefit that the employer might otherwise have granted.

CHARLES V. SCHALLER-KELLY:

First of all, I would like to make a brief comment on the kind of investigation that Mr. Marples made. This comment also applies, to some extent, to Mr. McGinn's paper.

It seems to me that the paper requires, in effect, a calculation for each employee or group of employees divided into age and service classifications. This seems to me, in certain circumstances, to take more time than is practical, and sometimes it is more expensive than it need be if you deal with a small group. When I refer to its taking time, I am thinking of such occasions as negotiations. Moreover, for the very small group, if the deposit administration or trustee approach is to be used—and I think that this is the most sensible way of funding pensions for a number of reasons—then a method such as the one proposed, or the one proposed by Mr. McGinn, might be more elaborate than is really justified or than can be borne by a group of, say, twenty employees.

We must remember that we have so many assumptions in pension costs at any time that they are always approximate. We are really charting our way through unknown future currents of interests and turnover and goodness knows what. I wonder whether it would not be desirable—and I shall bring this up again in a later portion of the whole program—to try to do precisely what Mr. Marples suggests he cannot do, that is to say, to look at the effect on model distributions so that we have some rules of thumb for evaluating costs of various pension benefits.

In the second place, there is the philosophical issue. I would like to stress that the views I am going to express are not specifically those of the United Automobile Workers. I have been called by my colleagues anything from a "crypto-conservative" to "that foreign socialist," so I do not think that my ideas should be taken to represent UAW opinion, except insofar as they happen to coincide with statements which have been made by people who can speak on behalf of the United Automobile Workers.

It seems to me that employee benefits should be looked at more from the employees' view than they commonly are by insurance companies for two reasons. First, the intention of employee benefit plans is to satisfy the employee. This is management's reason. I think that insurance com-

panies should therefore also look at them from the point of view of satisfying employees than of satisfying management. The employee benefit program comes from pressure by employees, either organized or unorganized pressure at the time of hiring.

I think that insurance companies are really, in another way, missing the boat insofar as there are so many large groups which are not covered by pension plans. In Canada I recently did a little study on this. The UAW alone in one of its regions has approximately 4,500 members not covered by pension plans. Multiplying this by 16 or 17 regions, and I don't know how many unions, it would seem to me that it would be desirable for insurance companies to start thinking in a slightly different direction.

Of course, you have the disadvantage that your image with unions is of such a nature that, if we have the choice, we tend to go through trust companies, which probably are just as reactionary but do not make so much noise about it.

Second, I think that employee benefits should be looked at from the employees' point of view because there are more employee voters than employer voters. If you cannot manage to provide what is required, then the same employee, in his anonymous capacity of citizen, will provide it for himself through the government, which is what happened with that quite conservative government in Ontario which provided the vesting legislation.

Third, of course, management is composed of employees, too, in 90 per cent of the cases today.

Fourth, I think what is good for management in the short run may not be good for the country. In fact, it quite frequently is not, because every management inevitably will attempt to do whatever is good for it in the short run, and there may be a tendency to forget about the country; it is not necessarily good for the insurance companies. I think that it is assumed that, if it is good for management, it must be good for insurance companies. I would question that assumption.

Let me briefly say that, as I see it, employee benefit plans should have at least four characteristics: (1) they should be adequate when required, that is, when earnings stop—in old age, upon disablement, and for dependents at death; (2) they should provide security of those benefits; (3) they should be provided in as cheap a way as possible; and (4)—this is an item which I think you should like—they should be flexible.

Government social security can provide all these things except flexibility, and I think that, if the private industry wants to stress something, it should stress its flexibility. That is what it can sell.

In regard to the first characteristic, arguing against vesting in the way in which Mr. Marples did is basically arguing against one aspect of adequacy. This is the most basic need of all pension systems, and if the private pension system cannot provide adequacy, then let us get rid of it. I know that this is heresy here; but if the private pension system cannot provide adequacy, it is not fulfilling a function any more than a dinosaur is.

Second, there is security. Try to tell a Studebaker employee that the private system has done well in security! There are still too many cases—and I come across them, unfortunately, all too frequently—in which the private system turns out not to have provided the pensions that were promised. And then the insurance industry opposes the Hartke Bill. It may not be a perfect bill; if you can come up with something better, splendid. I am sure even Senator Hartke would be delighted. Otherwise, there is another gap.

Third, there is a question of the provision of employee benefit plans in the cheapest way. A nonfunded plan—or almost nonfunded plan, such as the government plans—is probably cheaper in many ways. However, this can be counteracted in the private system by investing in stocks which go up in capital gain and ultimately enable higher pensions to be paid. So, I am prepared to go along on that.

Finally, as I say, the private system has flexibility. I do appreciate it. Just recently I became involved in a situation where we had some problem because of inflexibility.

I believe that the words of the new Republican insurance commissioner at a recent Michigan Actuarial Society meeting are also applicable in this case:

The political process from time to time reallocates the risks borne by various parts of our society; for example, it shifted the risk of enterprise from the employee to the employer in the field of employee injuries. This risk allocation is a political and social decision based upon a determination of what segment of society should bear a particular risk at a particular point in time. The role of insurance is to spread these risks, once they are allocated, by social, political, and legal mechanisms.

There you have a Republican view of the same kind of thing.

The private system should therefore create mechanics to overcome its shortcomings, some of which I have mentioned, in the field of security and portability. In this connection I should add one further shortcoming in portability; that is that since vested rights do not increase in value once a person has left they tend to get left behind in much the same way as in a career average plan, even when, in fact, a man may have been

covered by a succession of final-average plans. This is another item which I think the insurance industry should put its best brains to.

(AUTHOR'S REVIEW OF DISCUSSION)

WILLIAM F. MARPLES:

As I expected, a major part of the discussion related to the philosophical aspect of vesting, and I am grateful to those who extended the scope of my remarks; in particular, to Robert Link for his contribution on mandatory vesting provisions, to William K. White and Wilmer Jenkins for their expansion of the subject, to Frank Griffin for his support of my views, and to Harry Sarason for his advice on procedures.

I have to confess that I am an unreconstructed member of the Do-It-Yourself Club, and as such I have a fundamental objection to government-imposed rules. I did think that I made it clear that I considered vesting to be a feature of a well-designed pension plan, and I believe that plans should assume a position in this matter sufficiently commanding to guard the "pass," in Crawford Laing's words. However, I confess to being disappointed grievously in the failure of any participant in the discussion to indicate where he thinks this position might be. The position has to be found—in some place where it commands substantial support—perhaps further forward than in some plans and possibly not so far forward as might be advocated by some extremists.

I appreciate the contribution of Dan McGinn particularly, since his time must have been almost totally absorbed in writing his own paper, and I regret that I could not repay the compliment by discussing that paper. However, I have to go on record that I disagree with some of his comments on my paper, lest his summary of points in my paper lead others astray.

My approach is fully adaptable to all circumstances except one, in which it produces a zero answer; that one is the case in which withdrawal rates are not employed in the cost calculations. Is this the "smaller pension plans" to which he refers? Second, my approach is not limited to measuring the increase in the normal cost alone, as will be seen from paragraph 5. Finally, I add in formula form, not the arithmetical form, the demonstration he requests as to the use of my form of ratios. In a formula similar to those for policy values, the reserve value for a pension is:

$$\begin{aligned} & (\text{Projected benefit}) \times (\text{Value of pension of 1 at retirement}) \\ & - (\text{Amount of normal cost}) \times (\text{Temporary} \\ & \quad \text{amount to retirement age}). \end{aligned}$$

In his symbols, for attained age x and entry age w ,

$$(AL)_x = B_r \cdot \frac{D_r}{D_x} \cdot \ddot{a}_r^{(12)} - P_w \cdot (\text{Actual salary})_x \cdot \bar{a}_{x:r-x}.$$

Let the vesting cost ratios be $1 + k_x$ and $1 + k_w$, respectively. Then the reserve value, including the cost of vesting, is

$${}^v(AL)_x = (1 + k_x) \cdot B_r \cdot \frac{D_r}{D_x} \cdot \ddot{a}_r^{(12)} - (1 + k_w) P_w \cdot (\text{Actual salary})_x \cdot \bar{a}_{x:r-x},$$

from which the vesting reserve can be determined if necessary.

Mr. McGinn gives the ratio ${}^v(AL)_x / (AL)_x$, and it may be noted that over certain ages the ratio is less than unity. The explanation of this feature is that k_x becomes zero at the attained age z , at which age the withdrawal rates become zero, while k_w remains at its entry age value. In cases where P is obtained as the normal cost rate to balance liabilities, the adjustment by $1 + k_x$ produces a weighted increase in P , automatically including the effect of k_x equaling zero at certain ages.

The accrued benefit method would use the attained-age ratios. Incidentally, the total reserve, that is, the reserve for active participants plus the reserve for terminated vested participants, will theoretically always be greater than the unadjusted reserve for active participants. This offers an explanation which Mr. Bloch can give legitimately to those of his clients who examine his worksheets. On Mr. Bloch's formula, I present the following reconciliation with my own.

The expression inside the summation symbol in my formulas (7) and (8) can be written

$$\frac{w_{x+t}}{l_r} \cdot \frac{l'_r}{l'_{x+t+1/2}}.$$

To proceed on Mr. Bloch's basis, we have to use the following formula:

$$l_{x+k} = l_x \prod_{s=0}^{k-1} [1 - q_{x+s}^{(d)}] [1 - q_{x+s}^{(w)}],$$

from which

$$w_{x+t} = l_{x+t} [1 - q_{x+t}^{(d)}] q_{x+t}^{(w)},$$

assuming deaths subtracted first, and

$$l_r = l_{x+t} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}] [1 - q_{x+t+s}^{(w)}],$$

and

$$\frac{l_r'}{l_{x+t+1/2}'} = [1 - q_{x+t}^{(d)}]^{-1/2} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}].$$

Then

$$\begin{aligned} \frac{w_{x+r}}{l_r} \cdot \frac{l_r'}{l_{x+t+1/2}'} &= \frac{l_{x+t} \cdot [1 - q_{x+t}^{(d)}] q_{x+t}^{(w)}}{l_{x+t} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}] [1 - q_{x+t+s}^{(w)}]} \\ &\times [1 - q_{x+t}^{(d)}]^{-1/2} \prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(d)}] = \frac{q_{x+t}^{(w)} [1 - q_{x+t}^{(d)}]^{-1/2}}{\prod_{s=0}^{r-x-t-1} [1 - q_{x+t+s}^{(w)}]}, \end{aligned}$$

which differs from Mr. Bloch's formula only by the factor $[1 - q_{x+t}^{(d)}]^{-1/2}$.

It is now clear that Mr. Bloch has achieved identical results by different methods so far as the primary vesting ratio is concerned, and I assume that he has taken the further steps to allow for the accrued benefit and the salary-scale benefits.

I thank Mr. Bleakney for responding to my call for figures on the question of claim rates for vesting benefits in contributory plans, and I agree with his comment on the cost of vesting if contributory plans should become more fashionable. An alternative approach preventing contribution refunds is that of the Canadian government, which has prescribed a "locked in" vesting benefit in which the participant cannot retrieve his contributions and the employer must provide the vested pension benefit. The length to which this government regulation has gone may be thought on investigation to exceed reasonable limits and, if so, any osmosis south of the border needs to be resisted.

To refer now to the question of the difference between estimates and real costs raised by William K. White, the effect of the technique that I have set out in the paper is to develop, from age series of probabilities of death, withdrawal, and disability, the results which would have been obtained for each entry age separately had the appropriate rates been selected for each attained age and allowing for the provision of a vested benefit to the expected terminations after qualifying for such a benefit. It is usual to find in projected benefit actuarial valuation methods that the reserve value in respect to benefits accruing between the valuation date and the normal retirement date is positive. Hence, terminations tend to produce small valuation releases when the past-service benefit is vested,

so that differences in numbers and average benefits do not produce valuation strains. It is to be noted that the unfunded liabilities for these people still have to be paid up even though they are now not in the company's employment.

In the case of the accrued benefit valuation method, if liabilities for pensions accruing before the valuation date have been calculated on a mortality table basis only, then no valuation strain occurs. If withdrawal rates have been employed, there may be a valuation strain in 100 per cent vested cases equal to the difference in the single-premium costs without and with withdrawal rates. If the vesting percentage is less than 100 per cent, there should be some offset, because the reserve required for vested cases is reduced by the vesting percentage and the full reserve on the withdrawal rates is subtracted. These remarks apply at ages where withdrawal rates are used, and the effect of withdrawal rates should be substantially eliminated beyond the vesting qualification limits.

To go to the question of the increase in annual cost as the result of vesting, let us take the valuation methods under the two heads of accrued benefit method and projected benefit method. There are two elements in the additional cost, the theoretical increment, and the practical increment.

Accrued Benefit Method

If the cost calculations have been made on the basis of a mortality table only, there will be no theoretical or practical increase in the total liabilities for pension accrued prior to the valuation date. If withdrawal rates have been used with the mortality tables, the introduction of a vested benefit will involve an additional lump-sum supplemental cost. The theoretical increment for prior service is thus zero or positive. There will be no practical increment in respect to prior service.

In developing the company contributions for the new year, however, we have an amount equal to:

- a) the normal cost calculated on the old actuarial cost determinants, *plus*
- b) the theoretical increment for vesting in the current year's benefits, *plus*
- c) a payment to amortize the theoretical increment for vesting in respect to prior-service benefits, *less*
- d) reserves released by termination in the valuation year, *plus*
- e) the reserves required for vested benefits claimed and granted during the valuation year, *plus*
- f) a payment in respect to any unfunded liabilities.

Before the vesting benefit was incorporated into the plan benefits, the company contribution would have been calculated as $a - d + f$; with vesting, there has to be added $b + c + e$.

The theoretical increment is $b + c$, and the practical increment is e less the part of item d related to former participants who terminated and qualified for some vested benefit.

Projected Benefit Methods

These methods are divided into two main groups: Group A, in which unfunded liabilities are "frozen" and the whole of the experience of the plan in the year is reflected in the change in the normal cost rate, which is recalculated at the end of the year; and Group B, in which the normal cost rate is calculated theoretically at the end of each year and all experience of the plan in the year is reflected in the amount of the unfunded supplemental liability at the end of the year.

In Group A, we shall have the theoretical increment in respect to both *prior- and future-service* benefits. These may be separated and the amount for prior-service benefits added to the unfunded supplemental liability. The reserves for vested benefits to inactive participants would be set up during the valuation and would form part of the total liabilities. The new normal cost rate is now obtained as the rate required to balance assets and liabilities, that is, to meet the balance of the liabilities after deducting the fund and the unfunded liability. The normal cost thus is affected by all experience gains and losses during the year and, in this particular instance, will be increased on account of the theoretical increment for future-service benefits and may be for prior-service benefits also.

The adjustment for practical increment of reserves expected to be released by termination and reduced by reserves set up for vested inactive participants is also automatically included in the calculation. The analysis of theoretical increment and practical increment is the same as for the unit credit method, but the amounts are the equivalent normal cost rates.

In Group B, the theoretical increment for total benefits will be incorporated in the new normal cost calculation. The liabilities for prior- and future-service pensions will also be increased by the appropriate theoretical increment. The reserves for vested benefits to vested inactive participants will be included in the total liabilities of the plan. The residual unfunded supplement cost will thus be adjusted automatically by:

- a) the theoretical increment on pension liabilities, *less*
- b) the present value of the theoretical increment on the normal cost, *less*
- c) reserves released on terminations during the year, *plus*
- d) reserves set up for vested benefits to vested inactive participants.

The theoretical increment is $a - b$ and may be negative (see Dan McGinn's tables), and the practical increment is d less the part of c related to vested cases.

REPORTS ON TOPICS OF CURRENT INTEREST

REPORT OF THE COMMITTEE ON THE FUTURE COURSE OF THE SOCIETY*

Some day the histories of the United States and Canada will mark the past few years as significant for the extent of governmental inauguration of social plans and planning in fields directly or closely related to the actuary's work in pensions and insurance. As individuals we may embrace, accept, or reject the political philosophies underlying these current activities. As individuals we may agree on the broad social objectives being sought but disagree fundamentally on the best means of reaching these desirable goals. We may look upon some of these measures as overdue or, alternatively, as ill-advised attempts to legislate a general state of well-being that would be more safely and surely reached by an evolutionary process. One thing, though, we cannot do is to ignore the issues of our times, and, in so saying, I add quickly that I do not infer that actuaries, as individuals, are unmindful of or disinterested in their civic responsibilities.

This is not the occasion for casting up a complete account of the ways in which actuaries are now bringing their professional talents to bear on public questions. Perhaps the most apparent way is through service on committees appointed under the auspices of government, universities, chambers of commerce, national business organizations, as well as the trade associations of the life insurance business. To take, for example, just one recently appointed committee from each of these five categories, and limiting ourselves to committees dealing with national pension problems in the United States, we can count the names of more than a score of Fellows of this Society who are giving committee service outside of their regular employment. The influence of these individual efforts is unquestionably great, but nevertheless, the question that we face is whether or not the actuarial profession should have a voice of its own. This is particularly compelling when the matters dealt with are important from a public standpoint and clearly within the competency of the actuarial profession.

With the growth in general appreciation of the nature and responsibilities of the actuarial profession, it is not surprising that increasingly in the last few years we hear the question, "What do the actuaries think of

* Discussion of this report appears in Part II of this volume.

this or that proposal?" Unlike other professions, ours has no clearly accepted or clearly recognized procedure for giving answers. You are all aware that our President and Board of Governors have had to entertain this kind of question and that the source of the inquiries and the subject matter have been too important to ignore. Among other things, the President specifically requested the Committee on the Future Course of the Society to examine the problem of whether or not the Society should have any collective voice on public questions and, if so, under what conditions it should be heard.

In terms of numbers of members, ours is a small profession. Nevertheless, it would be foolish to suppose that all of us would ever totally agree on the best answers to the kind of public questions that arise. Should this foreclose any kind of collective statement of actuaries in an appropriate situation and under circumstances directly related to a major professional concern of large numbers of actuaries? After careful deliberation, the decision of our Committee was that there can be situations in which the profession, through the Society, has a public duty to respond. We considered that it was more reasonable to expect an increasing measure of public and legal recognition for the profession if, as a profession, we had a mechanism for expressing our views on questions of major public interest within our special fields of competence.

The Committee was highly conscious that over the years the Society has carried forward a strong tradition of our membership being a purely individual matter. This tradition, which runs counter to the idea of collective expression of opinion, finds its support in our requirements for membership, our procedures for electing officers and governing board, the sources of our revenues, and the nature of our meetings. Central to this basic aspect is the constitutional provision, carried forward unchanged for seventy-seven years, that no resolution expressive of opinion shall be entertained at any meeting of the Society. The account in *TASA*, Volume I of the founding of the Actuarial Society of America clearly indicates that the provision was included in the original constitution to avoid having the aborning Society become the instrument for any criticism of individual companies or individual plans of business. The discussion of the founders did not contemplate today's situation of a well-established actuarial organization in a world of insurance and pensions in which "business" as then referred to is now but a part. Our Committee concluded that some change was desirable.

If we look only to the letter and not to the spirit of our constitutional provision, the proposition can be made that, as matters stand, all manner of expression of opinion is not, in fact, barred. Because of the major

change of philosophy now contemplated, however, our Committee did not favor new procedures based on a legalistic interpretation but recommended that they be based on specific constitutional authority approved by the Fellows of the Society. Accordingly, our Committee recommended that the Constitution be appropriately amended to provide that neither the Society of Actuaries, nor the Board, nor a committee shall publicly express an opinion, except with respect to matters of public interest involving actuarial principles and then only in accordance with procedures determined in each instance by the Board of Governors.

Under the terms of our Committee's proposal, the Board itself would have to consider any requests or suggestions involving a statement of opinion. It was the Committee's view that the authority vested in the Board should be exercised sparingly and only in matters judged by the Board to be of vital and fundamental importance. Although the Board would have the power to assign a particular subject to a committee, our Committee recommended that, whenever this is done, the Board set the conditions under which the committee may make reports or statements or give testimony. For example, the committee could be instructed to preface such reports, statements, or testimony by a complete statement of how and why the committee was appointed. A disclaimer to the effect that the report or statement represented no more than the views of the committee so appointed might, or might not, be specified as a part of the required procedure in a particular case. Our Committee also contemplated that in a matter of great importance the Board might charge a committee to return its findings to the Board for review and disposition. The report, statement, or testimony in a case of this kind might well be made by the Board or by the President on behalf of the Board. These latter procedures come close to paralleling those utilized by the Institute of Actuaries on the rather infrequent occasions when that body has dealt with public questions. In no event could any opinion be expressed on behalf of the Society, as a whole, without a vote of the Fellows.

The Committee has been advised by our legal consultants that our recommendations, if adopted, do not endanger our incorporation under The General Not For Profit Corporation Act of Illinois, nor need they endanger our general exemption from taxation. To continue to qualify for tax exemption under the Internal Revenue Code, we have to be sure that no substantial part of our activities is directed to influencing legislation. This requirement strengthened the Committee's view that utilization of the proposed constitutional authority should be undertaken only in comparatively rare instances of fundamental importance and widespread public interest. To insure that consideration of these matters

would always have full and deliberate attention when they first arise, we have recommended that the revised Constitution provide that the Board must determine in each instance the procedures to be followed. Accordingly, our Committee further recommended that, if the Constitution is so amended by vote of the Fellows, the Board of Governors should amend its resolution with respect to the powers of the Executive Committee so that that committee will not have the power to express opinions.

The Board of Governors at its meeting on October 30, 1966, approved the Committee's recommendations just described. Accordingly, the Fellows of the Society will, in due course, be given notice of proposed constitutional amendments to be voted on at a later meeting of the Society.

The President has asked me to say that you will have an opportunity to discuss this matter at the general session on Wednesday morning after the Panel on Private Pensions in the United States and Canada.

WALTER KLEM

FINANCIAL REPORT

FOR FISCAL YEAR ENDED MAY 31, 1966
(Cash Basis)

INCOME

Membership Dues.....	\$ 84,460.00
Members' Payments for Bound Copies of Publications.....	8,961.00
Sale of Publications.....	54,163.75
Interest on Bonds.....	7,954.14
Registration Fees.....	19,685.00
Examination Fees.....	71,839.00
Study Notes.....	11,605.60
Income from Aptitude Tests.....	567.15
Royalties Received.....	241.19
Income from American Academy of Actuaries.....	1,312.46
Collection and Exchange Received.....	56.85
Miscellaneous Income.....	41.00
Total Income.....	<u>\$260,887.14</u>

DISBURSEMENTS

Salaries.....	\$ 38,809.17
Rent.....	10,616.50
Printing.....	5,065.18
Postage.....	24,798.04
Telephone, Telegraph, Electricity.....	580.69
General Office Expense.....	7,755.82
Insurance.....	1,552.03
Employees' Retirement Plan.....	4,320.14
Education and Examination Expense.....	50,359.13
Meeting Expense.....	21,292.68
Printing of Study Notes.....	11,687.19
Cost of Members' Copies of Publications.....	33,647.03
Cost of Complimentary Copies of Publications.....	501.22
Cost of Publications Sold.....	18,319.64
Taxes.....	1,212.26
Gifts.....	236.20
Collection and Exchange Paid.....	1,565.88

DISBURSEMENTS—*Continued*

Adjustment in Value of Inventory.....	\$ 10,439.46
Loss on Accounts Receivable.....	8.60
Public Relations Expense.....	5,715.20
Royalties Paid.....	257.26
Traveling Expense.....	118.46
Donation to New York Insurance Library.....	500.00
Miscellaneous Expense.....	32.55
Total Disbursements.....	\$249,390.33
Excess Income over Disbursements.....	\$ 11,496.81

ASSETS—MAY 31, 1966

Cash in Bank (Including \$15,156.56 of Canadian Deposits in Canadian Dollars).....	\$ 31,212.09
Petty Cash.....	50.00
Interest on Deposit.....	2,096.88
Bonds—at Cost (Including \$19,775 of Canadian Bonds in Canadian Dollars).....	194,158.76
Treasury Bills.....	9,754.20
Accounts Receivable—Invoices.....	2,818.05
Accounts Receivable—Committee Reports.....	5,192.36
Insurance—Withheld.....	2.50
Publications on Hand.....	40,150.53
Advance Payments on Publications.....	6,753.45
Furniture and Fixtures.....	\$2,219.78
Less: Depreciation.....	394.00
Total Assets.....	\$294,014.60

LIABILITIES AND OPERATING FUND—MAY 31, 1966

Accounts Payable.....	\$ 12,760.49
Advance Subscriptions to Publications.....	9,191.75
Taxes Withheld at Source.....	603.83
Permanent Committee for International Congress of Actuaries...	1,451.87
Suspense.....	1,048.30
Total Liabilities.....	\$ 25,056.24
Operating Fund.....	268,958.36
Total Liabilities and Operating Fund.....	\$294,014.60

RECONCILIATION OF OPERATING FUND

Assets Beginning of Year.....	\$283,409.22
Liabilities Beginning of Year.....	25,947.67
Operating Fund Beginning of Year.....	<u>\$257,461.55</u>
Income from Current Year.....	\$260,887.14
Disbursements for Current Year.....	<u>249,390.33</u>
Net Gain.....	<u>11,496.81</u>
Operating Fund End of Year.....	\$268,958.36

[Signed] HERBERT L. DEPRENGER
Treasurer

We have examined the balance sheet of the Society of Actuaries as of May 31, 1966, and the related statement of income and disbursements for the year then ended. Our audit was made in accordance with generally accepted auditing standards and accordingly included such tests of the accounting records and other auditing procedures considered necessary in the circumstances.

The inventory of publications on hand was not verified by physical count. Cash in bank and investments include funds in a Canadian bank and Canadian bonds valued in Canadian funds \$2,353.81 in excess of the valuation in United States dollars at May 31, 1966.

In our opinion, except for the above statement regarding inventory of publications and valuation of items in Canadian funds, the accompanying balance sheet and statement of income and disbursements present fairly the financial position of the Society of Actuaries as of May 31, 1966, and the results of its operations for the year then ended in conformity with generally accepted accounting principles applied on a basis consistent with that of the preceding year.

[Signed] H. W. DEININGER, C.P.A.
Auditor

BOOK REVIEWS AND NOTICES*

M. W. Latimer, *The Relationship of Employee Hiring Ages to the Cost of Pension Plans*, pp. i, 209, U.S. Department of Labor, Bureau of Labor Statistics, July, 1965.

BACKGROUND

Responding to a requirement of the Civil Rights Act of 1964 to "study . . . the factors which might tend to result in discrimination in employment because of age," the Secretary of Labor submitted to Congress on June 30, 1965, the comprehensive report, *The Older American Worker: Age Discrimination in Employment*. As background for one phase of this study, the report, which is the subject of this review, was prepared for the Bureau of Labor Statistics by Mr. Murray W. Latimer, a consulting actuary and former chairman of the Railroad Retirement Board.

The purpose of Mr. Latimer's study is evident from its title: *The Relationship of Employee Hiring Ages to the Cost of Pension Plans*. It will, of course, be of particular interest to pension actuaries, who are frequently asked to make similar analyses of their clients' plans. The report should also be of interest to other actuaries because of the techniques employed to analyze this surprisingly complex subject.

SUMMARY OF RESULTS

The conclusions of the study are concisely expressed in the Secretary of Labor's report to Congress:

The annual cost of a given pension increases with age of entry because the plan's obligations must be met with fewer installments and with smaller savings from experience factors. However, cost evaluations for a variety of illustrative plans indicate that both cost levels and age-cost differentials are sharply reduced or even eliminated by a variety of plan provisions.

Benefit formulas influence age-cost differentials by the extent to which they curtail the pensions of older entrants as compared with those of younger entrants. Formulas in common use among single employer plans accomplish this by relating pensions to length of service or to length of service and compensation. The differential between entrants is about cut in half, even at relatively young ages, if pensions are proportional to the workers' length of service. If pensions are made a function of both service and compensation, further reductions usually occur because of the older entrants' lower average earnings under the assumptions used.

Other plan provisions also have an impact on age-cost differentials. The addition of a disability pension affects these differentials very little, although costs are increased. Vesting, on the other hand, increases the relative cost of the young entrant by eliminating a major portion of the savings that otherwise result from withdrawals. The incidence of withdrawals is greatest among young entrants. For that reason, voluntary

* Books and other publications noted with an asterisk (*) may be borrowed from the library of the Society of Actuaries under the rules stated in the *Year Book*.

early retirement also reduces age-cost differences, although it does not have a significant cost impact if vesting is provided and if the pension is actuarially reduced. Finally, service requirements and involuntary retirement provisions have some effect. Service requirements reduce both annual costs and age-cost differences while involuntary retirement increases costs without significantly changing age differentials.

In conclusion, as between a young job applicant (age 27) and an older applicant (age 47), the cost differences against the older applicant under the conventional assumptions used in this report average about \$80 a year for typical single employer plans. In some cases the difference would be greater; in others it would be nominal. The differences would average about \$100 if withdrawal rates or benefit levels are somewhat higher than those in this report. More significantly, as between a hiring policy which concentrates on young applicants only and one which includes a cross-section of the available labor force, perhaps even favoring older workers, the cost difference against the latter group would usually average between \$50 and \$100 per new entrant. However, the complete elimination of age-cost differences would rarely be compatible with providing a reasonable pension for older entrants.

TECHNIQUE

Contribution requirements were computed on the entry-age-normal method for a variety of benefit formulas, using an interest assumption of $3\frac{1}{2}$ per cent per annum and mortality assumptions derived from the *a-49*, *Ga-51* and 1956 Railroad Retirement Board Disability Tables. Details about other experience assumptions and of the plan variations studied, of particular significance in analyzing the results obtained, are summarized below.

1. *Ages of employment.*—Separate sets of computations were made for each of two very different hiring patterns: a “young-hire” pattern and an “old-hire” pattern. The “young-hire” distribution was based on the experience of a large western public utility company which confines its employment principally to recent graduates of high school or college. In this pattern half of the new employees were under 22, three-quarters less than 26, and almost 90 per cent were less than 30. The median hiring age was 21.97.

The “old-hire” distribution is that of laborers employed by governmental units—state, county, municipal, special district—in an eastern state in which there is a statutory prohibition against discrimination in employment on the basis, among others, of age. Over half of these employees were over 40 when hired and more than one-quarter over 50. The median hiring age was 41.11.

The author believes calculations on the basis of these two assumed groups of new entrants “probably overstate the possible range of pension contributions which could result from, on the one hand, a policy of complete non-discrimination and, on the other hand, an almost complete preference for employees under 30.”

2. *Terminations of employment other than by death, disability, or retirement.*—Again, separate sets of calculations were made designed to cover the normal range of variations in pension plan experience. One calculation utilized a moderate turnover scale intended to produce experience gains in the typical case.

The other employed a scale derived from an industry with high turnover whose use in setting funding levels would typically result in funding losses.

Both of these scales were on a select and ultimate basis. In the moderate turnover scale, first-year termination rates were in the 15–10 per cent range, with ultimate rates grading from about $6\frac{1}{2}$ per cent at age 20 down to 0 at ages 54–64. The heavy turnover scale showed first-year termination rates varying from 50 to 20 per cent, depending on age at hire, with ultimate rates ranging from 16 per cent at age 20 to a low of 1.66 per cent at age 54 and then increasing gradually to 4.49 per cent at age 64.

3. *Starting salary and rates of compensation increase.*—Contributions under all salary-based plans were calculated on the basis of an annual initial compensation of \$3,600, some of them also at \$4,200 or \$4,800. These starting salaries are associated with each of three assumptions about rates of salary increase: uniform annual increments of \$75, \$100, or \$150. The author believes the use of uniform increments rather than the more complex conventional salary scales is satisfactory for the purpose of observing the effect on contribution requirements of changes in compensation levels. Certainly, the interpretation of results would otherwise have been made considerably more difficult.

4. *Rates of retirement.*—Under plans providing for voluntary retirement after age 65, retirement rates for ages 65 and over ranging from 47 per cent (age 65) to 31 per cent (age 68) were employed; almost all employees were assumed to retire by age 70. Prior to age 65 it is assumed in the study that pensions will be the actuarial equivalent of the age 65 pension and that, because of this actuarial reduction, retirement rates at ages under 65 will be very low.

The author points out that if the pensions payable on early retirement are larger than the actuarial equivalent of the age 65 pension the assumption about low rates of retirement under age 65 would no longer be valid. He believes, however, that if an employer has a need for a pension plan which makes early retirement attractive he will be unlikely to be interested in hiring individuals at older ages; thus the study excludes consideration of plans with “no actuarial reduction” early retirement features.

5. *Other assumptions.*—No explicit distinction was made by sex, since the author felt the use of the two service tables provided a satisfactory measure of the effect of differing experience levels. No allowance was made for expenses, since, except for small plans, the allowance would be small and, in any event, would not change the relative contributions requirements at different ages.

6. *Plan variations.*—Calculations were made for four broad classifications of plans: a flat monthly amount; monthly amount per year of service; salary based plan without Social Security break-point adjustment; and salary based plan with adjustment at Social Security break-point.

Variations in plans were introduced depending on whether retirement occurs because of age alone or because of age and disability; whether or not vesting is provided; by changing the Social Security break-point assumption and the per-

centages applicable to compensation above and below the break-point; and by providing for offsets to adjust for retirement benefits (Social Security essentially) provided outside the plan. In total some eighteen different plans were studied.

A number of variants of each of the eighteen basic plans was also studied. These variants related to length of service required at retirement to qualify for pension; whether age retirement is compulsory or voluntary; whether the formula is career-average or final average in the salary-based plans; different combinations of benefit/salary ratios in salary-based plans with salary-break points; and different "Social Security offset" amounts.

Within each of the plan variants per capita contribution requirements were determined for representative ages at entry, usually 24, 47, and 57, and for the "young-hire" and "old-hire" groups described above. Cost differentials between young and old ages and groups were computed on a per capita and percentage basis.

Pension actuaries today are usually ready to agree that in many situations a proper measurement of the extent of a pension-plan commitment is best achieved by making valuations of the plan under several alternative assumptions about the course of future experience. It is not often, however, that they are called upon to apply this technique, as Mr. Latimer was, to measure cost variations for virtually the entire universe of private pension plans. The scope of such a task is evident from a few statistics about the text of the report: eighteen basic types of plans, hundreds of variants of the basic types, many thousands of different cost appraisals. These voluminous data are presented concisely in some twenty-seven tables in Mr. Latimer's study and still further summarized in seven tables in the research materials volume of the Secretary of Labor's report. For general actuarial reference—and admittedly the study was not prepared for this purpose—the report suffers somewhat from this very conciseness, since the coding employed to identify individual plan variants was difficult to keep in mind and the code keys, dispersed throughout the text, were difficult to locate.

The study is enhanced by the author's review, at the outset of the report, of the significance of the actuarial assumptions employed in making his calculations and the considerations which led to the particular choices he made.

A. CHARLES HOWELL

*W. Haber and M. G. Murray, *Unemployment Insurance in the American Economy*, pp. xx, 538, Richard D. Irwin, Inc., Homewood, Ill., 1966.

The publication of *Unemployment Insurance in the American Economy* has occurred at a most propitious time. Perhaps in no other period in the history of unemployment insurance in the United States has there been as much public interest in this highly technical and little understood branch of our Social Security system. The introduction of the Administration's bills (H.R. 8282 and S. 991) in 1965 evoked a torrent of discussion on the merits of the changes

proposed and made many heretofore disinterested persons aware of large areas of basic disagreement in the program. Answers to questions that arose were not usually easy to find. The mass of literature on unemployment insurance is diffuse, and the statistics, though voluminous, often fail to provide data in depth. This volume would justify itself if for no other reason than on the basis that it contains in a single volume discussions of every phase of unemployment insurance, listing, wherever necessary, source material for those who wish to pursue any particular subject in more detail than is possible in a book of this length. Moreover, the authors have been able to incorporate some developments that have occurred within the past year or two, adding greatly to the value of the book for reference purposes.

The book was made possible through financial grants by the W. E. Upjohn Institute for Employment Research, an organization responsible for a long series of reports on a variety of subjects pertinent to employment, unemployment, and unemployment insurance.

The authors have chosen to subdivide their book into five sections: Part I, "Unemployment and Its Insurability"; Part II, "How Unemployment Insurance Developed"; Part III, "Issues in Unemployment Insurance—Coverage and Benefits"; Part IV, "Financing Issues and Problems"; and Part V, "Problems in Relationships." Each chapter closes with a summary of its contents, and, finally, the book concludes with an over-all summary of progress and problems, together with the authors' recommendations for improving the present program.

As stated previously, unemployment insurance remains a controversial subject even though it is now accepted by all elements of our national community as a necessary and highly desirable social instrument for alleviating some of the loss incurred by persons who become unemployed. Unfortunately, much of the controversy that exists does so because of deep-seated and often prejudiced viewpoints held by all discussants—government, labor, employers, and the general public. Completely objective viewpoints are almost nonexistent. It is a credit to this book's authors that they have endeavored to present fairly all sides of each question discussed. This is not to say they do not have definite leanings in many of the topics discussed in this volume. This reader, at least, obtains the impression that the authors are inclined to favor a federally administered unemployment insurance program as opposed to the present federal-state structure, although in many places throughout their text they do point out the case in favor of state administration.

Most of the provisions of H.R. 8282 appear to be favored by the authors, including the proposal to change the experience rating standards. There are many in industry who would disagree vehemently with the arguments advanced by the authors in support of a change in the experience rating mechanisms. Uniform reductions in rates for all employers, regardless of their unemployment experience record, would give to the unstable employer an advantage at the expense of the positive account, stable employer. Negative account employers (i.e., those who are charged more in claims than they pay in unemployment insurance taxes) would be inclined to ally themselves with labor, the traditional

enemy of merit rating, in order to exert legislative pressure for uniform reductions in rates.

A sizable amount of state and national legislation of all types arises out of crisis situations, and the history of unemployment insurance legislation has been no exception. Much of the unemployment insurance legislation has been aimed at specific situations requiring immediate attention, and, as the authors state, a particular provision might have been included in a "horse-trading" deal as the only means of obtaining passage of some other more desirable provision. A great deal of information on the why and wherefore of these dealings is given by the authors. In fact, commencing with chapter vi, and continuing through chapter xix, the authors present a wealth of material on every issue pertaining to unemployment insurance that has arisen during the more than three decades the unemployment insurance laws have been on the statute books. Chapter x, "Issues in Coverage Extension," is given a thorough and scholarly analysis, as are also the subsequent chapters dealing with issues arising in problems of adequacy of benefits, duration of benefits, extended benefits, testing of labor market attachment, trends in disqualification, voluntary "quits," and the huge and pressing problem of financing the unemployment insurance program.

Recent events in Congress in the field of unemployment insurance legislation demonstrate that careful consideration and full debate, based on intelligent analyses of all facets of the problems, are needed. The hearings and debate on H.R. 8282, the subsequent rewriting and introduction of H.R. 15119 by the Committee on Ways and Means, and, finally, the hearings and amendments in the Senate illustrate both well- and ill-conceived actions on the part of our legislative bodies. It is to be hoped that this work of Messrs. Haber and Murray will be used by those uninformed members of Congress who must determine the course of unemployment insurance legislation in the forthcoming months.

Students of unemployment insurance will find this book a welcome addition to the literature, regardless of whether or not they agree with the authors' viewpoints in all cases. It is especially valuable because of the wealth of historical detail that forms the background for each topic discussed. It is often difficult to appreciate some of the idiosyncrasies of unemployment insurance unless one is fortunate enough to have access to source material detailing its development. The authors have done an excellent job in condensing and organizing such background material.

Although unemployment insurance may not provide as broad scope for actuarial analyses as some other types of insurance, nevertheless, in recent years it has become of increasing interest to many actuaries. Cognizance of this fact is indicated by inclusion of unemployment insurance in the required course of study of the Society of Actuaries. *Unemployment Insurance in the American Economy* appears to be an excellent possibility for inclusion, in whole or in part, in the syllabus of required reading for the Society's examinations.

A few errata, mostly in dates, were noted by this reader. Undoubtedly, they have been detected and will be corrected.

K. ARNE EIDE

SELECT CURRENT BIBLIOGRAPHY

In compiling this list, the Committee on Review has digested only those papers which appear to be of direct interest to members of the Society of Actuaries; in doing so, the Committee offers no opinion on the views which the various articles express. The digested articles will be listed under the following subject matter classifications: 1—"Actuarial and Other Mathematics, Statistics, Graduation"; 2—"Life Insurance and Annuities"; 3—"Health Insurance"; 4—"Social Security"; 5—"Other Topics."

The review section of the *Journal of the Institute of Actuaries* contains digests in English of articles appearing in foreign actuarial journals.

ACTUARIAL AND OTHER MATHEMATICS, STATISTICS, GRADUATION

M. V. Dixon, Editor, *BMD Biomedical Computer Programs*, pp. viii, 620, Health Sciences Computing Facility, Department of Preventive Medicine and Public Health, School of Medicine, University of California, Los Angeles, 1965.

This text was "prepared to describe in a unified manner the various computer programs which have proved useful in many types of medical research projects at UCLA." They include programs of description and tabulation which calculate means, variances, correlations, chi-square tests, and frequency tables and also plot data curves and histograms. The section on multivariate analysis includes programs for principal component, factor, discriminant, and canonical analysis. Under regression analysis there are programs for simple linear, stepwise, multiple, polynomial, and periodic regression as well as harmonic analysis. There are also programs to prepare life tables, to calculate survival rates, to do contingency analyses, to analyze time series, and to prepare various analyses of variance including factorial designs, covariance, the general linear hypothesis, contrasts, and multiple range tests.

All programs, with some minor exceptions, are written in Fortran II for the IBM 7090/94. For each program there is (1) a general description which includes information on the input, output, and data limitations; (2) a section on the order of cards in the job deck; (3) a section on card preparation; (4) a computational procedure; (5) a reference list; and (6) a sample problem.

HEALTH INSURANCE

L. S. Reed, *The Extent of Health Insurance Coverage in the United States*, Research Report No. 10, pp. 72, Social Security Administration, Washington, July, 1965.

This report presents estimates of enrollment in various types of health insurance plans, based on available data from reported enrollment and from household surveys. Analyses are made not only for the United States as a whole and for the major types of coverage but also by regions of the country and by state and for some types of coverage other than the major ones of hospital care, surgical services, and physician in-hospital visits.

L. S. Reed, *Financial Experience of Health Insurance Organizations in the United States*, Research Report No. 12, pp. 70, Social Security Administration, Washington, December, 1965.

This report presents and analyzes a wealth of data on the financial experience of Blue Cross and Blue Shield plans, insurance companies, and independent plans operating in the health insurance field for the period 1962-64.

Division of Information and Research, *Health Insurance and the Effectiveness of Health Care*, Health Insurance Association of America, New York, 1965.

This study paper deals with the following areas which have direct bearing on the effectiveness and quality of health care: professional education; supply and distribution of personnel and facilities; organization of health care services; licensure requirements; professional society membership standards and qualifications for physician hospital staff privileges; accreditation; maintenance of medical records; tissue, medical audit and utilization committees in hospitals; medical society review committees; area-wide planning of health resources; pricing practices; health insurance benefits patterns; research; consumer awareness.

"The relationship of health insurance to these areas of consideration has been scrutinized from the following standpoints: insurance contract provisions and benefit design; claims management or conservation programs; development of statistical information and improved measurement of the quality of care; consultation, collaboration, and negotiation with professional organizations; technical assistance to professional organizations, purchasers of group insurance, or public bodies; financial assistance to others in the interest of improved effectiveness of care; support for appropriate legislation; and publicity, public support of professional and hospital associations, and education of the public."

B. L. Gordon, M.D., Editor, *Current Procedural Terminology*, pp. xx, 172, American Medical Association, Chicago, 1966.

"*Current Procedural Terminology (CPT)* is a system of standard terms, provisional eponyms, and descriptors developed for the convenience of physicians and designated personnel to expedite the reporting of therapeutic and diagnostic procedures of surgery, medicine, obstetrics, psychiatry, roentgenology, clinical laboratory sciences, and physical medicine. The system also includes terms for supplementary and/or related services, as visits, consultations, examinations, evaluations, and miscellaneous studies and/or management. The names and codes were derived from the major systems of procedural terminology, current literature, and latest editions of textbooks, and co-ordinated to attain maximum uniformity and currency.

"A feature of *CPT* is the listing of terms alphabetically and numerically to provide simplified access to the names of procedures by term or code."

Where applicable, the corresponding codes of the "Standard Nomenclature of Diseases and Operations" and the "International Classification of Diseases, Adapted" are given.

U.S. National Center for Health Statistics, *Identifying Problem Drinkers in a Household Health Survey*, pp. 46, Public Health Service Publication No. 1000, Series 2, No. 16, Washington, May, 1966.

"Is it possible, by asking the respondent in a household survey sample a few simple questions about his own and other related household members' drinking behavior, to distinguish those persons who have been labeled 'alcoholic' or who are likely candidates for the label and to do so with reasonable accuracy?"

The findings of this study identified three out of four known alcoholics of lower socioeconomic status but only one out of three of those with more income and education. However, the concept of alcoholism remains ill defined, there is a misclassification problem, and measures developed in one area of the country remain to be tested in other areas.

U.S. National Center for Health Statistics, *Conceptual Problems in Developing an Index of Health*, pp. 18, Public Health Service Publication No. 1000, Series 2, No. 17, Washington, May, 1966.

"The problem of measuring levels of health is discussed with a view to identifying measures indicative of changing health status in the United States population. Traditionally, mortality rates have been used for this purpose but changing health problems and programs have impaired their utility as a measure of the need for and adequacy of health programs. If new indexes are to be devised the question of what to measure must be resolved."

U.S. National Center for Health Statistics, *Characteristics of Patients of Selected Types of Medical Specialists and Practitioners, United States, July 1963-June 1964*, pp. 63, Public Health Service Publication No. 1000, Series 10, No. 28, Washington, May, 1966.

Persons who consulted pediatricians, obstetricians or gynecologists, ophthalmologists, otolaryngologists, psychiatrists, dermatologists, orthopedists, chiropractors, optometrists, and podiatrists are tabulated in relation to age, sex, residence, geographic region, family income, usual activity status, color, and education of the head of family.

"As education of the family head increased the proportion of the population that was seen also increased, except for chiropractic patients. Family income was directly related to the percentage of the population reporting visits. In general, proportionally more of the population residing in metropolitan areas reported visits than did those living elsewhere. In every instance the white population had more people using the services of the professions studied than did the nonwhite population."

U.S. National Center for Health Statistics, *Hospital Discharges and Length of Stay: Short-Stay Hospitals, United States, July 1963-June 1964*, pp. 66, Public Health Service Publication No. 1000, Series 10, No. 30, Washington, June, 1966.

"Among the civilian population not residing in institutions, the rate of discharge from short-stay hospitals during the average 12-month period was 133.7 per 1,000 persons. The average length of stay per discharge was 8.4 days. The discharge rate for females, 157.5 per 1,000, was considerably greater than that for males, 108.3 per 1,000. However, if hospitalizations for delivery of mothers are excluded, the sex difference in rate is reduced substantially, amounting to a rate for females of 117.7 or 8.7 percent greater than the rate for males. The average length of stay for males was 9.9 days per discharge, and for females, 7.4 days for all discharges and 8.5 days for discharges other than for delivery."

Data are available for other demographic variables and are adjusted to include information for deceased persons.

U.S. National Center for Health Statistics, *Oral Hygiene in Adults, United States, 1960-1962*, pp. 30, Public Health Service Publication No. 1000, Series 11, No. 16, Washington, June, 1966.

"Women were found to have cleaner teeth than men, as were white men and women in comparison with Negro adults. Oral hygiene was also found to improve with increasing education and with increasing family income. On the other hand, oral cleanliness worsened with advancing age because of a gradual increase in the amount of calculus."

Oral hygiene was more closely associated with education than with family income, and to both these factors is attributed a large part of the difference by race.

U.S. National Center for Health Statistics, *Episodes and Duration of Hospitalization in the Last Year of Life, United States—1961*, pp. 48, Public Health Service Publication No. 1000, Series 22, No. 2, Washington, June, 1966.¹

"In this report statistics are presented on the number of episodes of care and the average number of days of care persons received in short-stay hospitals and resident institutions during the last year of life."

"Of those persons dying in 1961, 72 percent received some care in hospitals or institutions during the 12 months preceding death," and, of those hospitalized, "three-fifths had one episode of care, about one-fifth had two episodes, and just less than one-fifth had three or more episodes." These persons averaged 63 days of care. "The episode rate for persons dying of malignant neoplasms was three times that of persons dying of accidents."

SOCIAL SECURITY

R. M. Ball, *Policy Issues in Social Security*, Social Security Bulletin, June, 1966.

This article, by the Commissioner of Social Security, expresses significant views in regard to the OASDHI program and should be read by all interested students of the subject. Discussed in particular are the level of benefits, automatic adjustment of benefits with changes in wages or with changes in prices, similar automatic adjustment of the earnings base, and the possibility of contributions from general revenues.

R. J. Myers, *Mortality Study of Charter Beneficiaries*, Actuarial Note No. 30, pp. 6, Social Security Administration, Washington, July, 1966.

This note presents a mortality study of persons who have received OASDI benefits continuously since January, 1940, and who are thus now at least 90 years old. The projected survivors (according to population mortality tables) agreed quite closely with the actual survivors. Continued study of this closed group in the future should provide interesting and valuable data in regard to centenarians.

G. I. Kowalczyk, *Enrollment for Supplementary Medical Insurance, July 1, 1966*, Actuarial Note No. 31, pp. 3, Social Security Administration, Washington, August, 1966.

This note gives data on the enrollment under the Supplementary Medical Insurance program as of July 1, 1966, by states, showing both the numbers involved and the percentage of those eligible to enroll who actually did so. For the United States as a whole, the participation rate was 91 per cent, and there was a relatively narrow spread around this figure for the various states.

W. Hasenberg, *Income-Tax Treatment of Old-Age Pensions and Contributions Here and Abroad*, Social Security Bulletin, August, 1966.

This paper presents interesting information on the income-tax treatment of benefit payments and contributions under old age pension programs in more than twenty foreign countries and discusses the policy considerations involved.

OTHER TOPICS

U.S. Bureau of the Census, *1960 Censuses of Population and Housing: Procedural History*, pp. 387, Washington, 1966.

This report is divided into three parts: Part I, "1960 Population and Housing Censuses"; Part II, "Survey of Components of Change and Residential Finance"; and

¹ See *TSA*, XVII, 590.

Part III, "Population and Housing Census Items." There are nine appendixes and an index, and most chapters are followed by detailed bibliographies.

Part I presents a brief history of the census, describes the organization of the Bureau, and discusses planning, sampling, and estimation, geographic work, preparatory work, and field organization and training; then enumeration, processing of data, the publication program, and the evaluation and research program are analyzed in detail. A description of the census in Puerto Rico and other outlying areas follows.

Part II relates the detail of the Survey of Components of Change and Residential Finance that "was developed to provide information about the types of changes that had occurred in the nation's housing and about the financing of residential properties for the use of Federal and private agencies engaged in planning, analysis and research in housing and related fields."

Part III details the items that were included on the questionnaires used in Parts I and II and defines the terms used thereon.

U.S. National Center for Health Statistics, *Seasonal Variation of Births, United States, 1933-63*, pp. 59, Public Health Service Publication No. 1000, Series 21, No. 9, Washington, May, 1966.

The seasonal pattern of births in the United States is characterized by a peak in September and a trough in April and May, with the magnitude of the former increasing over the years. The pattern is compared with that of other countries, and environmental and social theories are presented to try to explain the variation. Demographic factors and registration practices are also studied for their possible effects.

Obituary

ANDREW J. BLACKMORE
MALVIN EDWARD DAVIS
SAMUEL M. DAVIS, JR.
JAMES HECTOR LITHGOW
REGINALD CAMPBELL McCANKIE
JOHN EDWARD MORRISON
ALEXANDER MUTCH
WAYNE B. RULE
BRUCE EDWIN SHEPHERD

1894 . . . Andrew J. Blackmore . . . 1966

Andrew J. Blackmore died on September 12, 1966, at the age of 72. He was born in Brooklyn, New York, on February 28, 1894, but since early boyhood had resided in Vermont. World War I interrupted his attendance at Wesleyan University in Middletown, Connecticut, and he spent two years as a radio operator in the navy. Following the war, he completed his undergraduate work and then taught mathematics for a year in the New London, Connecticut, high school.

In 1921 he joined the actuarial department of National Life Insurance Company and became an Associate of the Society of Actuaries in 1936. The following year he was named assistant actuary of the company.

He assumed the duties of assistant secretary of the company in 1946 and three years later was named secretary. In 1955 he was made vice-president and secretary of the company, which position he held until his retirement in 1959.

Mr. Blackmore was a long-time member of the Montpelier School Board and had served as its secretary. He was a director of the Capital Savings Bank and Trust Company and of the Vermont Mutual Fire Insurance Company and was a trustee of the Boutwell Masonic Building, Inc. He is survived by his widow, two sons, a daughter, and ten grandchildren.

1901 . . . Malvin Edward Davis . . . 1966

The death of Malvin Edward Davis on August 26, 1966, at the age of 65 is a loss deeply felt throughout the insurance industry and the actuarial profession. His leading contributions and executive vision had significant effect upon the business and will continue to influence it for some time to come. It was characteristic of his vigor and devotion that his activity during his last years remained undiminished in spite of poor health.

His brilliance and energy were apparent from the first. He completed his undergraduate studies at Wesleyan University in three years, graduating as a Phi Beta Kappa member. He joined the Metropolitan Life Insurance Company in 1923 and completed his actuarial examinations three years later to become a Fellow of the Actuarial Society of America. In 1930 he was appointed an officer of the Metropolitan and rose to become senior vice-president and chief actuary on January 1, 1960. He was made executive vice-president on January 1, 1963, and held this post until his retirement in mid-1965.

Distinguished in professional circles, Mr. Davis was president of the Society of Actuaries in 1956-57, having previously served on the Board of Governors and several of the Society's committees. He was president of the XVth International Congress of Actuaries when it convened in New York City in 1957. He was also a member of the Casualty Actuarial Society, the American Mathematical Association, and numerous business organizations.

He brought scholarship and a remarkable thoroughness to his work. A specialist in industrial life insurance, he authored numerous papers on the subject. He presented testimony on the conduct of the industrial insurance business before the Congressional Temporary National Economic Committee at its 1939 hearings. In 1944 he published *Industrial Life Insurance in the United States*, which won the Elizur Wright Insurance Literature Prize as the best insurance book published that year.

He also brought foresight. After World War II he was among the first to recognize the potential to the industry of electronic data-processing machinery, and he forcefully pursued a program to test, apply, and guide the development of these new devices. His work in this gained him such prominence that his views on the subject were continually sought both at home and abroad. In 1948 he was appointed chairman of a new committee of the Society whose task was to examine electronic data-processing. The committee's pioneering reports gave guidance to the entire insurance industry during the formative years of this revolutionary

development and made a lasting impact upon the conduct of the insurance business.

He was a man of many intellectual attainments, not the least of which was an abiding love and knowledge of music, which brought him comfort and pleasure throughout his life. He traveled widely and at one time had a second home in Bermuda.

He was married in 1932 to Mildred Lamb, who died in 1959. There are no immediate relatives.

Mal Davis' alert competence, sound judgment, and unshakable integrity earned him the respect, admiration, and affection of all who worked with him. He left his mark on our profession, on his company, and in our hearts.

1921 . . . Samuel M. Davis, Jr. . . . 1966

Samuel M. Davis, Jr., a lifelong resident of Dallas and associate actuary at Southwestern Life Insurance Company, died May 23, 1966, after an illness of two months. He was a Fellow of the Society of Actuaries.

A resident of Richardson, Texas, he would have marked nineteen years with Southwestern Life in June.

He was graduated from the former Terrill Preparatory School, of which his father, Samuel M. Davis, Sr., was headmaster. He received his Bachelor of Business Administration degree from Southern Methodist University in 1942 and his Master's degree from the University of Michigan in 1947. He became associated with Southwestern Life Insurance Company in June of that year. In September, 1957, he was appointed actuarial assistant, in March, 1959, he became assistant actuary, and in March, 1961, he became associate actuary.

Mr. Davis played professional baseball in the West Texas-New Mexico and Texas leagues prior to entering the Marine Corps in 1942. A first lieutenant, he was a B-25 pilot and was awarded the Air Medal with four gold stars and the Distinguished Flying Cross. He was a member of the Messiah Lutheran Church, Richardson, Texas, and the Canyon Creek County Club.

Although Sam's professional career was relatively brief, he commanded the respect of his fellow actuaries through his professional activities. He was always eager to be of service through Society activities and work for the Actuaries Club of the Southwest, where he served for many years on various committees.

He is survived by his widow, the former Bobbie Jean Peek, a daughter, Deborah Ann, 13, and his parents, Mr. and Mrs. Samuel Davis, Sr., of Dallas.

1890 . . . **James Hector Lithgow** . . . 1966

James Hector Lithgow, 75, retired chairman of the board and director of The Manufacturers Life Insurance Company, died on July 1, 1966, after a lengthy illness.

Mr. Lithgow was born in Bobcaygeon, Ontario, and attended Trinity College School.

He joined the actuarial department of The Manufacturers Life Insurance Company in 1908. Three years later he became an Associate of the Institute of Actuaries in Great Britain and in 1916 enlisted as a gunner in the Canadian Artillery. Something of the character of the man may be learned from a letter received by the company from the war zone asking for books so that he could pursue his actuarial studies "as the opportunity presented itself." He obtained his Fellowship in the Actuarial Society of America in 1923 and was made the company's actuary the following year. He was appointed general manager of his company in 1931, a director in 1936, president in 1951, and chairman of the board in 1956. Ill-health forced his retirement in 1959. As the responsible administrative officer of The Manufacturers Life for twenty-five years, Mr. Lithgow guided the company through a major growth period.

Mr. Lithgow was president of the Actuaries' Club of Toronto in 1921. He also served as president of the Canadian Life Insurance Association, the Insurance Institute of Toronto, and the Life Insurance Institute of Canada and was a former director of the Institute of Life Insurance in New York.

Mr. Lithgow was well known, throughout the industry as well as in his own company, for his integrity and ability but will perhaps be best remembered for a rather shy modesty that was happy to credit his associates with the accomplishments which were often the end result of his leadership.

He is survived by his widow, the former Mizpah Sussex, and three children, Colonel Charles H. Lithgow, Mrs. C. L. Rodgers (Janet), and Mrs. John de Pencier (Marni).

1890 . . . **Reginald Campbell McCankie** . . . 1966

Reginald C. McCankie, a Fellow of the Society of Actuaries and a past president of the American Institute of Actuaries and the Home Office Life Underwriters Association, died in Des Moines, Iowa, on September 17, 1966, after a long illness.

"Reg," as he was known to his many friends, was born in Edinburgh, Scotland, on May 6, 1890, and was educated at George Watson's College,

a school where many Scottish actuaries received their early mathematical training.

After serving an apprenticeship in the Scottish Metropolitan in Edinburgh, he came to the United States in 1911 as an assistant to a fellow Watsonian, John Campbell Cameron. Jack Cameron, with whom Reg had been associated at the Scottish Metropolitan, had come to the United States the previous year and joined the Great Southern Life Insurance Company of Houston, Texas.

Soon thereafter, Reg became actuary of the San Antonio Life Insurance Company in San Antonio, Texas, where he served until 1914. While he was on vacation in Scotland in August, 1914, war broke out. Cabling his resignation to the Texas company, Reg immediately enlisted in the Ninth Royal Scots.

In 1915 he was commissioned a second lieutenant in the Highland Light Infantry. He participated in many of the major engagements during the four years of the war and was wounded in 1916 and again in 1917. He was awarded the Military Cross and decorated at Buckingham Palace by King George V. At war's end he was discharged with the rank of captain.

Returning to the United States in 1919, Mr. McCankie joined the Shenandoah Life of Roanoke, Virginia. On December 20, 1920, he came to the Equitable Life of Iowa as an assistant to another fellow Watsonian, Robertson G. Hunter.

Mr. McCankie was appointed assistant actuary of the Equitable Life of Iowa in 1921, associate actuary in 1926, underwriting vice-president and associate actuary in 1938, and underwriting vice-president and actuary in 1943. In January, 1948, upon the retirement of Mr. Hunter, he was elected vice-president and actuary and a member of the board of trustees. He retired in 1955 but continued to serve on the board until January, 1957.

Mr. McCankie was successively secretary, vice-president, president (1935-37), and member of the board of governors of the American Institute of Actuaries and was also a member of the council of the Actuarial Society of America.

Selection of risks was the field of Mr. McCankie's greatest interest as an actuary, and he had served as president (1944-45) and member of the executive council of the Home Office Life Underwriters Association.

Concealed beneath a somewhat stern exterior, engendered perhaps by four years of military service as a commanding officer on the western front, was a warmly human personality, whose constant consideration of others and innate kindness endeared him to his many friends in the

Equitable Life of Iowa and throughout the life insurance business in the United States and Canada.

Surviving are his widow, Olive, and a daughter, Phyllis, both of Des Moines, and a sister, Mrs. Arthur Coburn, of Dallas, Texas, widow of another well-known actuary born in Edinburgh.

1911 . . . John Edward Morrison . . . 1966

John Edward Morrison, executive vice-president of The Great-West Life Assurance Company, died in Winnipeg, Canada, on Friday, October 14, at the age of 55.

Mr. Morrison was an actuary and executive of great ability who was warmly admired and respected by all who knew him. His services to the actuarial profession were widely recognized, and during his outstanding career he was honored by high office in many actuarial organizations. At the time of his death he was a vice-president of the Society of Actuaries and had for many years been active in its affairs as a member of the board and of various committees. He was also vice-president for Canada of the International Congress of Actuaries, a past president of the former Canadian Association of Actuaries, and played a prominent part in the formation of the present Canadian Institute of Actuaries.

Ted Morrison, son of the late Alexander S. Morrison and Bessie Morrison, was born in Souris, Manitoba, on July 23, 1911, and came to Winnipeg at an early age. He attended public schools in Winnipeg, graduated from the University of Manitoba in 1931, and became a Fellow of the Society of Actuaries in 1943.

His whole business career was spent with The Great-West Life Assurance Company. He joined the company in 1932 and became assistant actuary in 1943 and underwriting executive in 1947. He was named assistant general manager and actuary in 1952 and in 1958 became vice-president and actuary. Mr. Morrison was appointed executive vice-president in 1960 and was elected to the Board of Directors one year later. In all these positions he made a major contribution to the company's growth.

Because of his broad interests and intimate knowledge of all aspects of life and health insurance, Mr. Morrison was frequently requested to serve on industry committees, and during his business life he made many valuable contributions to the work of various insurance associations, including the Canadian Life Insurance Association and the Canadian Health Insurance Association.

Mr. Morrison also had a long record of community service. He was a founding member of the United Way of Greater Winnipeg and a member

of the board and past chairman of the Manitoba Cancer Treatment and Research Foundation. For many years he was active and held office in the Manitoba division of the Canadian Cancer Society and the Winnipeg Chamber of Commerce.

While accepting wide responsibilities in business and community affairs, Mr. Morrison continued to be an avid sportsman throughout his life. He was a skilled golfer, curler, and angler and was interested in other sports.

Mr. Morrison had a great capacity for friendship and helpful understanding of others. His untimely death is a great loss, and he will be long remembered by his many friends and associates.

Mr. Morrison is survived by his widow, Mary, by a son and a daughter, and by his mother.

1907 Alexander Mutch 1966

Alexander Mutch, an Associate of the Society of Actuaries and a member of both the Institute of Actuaries and the Academy of Actuaries, died on September 4, 1966, after a brief illness. Mr. Mutch was born at Aberdeen, Scotland, on May 14, 1907. He attended primary and secondary schools in Aberdeenshire and was an honors graduate from Aberdeen University, with an M.A. degree in mathematics. In 1930 he emigrated to Canada and entered the service of Sun Life Assurance Company in Montreal.

In 1937 he joined the United States Life Insurance Company in the city of New York, where he contributed heavily to the actuarial aspects of the company's operations, with particular expertise in statement and agency compensation areas. In 1958 he was elected associate actuary, a position which he held at the time of his death. A sports enthusiast, Mr. Mutch played championship amateur tennis for many years.

Mr. Mutch was a quiet, capable, skillful, and intelligent man with uncanny perceptiveness. His death was a heavy loss to his family, friends, and business associates.

Surviving are his widow, Ethel, in Glen Rock, New Jersey, three daughters, and a granddaughter. In addition, he leaves two sisters and two brothers residing in Scotland.

1912 Wayne B. Rule 1966

Wayne B. Rule, an Associate of the Society since 1940, died suddenly on August 30, 1966, following a heart attack.

Mr. Rule was born on December 27, 1912, in Fort Dodge, Iowa, where he received his early education. He attended the State University of Iowa and was graduated in 1936 with Bachelor of Arts and Master of Science degrees, receiving Sigma Xi honors.

After graduation he joined the actuarial division of the Metropolitan Life Insurance Company and spent most of his professional career in fields related to statement accounting and taxes. He progressed through various positions of responsibility in these fields and rose to administrative rank. At the time of his death he was manager of the statement and analysis unit in the company's actuarial department and ranked as an outstanding authority in his field. Mr. Rule had a keen and analytical mind and enjoyed the challenge of a complex problem. He also had a love and gift for language which resulted in lucid, concise exposition of his findings and conclusions. A man of highest personal and professional integrity, completely devoted to the standards of excellence, he won the respect and esteem of all his associates.

Wayne and his wife, Eva, who survives, were active in the work of their church, the Presbyterian church in Basking Ridge, New Jersey. They also shared and brought with them from Iowa a love of the soil. Wayne's hobbies of gardening and carpentry were marked by the same standards of excellence as he gave to his professional life.

Wayne Rule was a quiet and a modest man whose passing is deeply felt by all who knew him.

1900 . . . **Bruce Edwin Shepherd** . . . 1966

Bruce Edwin Shepherd, vice-president of the Society of Actuaries, born in Kalamazoo, Michigan, on March 28, 1900, died in West Orange, New Jersey, on September 25, 1966. This closed the distinguished career of a scholar, an actuary of recognized ability, a practical administrator, a fine executive, and a dedicated protagonist of the life insurance message.

Educated in the public schools of Kalamazoo, he entered the University of Chicago, where he earned his Ph.B. in 1922 and was elected to Phi Beta Kappa. He became a member of Alpha Delta Phi.

He qualified as a Fellow of the Actuarial Society of America and of the American Institute of Actuaries in 1927. Between 1928 and 1934 he served as a member and as vice-chairman and chairman of the Examinations Committee of the Institute and in 1934 became vice-chairman of the Joint Committee on Examinations of the Institute and the Society. In the Institute he was a member of the Program Committee in 1936, chairman of the Press Committee (1937-42), and a member of the

Board of Governors (1936-38). In the Society of Actuaries he was a member of the Committee on Frequency and Type of Meetings (1962-63), of the Committee on Public Relations (1957-66), and of the Board of Governors (1956-57 and 1960-62). He was elected a vice-president of the Society in 1965. He had been a member of the Senior Actuaries Club since 1946. He was a contributor to the *Transactions* of the Society and to insurance publications.

Mr. Shepherd's public career began with his appointment as chief assistant actuary of the Department of Banking and Insurance of New Jersey in 1927. Prior to that time he had served in various actuarial capacities with the Security Life and Trust, the Illinois Life, and the Missouri State Life Insurance Company. He entered the service of the New Jersey Department shortly after its reorganization following a legislative investigation and was one of a group selected to give the state a department with insurance sophistication and competence.

He was appointed actuary of the Association of Life Insurance Presidents in August, 1932. He served the Life Insurance Association of America until his retirement at the end of the year 1964. He became manager of the Life Insurance Association of America in 1945, executive vice-president in 1954, and president in 1962. Upon his retirement, he was elected to the Board of Directors of the Guardian Life Insurance Company and served that company until his death.

His interests carried him into areas remote from the life insurance business. His early interest in athletics found later expression in his enthusiasm for golf, and his inquiring mind found satisfactions in ornithology. He participated generously in community activities and in the work of his church, of which he was a trustee for many years.

As a protagonist of the life insurance message, his energies were directed toward the strengthening of the foundations of the business in a practical way. As a spokesman for the business, his words were received with respect and attention. His influence on life insurance was far greater than his public appearances and expressions suggested. The *Insurance Field* named him "Insurance Man of the Year" in 1964.

Mr. Shepherd was one of three brothers dedicated to the actuarial profession. His brother, the late Clinton, was a Fellow of the Society. His brother Pearce is a past president of the Society of Actuaries. He is also survived by his widow, Edna Michon Shepherd; a son, Scott Shepherd; a daughter, Mrs. Justine Shepherd Freud; a sister, Edith E. Shepherd; and five grandchildren.

MINUTES OF THE ANNUAL MEETING

of the

SOCIETY OF ACTUARIES

HELD AT THE AMERICANA HOTEL
BAL HARBOUR, FLORIDA

OCTOBER 31, NOVEMBER 1 AND 2, 1966

The meeting was called to order by the President, Mr. Gilbert W. Fitzhugh, on Monday, October 31, 1966, at 9:35 A.M. The following is a summary of the attendance at the meeting:

SUMMARY

Fellows.....	603
Associates.....	165
Guests.....	47
Total.....	815

The President introduced as a distinguished guest Mr. Harold E. Curry, president of the Casualty Actuarial Society.

The President also recognized Mr. James E. Hoskins, Past President, who became an Associate fifty years ago.

The President invited members of any sister organizations present to participate in all discussions.

Mr. Andrew C. Webster explained the new election procedures. The Society then proceeded to the election of officers and members of the Board of Governors. The elections were completed on Monday afternoon with the following results:

<i>President-Elect:</i>	Morton D. Miller
<i>Vice-Presidents:</i>	<i>Two-Year Term</i>
	Wendell A. Milliman
	David G. Scott
	<i>One-Year Term</i>
	William J. November
<i>Secretary:</i>	Fredrick E. Rathgeber
<i>Treasurer:</i>	Herbert L. DePrenger
<i>Editor:</i>	Robert T. Jackson

*Board of Governors:**Three-Year Term*

John M. Bragg
John K. Dyer, Jr.
Donald M. Ellis
Alton O. Groth
Edward A. Lew
Charles L. Trowbridge

Two-Year Term

George Ryrie

One-Year Term

Alton P. Morton

The President gave his presidential address as printed in this number of the *Transactions*.

The President advised that the Society had received word of the deaths of nine members since the last meeting: Malvin E. Davis, Samuel M. Davis, Jr., James H. Lithgow, Reginald C. McCankie, John E. Morrison, and Bruce E. Shepherd, Fellows; Andrew J. Blackmore, Alexander Mutch, and Wayne B. Rule, Associates. The President made special mention of the death of Mr. M. Albert Linton, Past President of the Actuarial Society of America, which had been announced at the Chicago meeting, and of Mr. Reginald C. McCankie, Past President of the American Institute of Actuaries. Obituaries of Mr. Malvin E. Davis, Past President; Mr. John E. Morrison, Vice-President; and Mr. Bruce E. Shepherd, Vice-President, were then read. The members stood for a minute in silent respect for the memories of these men. Obituaries are printed in this volume of the *Transactions*.

The minutes of the Regional Meeting held April 14 and 15, 1966, and the minutes of the Regional Meeting held April 28 and 29, 1966, as printed in the *Transactions*, Volume XVIII, pages 30-34, inclusive, were approved.

The proposed amendments to the Constitution, redesignating the present Article VIII as Article IX and substituting a new Article VIII entitled "Indemnification of Society Members," which had been approved at the San Francisco meeting, were ratified by the membership.

The Secretary presented the following summary of business transacted by the Board of Governors at a meeting held October 30, 1966:

1. Approved for admission as Associates of the Society Leonard G. Hall, David F. Howe, Kul B. Kohli, Geoffrey R. Minns, and Bernard P. Willcox, all Fellows of the Institute of Actuaries, and William S. Pretorius, a Fellow of the Faculty of Actuaries.

2. Approved a recommendation of the Committee on Admissions that steps be taken to amend the Constitution to remove the residence requirement from Article III, Section 2(c), entitled "Waiver of Examinations." This is a technical amendment, the reasons for which will be furnished to Society members at the time the constitutional amendment is submitted for action.
3. Approved a recommendation that the Society become an affiliate member of the Conference Board of the Mathematical Sciences.
4. Selected the following sites for future meetings:
 - a) *1970 Regional Meetings*
 - April 2 and 3Hotel America, Houston, Texas
 - April 16 and 17Hartford Hilton Hotel, Hartford, Connecticut
 - June 4 and 5Drake Hotel, Chicago, Illinois
 - b) *1970 Annual Meeting*

It had been announced previously that this meeting would be held at the Denver Hilton Hotel, Denver, Colorado. The dates will be November 9, 10, and 11.
5. Received a report from Mr. John H. Miller, Chairman of the Committee To Study Pension Plan Problems. (It was announced that Mr. Miller would present a summary of this report later that morning.)
6. Approved a recommendation from the Committee for a New Society Publication that steps be taken to introduce the new publication as quickly as possible. The Board was very pleased to be able to take this action, since there has been a large demand for such a publication. It will be an informal publication featuring news of special interest to actuaries as well as articles of current interest. The editorial board will be appointed by the President. With an initial publication schedule of ten issues per year, the annual cost will approximate \$10,000. The Board of Governors decided that, initially at least, the publication would be furnished to members as a service without specific charge. It is realized, of course, that the total cost of all services rendered to members must be taken into consideration in any future reviews of our dues structure.
7. Received an interim report from the Committee on the Future Course of the Society on the subject of expressions of opinion on behalf of the Society of Actuaries and approved a recommendation of the Committee that action be taken to initiate changes in the Constitution to reflect current needs of the Society. (It was announced that Mr. Walter Klem, Chairman of the Committee, would summarize his Committee's report later that morning and would describe the pro-

posed Constitutional amendments. It was also announced that there would be an opportunity for discussion of this subject on the following Wednesday morning.)

8. Received a report from the Committee To Report on Procedures in Connection with Disciplinary Action and approved a recommendation of the Committee that action be taken to institute some procedural changes in the Constitution. (It was announced that Mr. Andrew C. Webster, Chairman of the Committee, would summarize his Committee's report later that morning and would describe the proposed constitutional amendments.)

The Treasurer reported that on September 30, 1966, the membership of the Society stood at 1,544 Fellows and 1,302 Associates, making a total membership of 2,846. The Treasurer gave in summary form the financial report printed elsewhere in this number of the *Transactions*.

The President extended a welcome to the new Fellows and Associates who were attending their first meeting of the Society after attaining their present rank.

Mr. John H. Miller, reporting for the Committee To Study Pension Plan Problems, mentioned a proposed Manual of Actuarial Principles and Practices in Relation to Private Pension Plans, and the consideration which the Committee has been giving to proposals to modify the rules for integration of private pension benefits with Social Security benefits. Mr. Andrew C. Webster outlined the reasons for the proposed amendment to Article VII of the Constitution; there is a report on file with the Board giving complete details. Mr. Walter Klem presented a report of the Committee on the Future Course of the Society.

The President announced that, because of the delay in the distribution of papers for this meeting, discussions could be submitted up to December 1, 1966. The President then thanked Mr. Alton O. Groth, the retiring Editor, for the valuable services he had rendered the Society.

The meeting adjourned at 12:00 noon.

The meeting reassembled that afternoon at 2:00 P.M.

A panel discussion "The Future of the Actuarial Profession as It Appears to the Younger Actuaries" then took place. Mr. Andrew C. Webster was moderator, and panel members were Mr. Allan D. Affleck, Mr. Russell M. Collins, Jr., Mr. Peter R. Wilde, and Mr. Richard A. Winkenwerder. After the panel members presented their views, there followed discussion from the floor.

The meeting adjourned at 4:15 P.M.

On Tuesday, November 1, 1966, the meeting was called to order by President-Elect Harold R. Lawson at 9:35 A.M. Mr. Lawson then moder-

ated the panel on "Property and Casualty Insurance." Panel members were Mr. Harold E. Curry, senior vice-president, State Farm Mutual; Mr. Richard Humphrys; Mr. Harold Schloss, vice-president and actuary, Royal-Globe; and Mr. Seymour E. Smith, senior vice-president, The Travelers.

The meeting was then turned over to President Fitzhugh, who announced that Mr. John H. Miller had been named President-Elect of the American Academy of Actuaries.

Then followed the presentation and discussion of papers. The meeting adjourned at 12:40 P.M.

The meeting reassembled that afternoon at 2:00 P.M. in six simultaneous sessions.

Simultaneous Session I, "Group Life and Health Insurance," was moderated by Mr. Robert N. Stabler. Panel members were Mr. Donald D. Cody, Mr. William A. Halvorson, Mr. William W. Keffer, and Mr. Richard J. Mellman.

Simultaneous Session II, "Electronic Data Processing," was moderated by Mr. J. Stanley Hill. Panel members were Mr. Edwin F. Boynton, Mr. John M. Bragg, Mr. John J. Finelli, Mr. Robert B. Koch, Mr. Chandler L. McKelvey, and Mr. Julius Vogel.

Simultaneous Session III, "Annual Statements of Life Insurance Companies in the United States," was moderated by Mr. George H. Davis. Panel members were Mr. W. Harold Bittel, Mr. William A. Dreher, Mr. H. Edward Harland, Mr. Gathings Stewart, and Mr. Frederick S. Townsend.

Simultaneous Session IV, "Actuarial Principles and Practices in Relation to Private Pension Plans," was moderated by Mr. John K. Dyer, Jr. Panel members were Mr. Frank L. Griffin, Jr., Mr. John H. Miller, and Mr. Ray M. Peterson.

Simultaneous Session V, "Individual Life and Health Underwriting," was moderated by Mr. Charles A. Ormsby. Panel members were Mr. Edward A. Dougherty, Mr. Niels H. Fischer, Mr. Edward A. Lew, and Mr. Edward H. Sweetser.

Simultaneous Session VI, "Equity Life Insurance and Mutual Funds," was moderated by Mr. Kenneth R. MacGregor. Panel members were Mr. DeLos H. Christian; Mr. Adrian de Hullu; Mr. Carl A. Haase; Mr. Graham H. Holland, F.I.A., Actuary for Canada, Norwich Union Life Insurance Society; and Mr. Alfred N. Guertin.

On Wednesday, November 2, 1966, the meeting was called to order by Mr. Pearce Shepherd at 9:30 A.M. for a panel presentation on "Private Pensions in the United States and Canada." Mr. Shepherd was modera-

tor and panel members were Mr. Benjamin T. Holmes; Mr. Walter Klem; Mr. Dan McGill, Chairman, Insurance Department, University of Pennsylvania; and Mr. Wendell A. Milliman. After the panel presentation, there was discussion from the floor.

After a short recess, President Fitzhugh called the meeting to order. There followed a discussion on the Interim Report of the Committee on the Future Course of the Society on the subject of expressing opinions. Following the discussion, Mr. Morton D. Miller presided over the meeting. Topics of current interest were to be discussed. Mr. John K. Dyer, Jr., discussed the need for a treatise on private pension plans. Discussions had been prepared by Mr. John Mahder on Medicare, Mr. Lloyd G. Rollerson on health care in Canada, Mr. Richard J. Mellman on Title XIX, and Mr. George H. Davis on policy loan interest rates, but were not given due to the lateness of the hour.

Mr. Miller then turned over the podium to Mr. Andrew C. Webster. Mr. Webster paid a tribute to Mr. Fitzhugh and presented him with a silver tray on behalf of the Society. President Fitzhugh acknowledged the presentation and remarked on the great sense of pride he felt at having served as President.

President Fitzhugh then turned the gavel over to President-Elect Lawson, who closed the meeting at 12:25 P.M.

A Forum for Consulting Actuaries was called to order at 2:00 P.M. on Wednesday, November 2, 1966. The forum was in the form of a panel discussion moderated by Mr. Thomas P. Bowles, Jr. Panel members were Mr. Preston C. Bassett, Mr. Charles G. Bentzin, Mr. Alfred N. Guertin, Mr. Davis H. Roenisch, Mr. Kenneth H. Ross, and Mr. Conrad M. Siegel. After discussion and questions from the floor, the meeting adjourned at approximately 4:30 P.M.

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